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Characteristics of Undergraduate Students' Mathematical Proof Construction on Proving Limit Theorem

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Abstract

Ability to construct proof is compulsory for whoever involved in learning mathematics and mathematics education. However, many studies showed the result that most students still found it difficult to construct proof, especially, when it is related to constructing proof for function concept. Therefore, to find out why it is difficult for the students, it needs to charraterize the proof construction done by the students should be characterized. This studys aims at describing characteristic of proof construction by students using assimilation and accommodation framework by Piaget. This researchapplied qualitative method. This study had found out three charracteristics of mathematical proof construction when students attemptedto solve mathematical proof problem, that are (1) pseudo complete proof construction, (2) illogical proof construction, (3) likely logical proof construction.

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1. Introduction

The ability to construct proof is necessarily important for mathematics learning in mathematics education ([3, 4, 6, 10, 12, 17, 26]). However, most studies found out that proof and constructing proof are difficult concept for students ([11, 16, 25]). What is needed is close observations on student "in the act" of proving to find out the kinds of dificulties they have. For this, one can turn to research on how students learn to construct and work with proofs [16]. Knowing students' thinking process when they attempt to construct proof help us to recognize more about the problem faced by students and provide ways to help them.

Previous studies had been comitted related to tracing the process of constructing mathematical proof ([9, 12–14, 24]). [24] found out the students meet difficulties in constructing proof because they do not have three types of strategic knowledge, that are (1) knowledge of the domain's proof techniques, (2) knowledge of which theorems are important and when they will be useful, and (3) knowledge of when and how to use 'syntactic' strategies. [16] described the students' failure in constructing proof coused by two conditions, that are incomplete schema on accomodation process and complete but unrelated schemas on assimilation and accommodation process. [17] proposed three factors that made students' dificulties when they attempted to contruct proof, that are (1) student did not construct proof according to proof framework, (2) students were not able unpack conclusion, and (3) students could not use definition properly.

In this paper, I explore the issue of proof construction within the context of function, a central concept of limit function. While there has been some educational research within the domain function.The function concept has become one of the fundamental ideas of modem mathematics, permeating virtually all the areas of the subject [5]. [22] studied the construction process in solving the problem of composition function. [5] used a proof issue on the limit theorem to explain the workings of the proof framework. So far, there is no study of the proof construction process of functional based on knowledge schemes and assimilation and accommodation frameworks.The purpose of this paper is to address the following question, what are characteristics of mathematical proof construction done by the students based on assimilation and accommodation framework from Piaget.

Schema is a terminology of psychological field for describing mental structure. This term is not only used for complex mathematics structure but also for simple mathematics structure. A schema has two main functions: it integrates existing knowledge and it is a mental tool for the acquisition of new knowledge [20]. Everybody learns something depending on existing knowledge.Some one will learn a new thing more easily if he/she has some schemes needed for acquiring the new things. The newly learned knowledge is integrated into a new schema in the person's knowledge structure. It is suported by Neisser (in [1]) that construction of a schema is fruit of assimilation and accomodation

Assimilation involves the interpretation of events in terms of existing cognitive structure and accommodation increases knowledge by modifying structure to account for new experience [7]. Process of assimilation and accommodation is closely related to student's mental structure. Tracing the process of proof construction by student is held by observing and studying students schemas. Relation between process of assimilation and accommodation with schema forms a cyclus. There are two conditions when the

student is faced with a problem of proof, namely (1) the student has the same complete schema as the problem structure so that the student can assimilate the problem, and (2) the student does not have a complete schema on the problem so that the student does the accommodation for altering or adding a scheme that has been owned and can be continued with assimilation. Illustration for assimilation and accommodation would be as follows
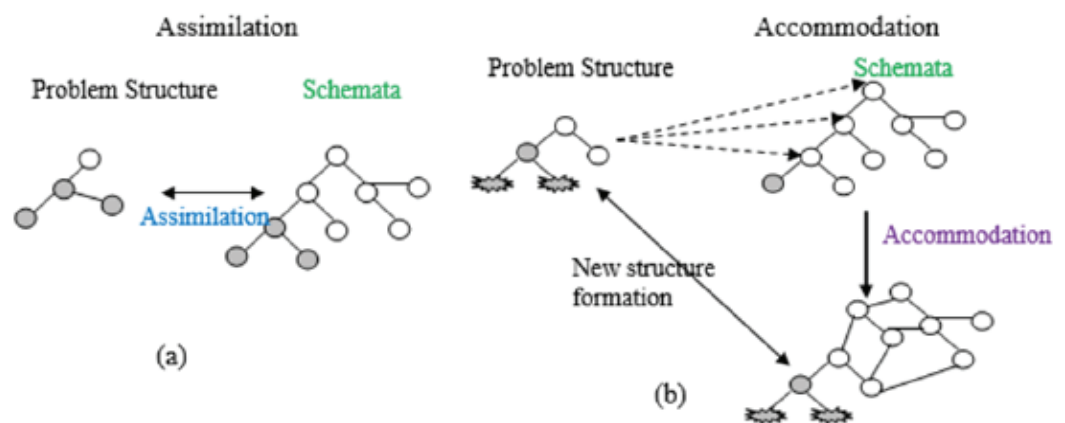

Figure 1: Assimilation and accommodation process (adopted [18]).

Figure 2(a) would explain that if the pattern of structure present in student's schemas, she/he will able to interprete the proof problem directly through assimilation process. Figure 2(a) also shows the pattern of schemas at problem structures in knowledge schemas that bring about the process of assimilation directly. Figure 2(b) explain, schema of pattern of problem structure that was not found in knowledge schemas, therefore it was necessary to do accommodation process to form a schema that suits problem structure. After the formation of similar schema structure, then students could progress to assimilaton process.

2. Method

This study uses qualitative method. The subjects were the students of Mathematics Departement at State University of Malang. For data collection 10 students who had passed Calculus subject were given proof problem task.They had to think aloud during working on the problem [2]. After they got through with proof problem, these students were going on interview for the need to discover their thinking process. All activities during data collected were recorded audiovisually, using video camera.

Proof problem task is as follows;

Solve the proof problem below. Given set A, where $A \subseteq \mathbb{R}$, $a \in A$ and two function f and g defined in A If f and g are continous functions at a, proof that $f + g$ is continous at a.

Ideally, the proof problem above could have been solved if the students possesed all schemas related to proof problem. These schemas were all about proof concept, set theory, function concepts, continous function definition and its laws of operation, limit function concepts and its the law of addition. If students posses schemas, it can be assumed that they only went through thinking proccss of assimilation. They were able to read and comprehend the proof problem easily. Students could make a direct connection with function schemas of f dan g, continous function etc. Students could find the main idea directly that he/she had to formulate that $f + g$ is continous at a. Students also were able to show that $\lim_{x \to a}(f + g)(x) = (f + g)(a)$ using property of function addition, law of addition of limit function and its value. Finally, students convinced that they had done things properly in reflection phase.

Data analysis was done by comparing the students' thinking structure in construct to proof with the ideal construction stage described above. The comparison can be show the characteristics of students' mathematical proof construction.

3. Findings and Discussions

Based on 10 results of proof construction done by the students, none of them was purely valid. The result of students proof construction can be classified into three categories: they are (1) pseudo complete proof construction, (2) illogical proof construction, (3) likely logical proof construction. Each type can be explained as follows;

3.1. Pseudo complete proof construction

Pseudo completeProof construction terjadi karena kondisi skema mahasiswa yang almost complete schema and awned by students was nearly valid. One out of three students' work who resulted in the almost/nearly valid construction was taken as sample data to be described. A student was female called Nani. Her thinking structure can be described as complete because it is nearly the same as problem structure. Nani was successful to pruduce proof construction that almost similar to that of scientific concept.

The result of proof construction by Nani's schemas was almost complete. In fact, during the process of interview Nani was not confident with her construction. It wa because Nani didn't us assimilation during the phase of determining main idea. She was

$$\lim_{x \to a} (f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

Karena f dan g Kontinu di titik a, yaitu $\lim_{x \to a} f(x) = f(a)$ dan $\lim_{x \to a} g(x) = g(a)$, maka

$$= f(a) + g(a) \quad //$$

Figure 2: Nani's Proof Construction Result.

inconfident with her property of function addition. Nani should have written $\lim_{x\to a}(f + g)(x) = (f + g)(a)$ but she was doubtful, so that she finally wrote as follows:

Akan ditunjukkan bahwa $f+g$ Kontinu di titik a, yaitu

$$\lim_{x \to a} (f(x) + g(x)) = f(a) + g(a)$$

However, in the work sheet she had tried to make sure herself about the function addition by writing the formula of function as below:

$$f(x) = x^2$$
$$g(x) = x$$
$$f(x) + g(x) = x^2 + x$$

She had function schemas, such as schemas of definition for function addition, schemas of rule of addition at limit and direct proving concept,but these schemas were still week.Week schemameans the concept of function has not been well understood so that she can not do improvization with it.

3.2. Illogical proof construction

This process of proof construction with incomplete schemas happened/occured on almost allstrudent. A femele student, called Yulia had been chosen as subject to be described about her thinking process. When she was reading the statement $f, g : A \to A$ continuous in $a \in A$, she became confused because she hardly ever knew the function symbol in general form. Yulia didn't have complete schemas about of $f, g : A \to A$. There were subschemas about function f and g that Yulia had, such as f and g represented

infinite function, defined at A. This is evidenced by the following interview excerpts (R: Researchers. Y: Yulia)

> R: While reading $f, g: A \rightarrow A$, you are silent and continue
>
> Repeating the statement reading up to three times, why was it so, what were you thinking?
>
> Y: I do not understand the function f and g because there is no function formula.
>
> R: ooo so, can you explain what you mean?
>
> Y: There is always a formula of itsfunction in all problems that I have solved, it's clear to me. If there is no function formula it's not clear to me, like something dark.
>
> R: Then how did you try to understand it?
>
> Y: I tried, but I could not, I decided to continue.
>
> R: ooo so, then you read the fuctions$A$ to A($f, g: A \rightarrow A$) up to three times, why did you do it?, what did you think?
>
> Y: Yes, I was also confused with that function, what is the meaning and what to do with other concepts.

Yulia continued the process of proving in spite of being in the condition incomplete shemas. She accommodated to interprete definition of continous function and continous definition of function addition f dan g at a because of her incomplete shemas of function f and g. She could not interpret well the properties and concept to do with the function f and g.

The following figure shows the unsistematical Yulia's proof construction.

Figure 3: Yulia's proof construction result.

When Yulia was successful to write the equation (inbox), she should have worried with unrecognized symbol. The next process of proof construction would be another fact. She

solved proof problem, showing $\lim_{x \to a} h = h(a)$ is true. In this context, students were in unrecognitized condition because of her incomplete schemas of function concept in general form.

[17] told that case by Yulia is coused by unability to use definition/theorems proprerly. In the same context [26] told that students didn't know when and how a theorem is used in a "proof". In this case it can be understood that the root of the problem was that being in incomplete schema, it makesstudents unable to construct valid proof.

[20] explained that someone can interprete a problem if he/she has complete and sifficient schema.In this same case as Yulia, incomplete schema happened becauseof forgetfulness.. There are two theories that discuss about "forget": Decay Theory and The Theory Of Interference (see [21])

3.3. Likely logical proof construction

This process is described based on the result of proof construction by student, named Rina. Rina initiated her proof contruction with assimilation when she read the statement f dan g continuous at a. In assimilation process,Rina interpreted continuous statement as "not discontinuous". In Rina's schema discontinuous function is understood as undefined of f dan g function value. As the result, Rina interpreted the function of f dan g continuous at a as f and g should have function value at a. Based on comparison between Rina's interpretation and scientific concepts it is found that the Rina's scheme of the concept of continuous function is incorrect or false. In scientifict concept,a function continuous at a should fulfill three conditions, (1) the function defined at a, (2) having limit value at a, and (3) limit value at a and limit function at a is similar

Rina has a strong schema about that continuous function at a point although it doesn't match with a scientific concept. It made Rina to ignore two other conditions of continious function. Although Rina had other schemas that match scientific concept as generalizing form of continuous function f dan g and concept of direct proof. However, the non-matching of Rina's scheme causes a wrong in applying the right schemas. Finally, the result of her proof construction was likely logical proof construction. The following figure shows the logical Rina's proof construction.

4. Conclusion

According to the results of analysis and exposure of data on the characteristics of the construction owned by the students, it can be summarized as follows. *First*, Pseudo
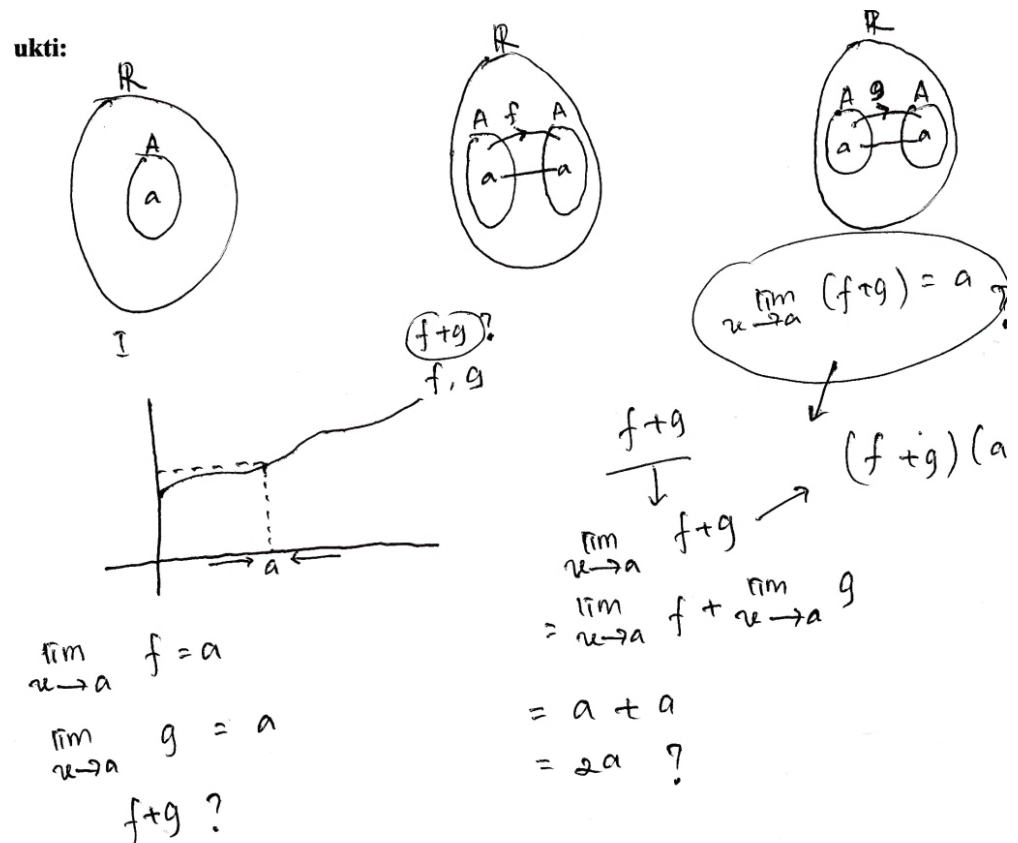
Figure 4: Rina's proof construction result.

complete proof construction, with the result of the construction of the proof close to valid. The schema of function *f* and *g* written in the general form *f: A → A* and *g: A → A* is not solid and consequently the student is unsure about the resulting construction. *Second*, Illogical proof construction is marked with the result of invalid proof construction.Students's schema about function *f* and *g* in general form *f: A → A* and *g: A → A* is incomplete, student only knows the symbol but did not understand the meaning behind the symbol of function *f* and *g*. Consequently, the process of constructing proof in unrecognized condition and the student is students can only write a series of proofs like the examples he has seen, in other words just copy it without understanding. *Third*, likely logical proof construction is marked by the invalid proof construction results, there is a scheme about the concept of continuous function in one point is not in accordance with the scientific concept, consequently wrong in applying to other schemes and the student is confident with the result of construction of the proofs produced. The series of proof construction seems logical but not valid because it starts with the wrong concept.

Based on the above conclusions it should be noted that the quality of students' schemes greatly affects the way students interpret a problem and, ultimately impacting the quality of proof construction. Ensure that in every lecture the students construct

the correct and solid schema. Students having a schema about certain mathematical concepts is not enough if the schema is not been strong. A strong schema will not be easily damaged and can be used more precisely in solving problems. In the future, a special learning model on the problem of prooving that takes into account the students' schema and proof construction quality can be designed

Acknowledgement

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