Research Article

Analysis of Student Errors in Solving Mathematical Induction Problems in Online Learning

Eliva Sukma Cipta[1,2*], Rippi Maya[3], Didi Suryadi[1], Sufyani Prabawanto[1]

[1]Department of Mathematics Education, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudhi No. 229, Bandung 40154, Indonesia
[2] Department of Basic Education Madrasah Ibtidaiyah, Universitas Islam Nusantara, Jl. Soekarno Hatta No. 503 Bandung 40286, Indonesia
[3]Department of Mathematics Education, Institut Keguruan dan Ilmu Pendidikan Siliwangi, Jl. Terusan Jend. Sudirman, Baros, Cimahi Bandung 40521, Indonesia

ORCID
Eliva Sukma Cipta: https://orcid.org/0009-0002-2865-7859
Rippi Maya: https://orcid.org/0000-0003-3518-7883
Didi Suryadi: https://orcid.org/0000-0003-0871-8693
Sufyani Prabawanto: https://orcid.org/0000-0003-2872-6535

Corresponding Author: Eliva Sukma Cipta; email: elivasukmacipta@upi.edu

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Abstract.
The research aims to analyze student errors in solving math induction-proof problems during online learning in the Covid 19 era. This was a qualitative research with a case study approach. This research was conducted at the Mathematics Education Study Program of UIN Sunan Gunung Djati Bandung in the academic year 2020/2021, and the subjects of this study being students of semester V Class 2018. The instrument in this study was a test, and data analysis were done using qualitiative anlysis. The results showed that the types of errors experienced by students in working on mathematical induction problems are conceptual errors and procedural errors.

Keywords: online learning, solving mathematical induction problems, student errors.

1. INTRODUCTION

The emergence of Corona Virus Disease (Covid-19) has greatly affected all levels of society so that the government has implemented an appeal for social and physical distancing. This condition requires all people to remain at home, whether they are working, worshiping or studying. For the world of education, the Covid-19 outbreak presents its own challenges, all learning and learning processes are carried out at home in accordance with the Government Form Letter No. 15 of 2020 concerning Guidelines for Implementing Learning from Home in an Emergency Era for the Spread of Corona Virus Disease (Covid-19). Teaching and learning activities that were previously carried out face-to-face are now carried out online. This activities needs to be implemented to minimize the spread of the virus. This pandemic situation requires educators and students to study via the internet network or study online. Online learning not only makes it easy for students digitally, but also allows students to actively follow changes and improvement in conditions [1].

Online learning using information technology is expected to be able to overcome constraints in the teaching and learning process so that it continues to run well even with the exposure to the Covid-19 pandemic. Information technology is making online learning easier as a new mode of learning at this time. Namely, students can access a growing online learning environment without the constraints of time and place by using the internet and various online learning platforms. In addition, this learning mode provides more flexibility in the learning environment because there is no limitation of time and space [2].

The use of information technology as a tool in online learning still has the same goals as face-to-face learning, namely achieving good learning outcomes. Online learning can increase efficiency in the interaction between teachers and students and have a significant impact on learning outcomes [3]. On the other hand, online learning has difficulties in terms of assessing learning evaluations through closed book exams. This happens because teachers cannot directly supervise students during exams and there is no adequate technology for this [4]. One of the difficulties in assessing student learning outcomes is in mathematics, because mathematics is one of the subjects that plays a role in helping develop logic [5] which is difficult to measure by doing written tests in online learning.

NCTM divides basic math skills into five basic skills, namely problem solving, reasoning and proving, communication, connection, and representation [6]. In terms of reasoning and proof abilities, there are several proof techniques that can be used by students to prove a mathematical statement, one of which is mathematical induction.

Mathematical induction is a special method of proof based on inductive reasoning that is used to prove the truth of a statement for each natural number through two stages, namely the basic stage and the inductive stage. In the basic step the statement is validated for the smallest value $n = n_0$ at the initial value P(n), which is an open sentence. Furthermore, the inductive step proves the implication of P(n) => P(n + 1) for any n random [7]. In the process, mathematical induction can improve understanding and logical thinking skills [8].

The process of proof in mathematical induction is clear and easy to do because it has regular stages of proof known as the Principles of Mathematical Induction. This makes mathematical induction different from other proving techniques that do not have a clear stage of proof. However, clarity in proof does not make it easy for students to prove using mathematical induction. Students still have trouble proving it.

In online learning, learning resources for students are available at home, be it in the form of textbooks, notes, library sources from the internet, and so on, making it difficult for teachers to conduct closed book exams. One of the strategies that can be used to evaluate student's proof ability with an open book exam is to package the proof of question in such a way that the problem is different from the questions given during face-to-face learning. The problem is modified in such a way that it demands other mathematical skills besides the proof ability expressed by NCTM. In this case, the students' conceptual understanding ability is tested before their proving ability (reasoning) is used to prove it. Students' answers are then analyzed for errors to see the types of student errors so that next steps can be determined to anticipate the same mistakes recurring and to anticipate future learning [9]. Based on the description above, the purpose of this study is to describe the students' mistakes in solving the modified proof questions.

2. RESEARCH METHOD

This type of research is a qualitative with a case study approach. This research was conducted at the Mathematics Education Study Program of UIN Sunan Gunung Djati Bandung in the academic year 2020/2021 with the subjects of this study were students of the fifth semester of the 2018 class using purposive sampling technique, namely 4 students. The instrument used was a mathematical induction proof test with two questions. After the data is obtained, then it is analyzed qualitatively. Qualitative data analysis is used to describe student errors in solving mathematical induction proof problems.

3. RESULTS AND DISCUSSION

Mathematical induction problem given to students presented in Figure 1:

The written test results were analyzed by means of qualitative data analysis by looking at the mistakes made by students in answering the questions given. Each error will be discussed in detail with respect to the stages of mathematical induction [7]. This aims to get an idea of the subject's ability in the thought process to solve the problem. The student answer showed in Figure 2:

PROBLEM

Is the statement: $P(n)$: $9^n - 8n + 1$ divisible by 64 for $n \in \mathbb{N}$ a true statement? If so, prove it by mathematical induction. If it is wrong, investigate the error, then correct it and prove the truth with mathematical induction.

Figure 1: Mathematical induction problem.

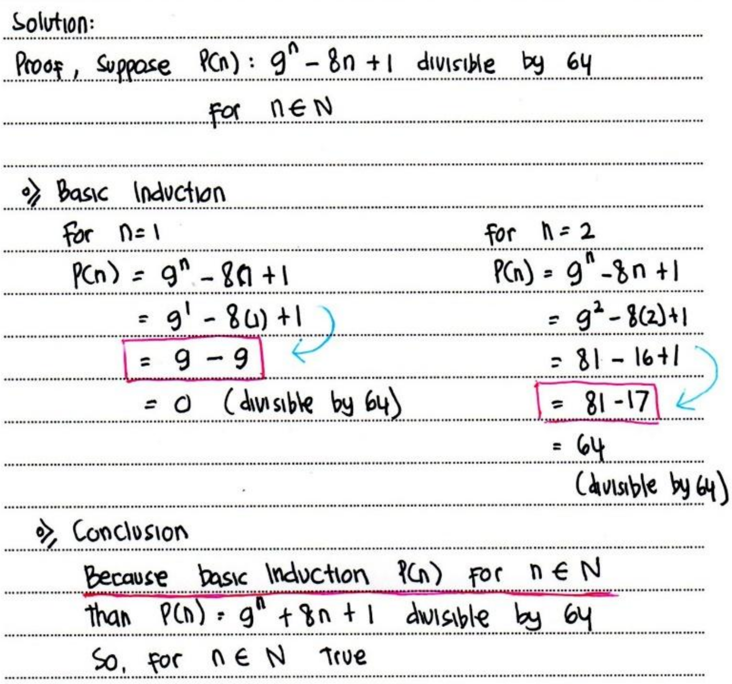
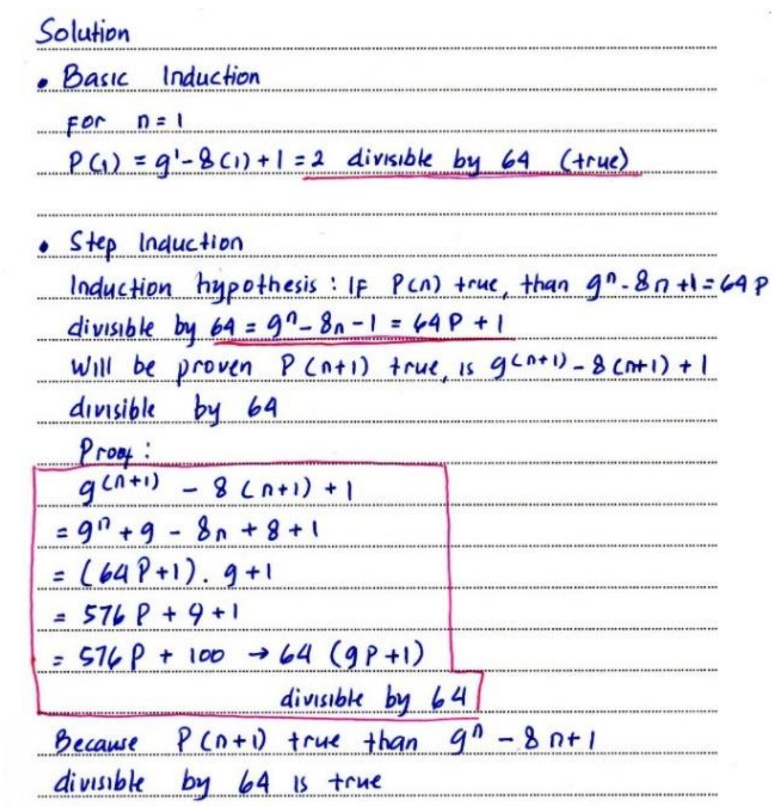


Figure 2: Answers from students 1.

Student 1's answer shows that there was a concept and procedure error, a conceptual error occurred when completing operations 9 - 8 + 1 = 9 - 9. The concept of the basic rule of doing mathematical calculations is that if in a mathematical sentence there are addition and subtraction arithmetic operations, then what is done first is arithmetic operations that are in front. Student 1 does not do this but does the arithmetic operation which is behind first. This means that student 1 made a concept mistake on the basic rules of doing math calculations. For procedural errors that occur from the basic induction stage which is considered correct by student 1 directly to the conclusion stage, even though according to the mathematical induction stage after the induction base is proven correct, the next step is to prove the induction step. Student 1 immediately generalizes the statement so that it ignores formal logic [10]. In conclusions, students are also inaccurate in making their statements, students should write because P (1) is true and P (n + 1) is true, so P (n) is true.


Solution
• Basic Induction
for n = 1
P(1) = 9¹ - 8(1) + 1 = 2 divisible by 64 (true)

• Step Induction
Induction hypothesis : If P(n) true, than 9ⁿ - 8n + 1 = 64 P
divisible by 64 = 9ⁿ - 8n - 1 = 64 P + 1
Will be proven P(n+1) true, is 9⁽ⁿ⁺¹⁾ - 8(n+1) + 1
divisible by 64
Proof :
9⁽ⁿ⁺¹⁾ - 8(n+1) + 1
= 9ⁿ + 9 - 8n + 8 + 1
= (64 P + 1) . 9 + 1
= 576 P + 9 + 1
= 576 P + 100 → 64 (9 P + 1)
divisible by 64
Because P(n+1) true than 9ⁿ - 8n + 1
divisible by 64 is true

Figure 3: Answers from students 2.

Student 2 made a mistake in the concept of the original number count operation, which confirmed that 2 was divisible by 64. Then the concept error also occurred in the next step, namely student 2 wrote that $9^n - 8n + 1 = 64p$ was divisible by $64 = 9^n - 8n - 1 = 64 + 1$. This shows a misconception of algebra, where student 2 does not understand what he is writing. Furthermore, in the part that is given a red box, there is a concept error of $9^{(n+1)} = 9^n + 9$ and the calculation operation in the next step. In the final stage, student 2 also made a wrong conclusion to conclude that the P (n) statement was true. Student 2 should first write that P (1) is true and P (n + 1) is true. Student 2 only writes “P (n + 1) is true, then P (n) is true” which means there is an error in the procedure and concept in concluding. to prove propositions that state P(n) is true for all positive integers n is discussed and answered by stating the principle of Mathematical Induction as a proof technique consisting of two steps: (1) Base step, show P(1), (2) Inductive step, show P(n)⇒ P(n +1) for every positive integer n [11].

Student 3 made a counting operation error, namely confirming that 2 was divisible by 64 so student 3 ignore the set of speech in mathematical induction is an integer [11], so that assuming that the basis for the induction was correct, the induction step was carried out. However, student 3 made a mistake in writing the statement he was going to prove. The next step students 3 are confused because they lack understanding of the

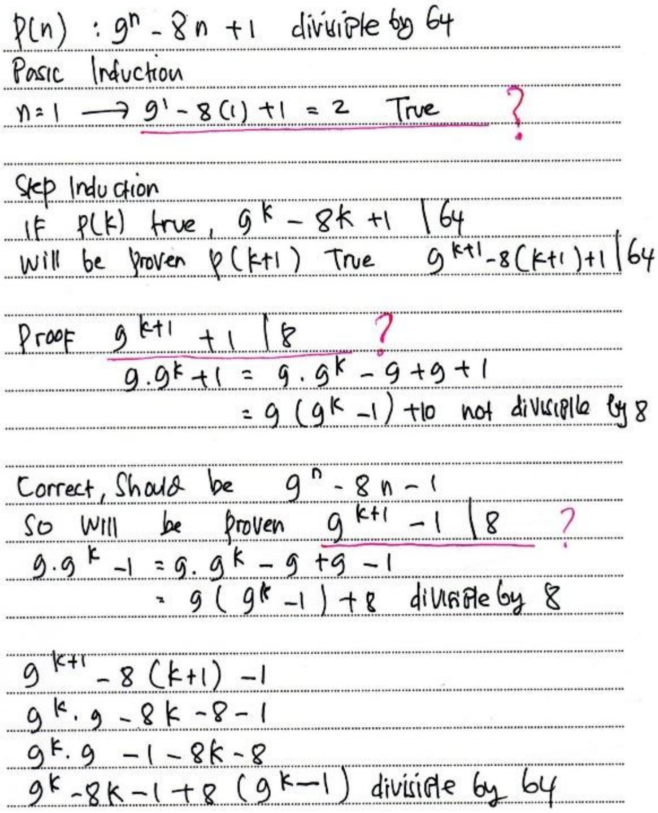
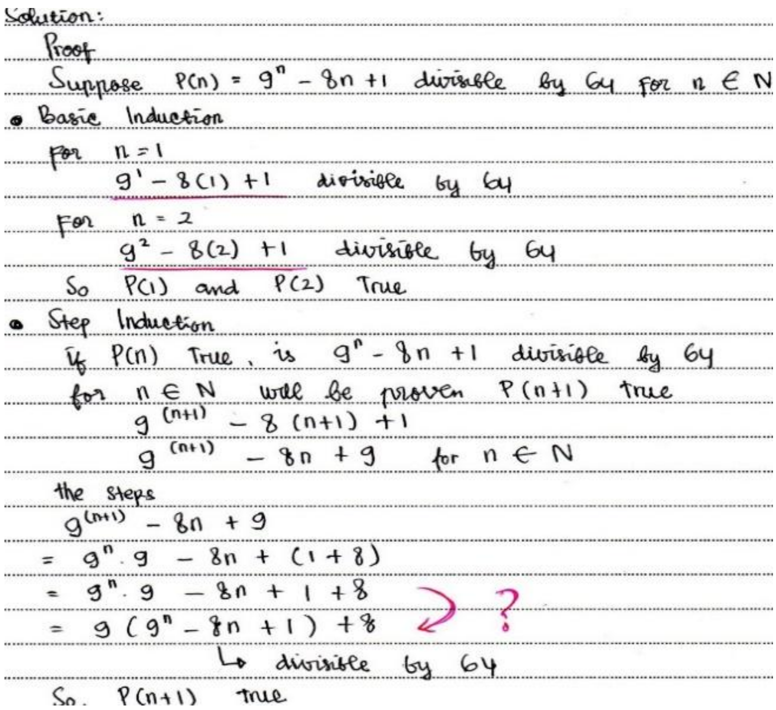


Figure 4: Answers from students 3.

concept of arithmetic operations and procedures in the mathematical induction proof stage. Student 3 was unsure of the answer he made so student 3 made a new statement, but just like the previous step, student 3 wrote down the error of the statement that had to be proven in the mathematics induction stage. Student 3 did trial and error in proof of mathematical induction because of the lack of mastery of prerequisite concepts in proofing.

Student 4 made the same conceptual error as student 1, namely the concept of the basic rules for working on mathematical calculations in the induction basis step. In the induction step, student 4 also made a mistake in the concept of the similarity operation. The answers made by student 4 are also incomplete, namely there is no final conclusion, meaning that student 4 does not understand the problem, student 4 only focuses on how to prove mathematical induction questions. Student 4's answers can be seen in Figure 5.



Solution:

Proof

Suppose $P(n) = 9^n - 8n + 1$ divisible by 64 for $n \in N$

- Basic Induction

For $n = 1$

$9^1 - 8(1) + 1$ divisible by 64

For $n = 2$

$9^2 - 8(2) + 1$ divisible by 64

So $P(1)$ and $P(2)$ True

- Step Induction

If $P(n)$ True, is $9^n - 8n + 1$ divisible by 64

for $n \in N$ will be proven $P(n+1)$ true

$9^{(n+1)} - 8(n+1) + 1$

$9^{(n+1)} - 8n + 9$ for $n \in N$

the steps

$9^{(n+1)} - 8n + 9$

$= 9^n . 9 - 8n + (1 + 8)$

$= 9^n . 9 - 8n + 1 + 8$

$= 9(9^n - 8n + 1) + 8$?

↳ divisible by 64

So. $P(n+1)$ true

Figure 5: Answers from students 4.

4. CONCLUSION

From the results of the identification of errors in working on mathematical induction problems, there were several mistakes made by students in proving, namely conceptual errors and procedural errors. There are three conceptual errors made by students, namely conceptual errors in the basic rules of mathematical calculations, errors in the concept of division operations, and errors in algebraic concepts. In the conceptual error of the basic rules for working on mathematical calculations, students incorrectly perform the sequence of operations which must take precedence. In the concept of operation for division errors, students make the wrong conclusions because the concept of the set of speech in mathematical induction is natural numbers or sometimes integers. In an algebraic operation error, the student incorrectly performs the similarity operation or other algebraic operations. In procedural errors, students have difficulty in connecting the concept with the processing steps/procedures. This error occurred maybe because the student started the proof with a conceptual error.

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