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# How Do Students' Method of Solving Proofs Change When Working in Groups?

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**Abstract.**
This study aimed to describe the shift in students' mathematical arguments from working on a proof individually to working on it in a group. This descriptive-exploratory study used a qualitative approach. Nine undergraduate students from a private university in Jombang, Indonesia, in the $5^{th}$ semester were the subjects of the study. They were selected because they had already learned the concepts of math logic, argumentation, the theory of numbers, and analysis. Hence, they were ready to construct arguments in dialogue and non-dialogue forms. A task of argumentation and an interview were used to collect data. The study had several stages. First, the students solved the given argumentation task individually. Second, they had to make small groups of 3 members and discuss the same task. The result found that four of them had a complementary shift in their arguments, while the other five students had a reconstructive shift in their views. The complementary change happened because they reconsidered their initial thoughts, complementing their previous thinking structure. On the other hand, the reconstructive shift occurred due to group discussion (i.e., dialogue), which made them reconstruct or even entirely change their previous thoughts. Furthermore, they initially used inductive reasoning and then shifted their reasoning to a deductive one.

**Keywords:** Mathematical Argumentation, Proving, Reconstruct, Complementary shift, Reconstructive shift

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## 1. Introduction

Proof plays an important role in mathematics ormathematics education[1]–[5]. Once someone is willing to learn math, he has to learn how to prove or, at least, understand the course [6]. Argument and argumentation have tight relation. These two terms reflect two definitions in which argument refers to product, while argumentation refers to process [7]. Furthermore, an arguer needs to have supports for his argumentation [7]–[14] and the arguer needs this argument to define, generate, and support a reasonable solution. Through argumentation, he may explain his reason behind selecting and using particular definition and postulate theorem, supporting or refusing an opinion, thought, or ideas. Once he has argumentation skill, he may justify this solution and measure. Furthermore, he may get rid of his hesitation in having argumentation. Additionally, he may feel freer to choose and even propose a reasonable solution.

To construct their arguments, arguers do not only correlate premises to solution. It needs Toulmin scheme to analyze more complex arguments. As presented in Figure 1, Toulmin scheme consists of data (D), claim (C), warrant (W), backing (B), rebuttal (R), and qualifier (Q) [15]. Data is actual facts the support claims. Claim is a statement which truth should be verified. Warrant is a hypothetical statement that bridges and justifies the procedures. Backing presents further arguments including legal basis through which the warrant relies on it. Rebuttal is a condition of exception for arguments. Qualifier reveals the level of strength which data has for claim by warrant.

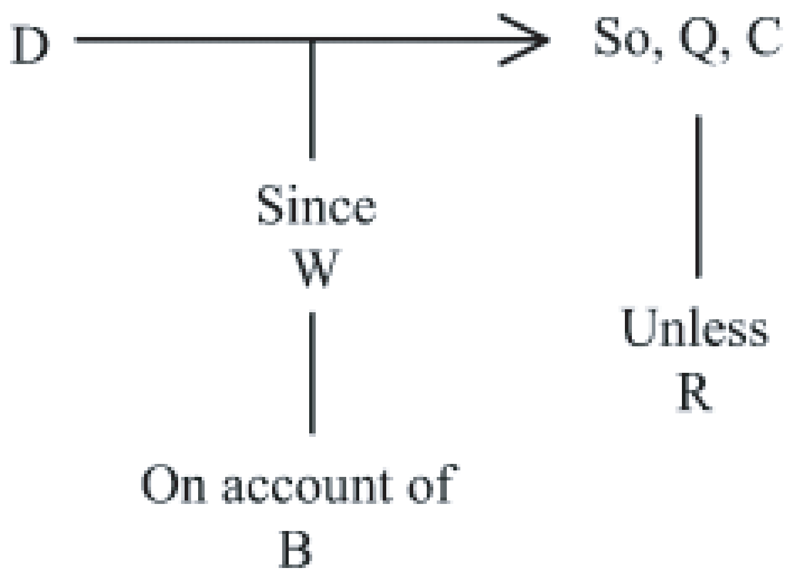


**Figure** 1: Toulmin Scheme.

Arguments may appear in both dialogue and non-dialogue context [16]. One example of arguments in non-dialogue context is problem planning and solving. This activity evokes self-interaction through which the same person alternately plays two roles; initiator and respondent. Someone may has self-approach and self-debate. This not-in-dialogue argument is important since the individual tries to show and ensure a correct view through the argument he presents to himself. He tries to convince himself, and thus, he has self-approach and self-debate. Whereas, the example of in-dialogue argument appears in a group discussion through which each of the members presents their arguments to others to justify their view. These in-dialogue arguments aim to make them speak perfectly that others may understand, accept, and finally feel convinced to their view [17].

This study is specifically focused on a shift of mathematical arguments from non-dialogue to dialogue. Previous studies only focused on describing the pattern of arguments and the dimension of scientific practices by high school students during discussion [11]. The concept of arguments by prospective teachers in facilitating their students to construct arguments [18]. [19] studied about the importance of rebuttal in mathematical argumentation; activities to develop high school students' argumentation under their teachers' guidance [20]; the method of analyzing the process of interaction in math class [21]. However, it did not mean that these all previous studies were not important. Otherwise, they were very fundamental as a basis to thoroughly investigate students' mathematical argumentation.

Once the arguer has not-in-dialogue argumentation skill, he/shewill has provisions in dialogue context [22]. The effect of dialogue in group discussion may imply an argument change from non-dialogue to dialogue one. It is a condition on which students recheck their mathematical thoughts and revise them. This shifting indicates that not-in-dialogue arguments is not quite strong to convince others in dialogue context.

In-dialogue argument is marked by critical discussion [16]. Some students propose their ideas to others who play as critics that respond to their ideas. Such reciprocal interaction in argumentation makes them able to monitor and control their ideas in addition to thethought when they have to work together to solve problems. Therefore, this study aimed to describe the shift of mathematical argument from dialogue to non-dialogue context.

## 2. Methods

### 2.1. Participants

It took nine undergraduate students of STKIP PGRI Jombang, Indonesia as the research subjects (participants). All participants have taken basic mathematics (set and proof), number theory, 2D geometry, 3D geometry, algebra, and basic calculuscourses. They were divided into three groups that consist of three students for each. The participants consisted of 5 women and 4 men. Their ages range from 20-21 years old. They After solving the given problem individually, they have to discuss and work together to solve the same problem.

## 2.2. Instrument

This study used two kinds of instruments including primary and supporting instrument. The primary instrument referred to the researcher self who acted as planner, data collector, data analyst, data interpreter, and reporter. Additionally, supporting instrument referred to the given task of problem-solving and the guidelines of interview.

### 2.2.1. Problem-Solving Task

The given task for this study was argumentation problem. It was related to decision-making which encouraged students to test, design a strategy, revise and evaluate the effectiveness of their previous solution before making final decision. At first, the students had chances to complete the given task individually and construct their arguments in 20-30 minutes. Then, they should make small groups consisting of three members for each to discuss the same task. The following was the problem-solving task related to decision-making that students should address.

**Investigate the truth of this statement!**

For each $n \in \mathbb{N}$, applies $1 + 2 + 3 + . . . + n = \frac{1}{2}(n + 1)$

**Figure** 2: Showed the solution of this given task presented in Toulmin Scheme.

### 2.2.2. Guidelines of Interview

Guidelines of interview were used as references for the researcher to conduct interview with the research subject. This aimed to confirm the result of students' *think aloud* and discussion. It applied unstructured interview in which the asked questions would be conditional although they had been previously arranged. Interview was conducted after the students had complete the given task in group.

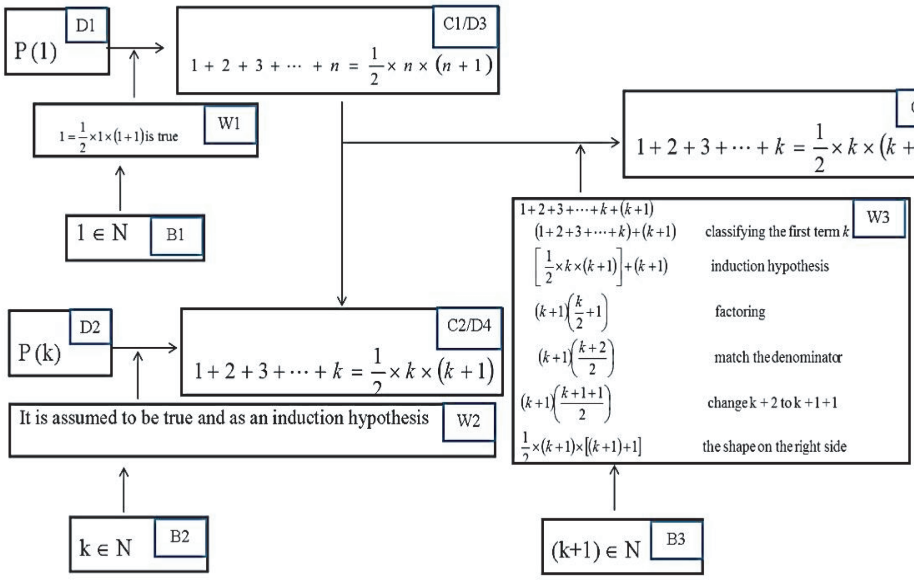


**Figure** 3: The Scheme of Complete Mathematical Argumentation.

### 2.3. Procedures

First, the students completed the given problem-solving task individually using think aloud method. In this stage, it resulted in not-in-dialogue argumentation. Second, they had to discuss and make collective decision in small groups consisting of three students for each to solve the same problem, while the researcher recorded their discussion. In this stage, it resulted in in-dialogue argumentation. Third, the researcher conducted task-based interview and simultaneously confirmed the students' mathematical arguments in solving the problem both individually and collectively.

### 2.4. Data Analysis

In data analysis, the researcher (1) did data transcription of students' think aloud and recording during group discussion and interview, (2) had data reduction that involved explaining, selecting and focusing on critical matters, discarding the unnecessary ones, and organizing the collected raw data, (3) had data coding that involved taking, categorizing, and labeling the collected data with specific terms, (4) describing the shift of mathematical argumentation in proving, and (5) making conclusion.

Towards investigating the shift of argumentation from not-in-dialogue to dialogue one, the researchers needed to consider two things; *think aloud* in solving problem individually and *students' dialogue* in discussing the given problem. In case of needing further information, the researchers conducted unstructured interview with the research subject.

Not-in-dialogue mathematical argumentation data was collected when the subject completed the given task individually while doing think aloud. It was analyzed using Toulmin scheme. On the other hand, in-dialogue data was collected when they had group discussion to solve the given problem. It was also analyzed using Toulmin scheme, as seen in Figure 2. To see whether or not their argumentation shifted, the researcher compared the schemes between not-in-dialogue and in-dialogue argumentation. Once the argumentation changed, the shift of argumentation from not-in-dialogue to in-dialogue was found.

# 3. Results

The result of this study about students' mathematical argumentation shifting in proving-problem consisted of "complementary" shift and "reconstructive" metacognitive shift.

1. **Complementary** shift referred to the shift of students' mathematical argumentation from not-in-dialogue to in-dialogue that happened when they rechecked their mathematical thoughts in solving the proving problem, and thus, they decided to complete their initial structure of thinking.

2. **Reconstructive** shift referred to the shift of students' mathematical argumentation from not-in-dialogue to in-dialogue that happened since the students rechecked their mathematical thought and then decided to reconstruct a new structure of thinking.

According to the result of analysis on the students' *think aloud* works, the recording of their conversation in group discussion and interview, it found that five of them had reconstructive shift while 4 others had complementary shift. The description of their arguments was as follows.

## 3.1. Complementary Shift of Mathematical Argumentation

### 3.1.1. The Scheme of Mathematical Argumentation by S1 when Individually Completing the Given Task

To solve the proving problem, S1 used mathematical induction to show the truth of a statement (i.e., deductive argument). However, his argument was incomplete. Figure 4 showed the written answer by S1 when he individually completes the given task along with his scheme of argument.

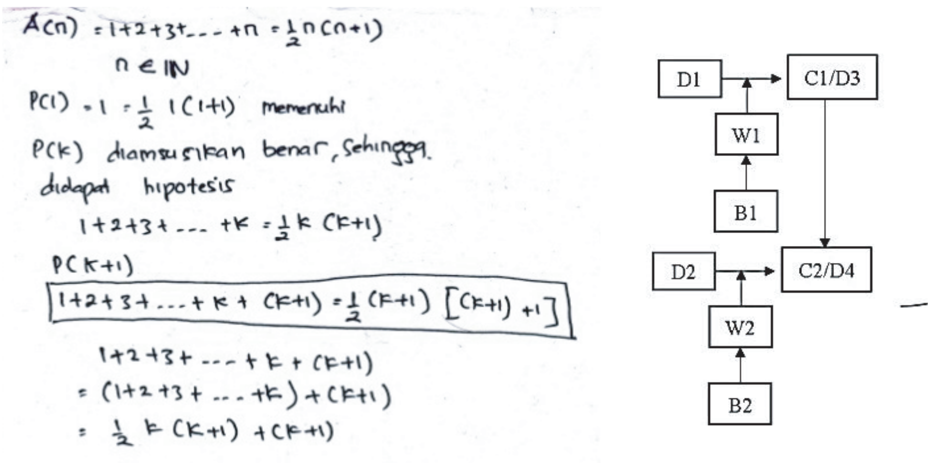


**Figure** 4: Not-in-Dialogue Mathematical Argumentation by S1.

As seen in Figure 4, mathematical argumentation by S1 referred the data (D) to 1, *n,* and *k+1* of natural numbers. The warrant (W) referred to mathematical induction that substituted 1, *n,* and *k+1* into equation $1 + 2 + 3 + \ldots + n = \frac{1}{2}(n+1)$ ” could not be generally concluded.

### 3.1.2. The Scheme of Mathematical Argumentation by S1 when Completing the Given Task by Group Discussion

After each of the subjects was asked to solve the given problem individually, they were asked to discuss in group to solve the same problem. Small group discussion could affect other members to recheck and complete their previous solution. Before having group discussion, S1 used deductive argument. However, his argument was incomplete. When one of his group partners whose initial was A proposed that the work by S1 could be furtherly completed through factoring (*k+1*), S1 considered this idea and decided to complete his previous solution. Figure 5 showed his written answer along with the scheme of his argument.

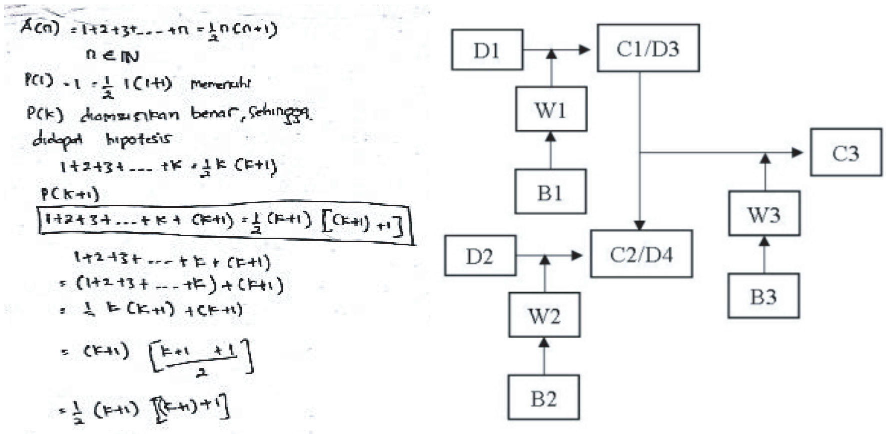


**Figure** 5: In-Dialogue Mathematical Argumentation by S1.

### 3.1.3. The Scheme of Complementary Mathematical Argumentation Shift from Individual to Social

Ideas and opinions from partners of discussion brought effect on S1. Group discussion evoked the shifting of mathematical argumentation from individual to social. It was a condition on which students rechecked their mathematical thought and decided to complete their previous solution to solve the given problem.

Comparing his individual and group works, it found that S1 had complementary shift on his mathematical argumentation. It happened due to group discussion (i.e., social) which made him decide to complete his previous solution. Figure 6 showed the scheme of students' complementary argumentation shift from individual to social.

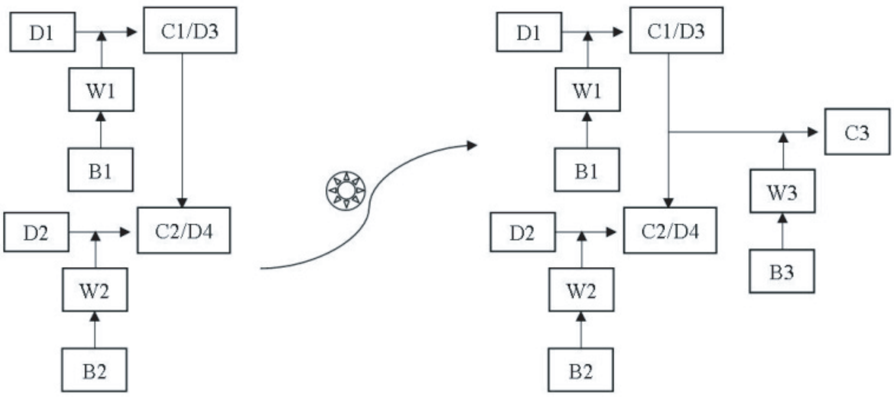


**Figure** 6: The Scheme of Complementary Mathematical Argumentation Shift.

Note: this symbol represented the process of discussion

## 3.2. Reconstructive Argumentation Shift

### 3.2.1. The Scheme of Mathematical Argumentation by S2 when Completing the Given Task Individually

The students read many times to understand what was asked before classifying the information they got into data (D). In argumentation, S2 used specific examples to show the truth of a statement (i.e., inductive argument). The following figure showed his written answer along with the scheme of his argument.

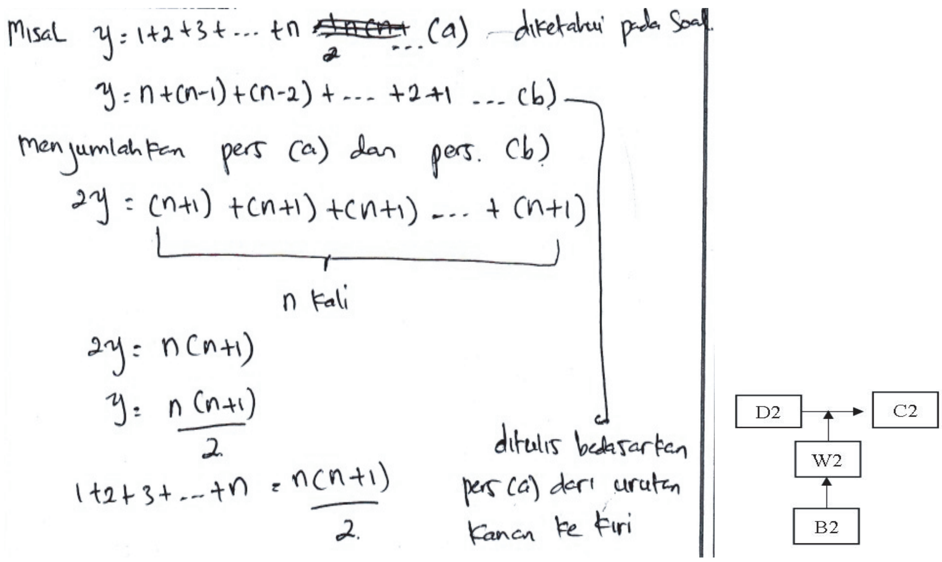


**Figure** 7: Not in Dialog Mathematical Argumentation by S2.

As seen in Figure 7, S2 defined the data (D) as 1, 2, 5, 6, 10, 11, the member of natural numbers. Furthermore, he used specific case as his warrant such as substituting 1, 2, 5, 6, 10, 11 into 1 + 2 + 3 + . . . + $n = \frac{1}{2}(n+1)$ " was true.

### 3.2.2. The Scheme of Mathematical Argumentation by S2 when Completing the Given Task by Group Discussion

After completing the given problem individually, the students were asked to make group consisting of 3 members to discuss the same problem. The time for completion was 30 minutes. The given problem was designed quite challenging that each member of the group should get involved in making decision, checking and revising their previous solution, and stimulating the involvement of mathematical argumentation. Small group discussion might bring effects on other members to recheck and revise their previous solution. The effect caused the shifting on students' mathematical argumentation from individual to social, and thus, they reconstruct or even thoroughly changed their previous path of procedures.

During group discussion, S2 tried to reconsider his mathematical argument. He realized that the qualifier of his mathematical argument was still probable, not certain. It was because his discussion partner (with initials B) wondered if 100, 1000, or the other big natural numbers could comply with the statement "for each $n \in \mathbb{N}$, applies $1 + 2 + 3 + \ldots + n = \frac{1}{2}(n + 1)$ ". Therefore, S2 did evaluation by considering warrant.

His mathematical argumentation shifted into deductive argument when he completed the task by group discussion. S2 used direct proof from the identified facts. In his argumentation, S2 set the identified data and did mathematical manipulation correctly to get conclusion. He concluded that "for each $n \in \mathbb{N}$, applies $1 + 2 + 3 + \ldots + n = \frac{1}{2}(n + 1)$ " was true. Therefore, the warrant he used was deductive [19]. Figure 8 showed the work by S2 in group discussion along with the scheme of his argument.

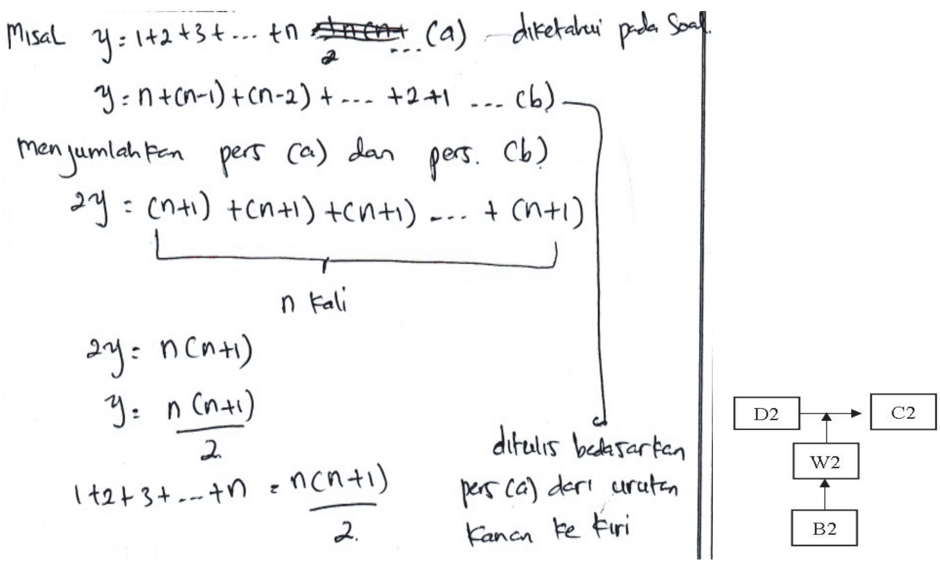


**Figure** 8: In Dialog Mathematical Argumentation by S2.

### 3.2.3. The Scheme of Reconstructive Mathematical Argumentation Shift from Individual to Social

Ideas and opinions from discussion partners brought effect on S2. Group discussion evoked the shifting of mathematical argumentation from individual (i.e., not-in-dialogue)

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to social (in-dialogue). It was a condition on which students rechecked their mathematical thought and decided to complete their previous solution to solve the given problem.

Comparing his individual and group works, it found that S2 had reconstructive shift on his mathematical argumentation. It happened due to group discussion (i.e., dialogue) which made him decide to reconstruct or entirely change his previous path of procedures. At first, he used inductive argument (i.e., nondeductive), then, he decided to reconstruct or change it to be deductive one. Figure 9 showed the students' reconstructive mathematical argumentation shift from individual to social in solving problem.

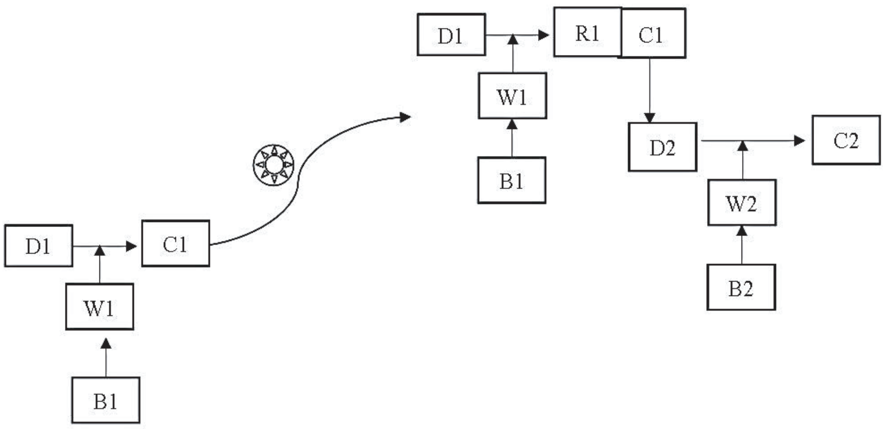


**Figure** 9: The Scheme of Reconstructive Mathematical Argumentation Shift.

## 4. Discussion

### 4.1. Complementary Shift of Mathematical Argumentation

In Not-in-dialogue mathematical argumentation, the subject used deductive approach to solve the proving problem individually. However, their arguments were incomplete. It was close to complete argument as seen in Figure 3. [23] called it as malformation 4 which was marked with incomplete argument. The subject could not complete their arguments until the time was over. It showed that the students felt difficult to construct arguments as they were confused at the beginning and had no idea of how to prove the problem [24], [25].

In a critical discussion, the subjects could complete their individual arguments. They were classified into social-based characterization, which interpreted various perspectives [26]. This characterization represented someone's thought stimulated by the other's mathematical approaches such as considering new information from his partners and tried to understand their mathematical explanation. The subjects' interaction in critical discussion potentially optimized their chance to re-test their thoughts and then revise their misconception.

## 4.2. Reconstructive Shift of Mathematical Argumentation

The subjects still used inductive approach to solve the given problem individually. They checked some specific case before finally making conclusion. They checked specific case 1, which was $n = 1$ substituted into$1 = \frac{1}{2} \times n \times (n+1)$, and concluded that $1 = \frac{1}{2} \times 1 \times (1+1)$. They checked specific case 2, which was $n = 2$ substituted into$1+2 = \frac{1}{2} \times n \times (n+1)$, and concluded that $1+2 = \frac{1}{2} \times 2 \times (2+1)$. They checked specific case 3, which was $n = 3$ substituted into $1+2+3 = \frac{1}{2} \times n \times (n+1)$, and concluded that $1+2+3+4+5 = \frac{1}{2} \times 5 \times (5+1)$. They checked specific case 4, which was $n = 6$ substituted into $1+2+3+4+5+6 = \frac{1}{2} \times n \times (n+1)$, and concluded that $1+2+3+4+5+6 = \frac{1}{2} \times 6 \times (6+1)$. They checked specific case 5, which was $n = 10$ substituted into $1+2+3+4+5+6+7+8+9+10 = \frac{1}{2} \times n \times (n+1)$, and concluded that $1+2+3+4+5+6+7+8+9+10 = \frac{1}{2} \times 10 \times (10+1)$. They checked specific case 6, which was $n = 11$ substituted into $1+2+3+4+5+6+7+8+9+10+11 = \frac{1}{2} \times n \times (n+1)$, and concluded that $1+2+3+4+5+6+7+8+9+10+11 = \frac{1}{2} \times 11 \times (11+1)$.

Furthermore, they made a general conclusion that $1+2+3+\cdots+n = \frac{1}{2} \times n \times (n+1)$. Therefore, the process of generalization by the subject was the pattern of generalization result [27]. The pattern of generalization result by the subject focused on the regularity of the result which could be visualized as: E1, E2, E3, ..., in which E was the parts of generalization on case 1, 2, 3, ... Hence, the subject's mathematical argumentation could be represented using Toulmin Scheme of generalization pattern, as shown in Figure 10.

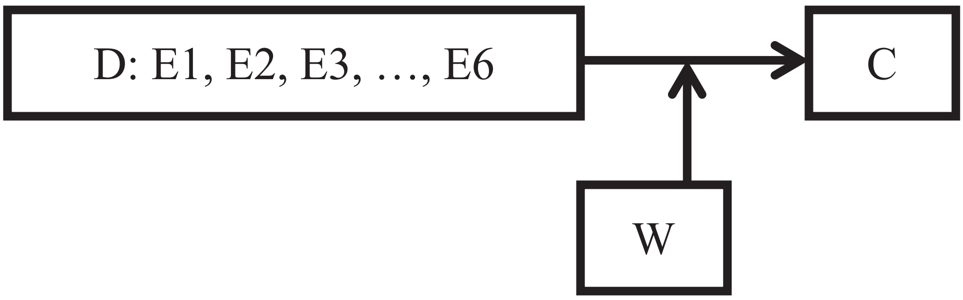


**Figure** 10: The Inductive Pattern of Generalization Result.

This inductive argument played an important role in developing students' mathematical argumentation.Although using inductive argument, however, the students should realizethat the qualifier of conclusion was still probable, which could only be applied to studied specific cases [19]. Furthermore, this inductive mathematical argumentation was only used to alleviate the uncertainty, not to eliminate it. The subject did not realize that they had generated invalid scheme of argument (inductive argument). It indicated that they could not differentiate between valid and invalid arguments [28]–[31]. In critical discussion, the inductive argument that the subject constructed when completing the given problem individually was not strong enough to convince (themselves and others). Their group discussion related to the qualifier of conclusion which was still in probable level influenced them to recheck and finally revise their mathematical thought. Furthermore, ideas from their discussion partners successfully made them realizethat inductive argument could not be used to convince others, since it could only be used to solely alleviate the uncertainty, not eliminate it [19]. Moreover, the subject was classified into social-based characterization; that interpreted various perspectives [26]. This characterization represented someone's thought stimulated by the other's mathematical approaches such as considering any new information that their discussion partners proposed, and then trying to understand their explanations.

The subjects used inductive warrant in their not-in-dialogue context. However, they shifted to deductive warrant in their in-dialogue context. It indicated that the students did not directly use abstract objects. Otherwise, they used some concrete objects, specific examples or cases as a bridge to understand the abstract ones. It was consistent to [32]–[34] that neither inductive nor deductive argument was independent, as inductive structure was used as a foothold to construct deductive structure. Moreover, inductive argument was invalid or informal, while deductive argument was valid [27], [33], [35]. It was due to lecturer's teaching that made students decide to use inductive mathematical arguments. Students' arguments depended on their learning culture in class, the nature of the given task, and the kinds of reasoning their teachers tried to emphasize [18], [36]–[38]. Teachers' measures encouraged students to explain, note, and justify their arguments in class.

In teaching a course related to argumentation, some lecturers in the research place considered their students' inductive thought. Hence, before giving a formal argumentation, the lecturers began with some specific cases. By considering students' inductive thought in teaching-learning process, the lecturers could assist their students to construct deductive arguments [22]. Students could learn and transfer what they had learned when the material was presented in examples [39]. Students' competence to absorb declarative knowledge developed after considering their inductive thought in teaching and learning process [40].

In developing students' mathematical argumentation skills, lecturers can use the CIRC type of cooperative learning model, Problem Based Learning, or infusion learning [41]–[43]. This learning provides many opportunities for mathematical activities for students to make arguments. Students are also directed to consider the components of Toulmin's schema in constructing arguments. Student in teractions with the learning environment such as group problem solving have the potential to maximize opportunities for stu-dentsto reexamine their thinking and correct their misunderstandings [44].The need to optimize student interaction at the university to improve academic performance [45]. Students are given the opportunity to actively participate and interact with lecturers and peers both to share knowledge horizontally and vertically.

This researchis still limited to students' arguments in solving evidentiary problems, there fore for further research, students' arguments can be identified in solving problems arising from real life. Solving real-life problems is important because the mathematics curriculum focuses on application, relevance to practice or other subjects. One of the practical activities and experiences in mathematics education is the implementation of mathematics projects in real life [46].

The participants of this research were prospective mathematics teachers. Therefore, it is recommended that similar research be conducted with participants from university students in the field of mathematics. This research includes mathematical arguments in solving the problem of proof individually, groups, and shifts. The results of these studies may be used by lecturers to design professional development program to in crease students' abilities in formulating mathematical arguments. It give positive impact on students' argument [47], [48]. In the direction of this research, pay attention to their mathematical abilities and the mathematical concepts that students have.

## 5. Conclusions

To conclude, this study found two types of students' mathematical argumentation shifts in proving. They are the shift of complementary and reconstructive mathematical argumentation. Discussionsbv in small groups have influenced the other members to check and revise their original solutions in solving the evidentiary problems. The influence in this group discussion resulted in a shift in the mathematical argument process. The shift in mathematical argumentsis a shift that occurs due to the influence of group discussions where students rearrange the initial procedure and complete their answers during argumentation in solving proof problems. The shift in complementary mathematical argumentation occurs because studentsre-examine their mathematical thinking in solving proof problems, so students complete the structure of their initial thinking. Meanwhile, the shift in reconstructive mathematical argumentation occurs because of the influence of group discussions (dialogue) so that students disassemble or change the path of their initial procedure as a whole. Initially, students used inductive (non-deductive) arguments, then they were disassembled or converted into deductive arguments. The shift in constructive mathematical arguments is more often done by students because students do not have sufficient knowledge to prove it.

It is hoped that future researchers who want to measure students' mathematical argumentation skills should modify the learning model by integrating arguments not in dialogue, and discussions that facilitate arguments in dialogue. It aims to determine the effectivenesss of the learning model in improving students' mathematical argumentation skills. The model that can be used as a reference by researchers is the PBL learning model, cooperative type CIRC and infusion learning. This research can also be used to develop other, more innovative learning models by integrating character education. Utilization of technology such as smartphones, gadgets, social media needs to be developed to support the learning process in improving students' mathematical argumentation skills.

This study covers the shift of non-dialogue arguments to dialogue arguments for 2018 undergraduate mathematics education students at Jombang University, East Java. This study provides an overview to the lecturer about the argumentation ability of undergraduate students.

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