KnE Social Sciences
ICESRE 2021
4th International Conference on Education and Social Science Research (ICESRE)
Volume 2022

Research article

Algebraic Reasoning in Marzano's Taxonomy Cognitive System

Mochamad Abdul Basir*, S.B. Waluya, Dwijanto, and Isnarto

Semarang State University, Indonesia

Abstract.
This article explores the relationship between algebraic reasoning and the cognitive system of Marzano's taxonomy. The reasoning is known as a thought process that connects premises to conclusions. The ability to solve problems related to learning and how to state generalizations about numbers, quantities, relations, and functions is part of algebraic reasoning. There are four indicators of algebraic reasoning ability – Knowledge Retrieval, Connecting Mathematical Representations, Pattern Recognition, and Reasoned Solving. Algebraic reasoning abilities can increase awareness of the knowledge process, help in constructing or using knowledge, and develop one's self-confidence while engaged in tasks through assignments to Marzano's taxonomy. Marzano's taxonomic cognitive system not only explains how a person makes a decision to engage in a new task but also explains how information is processed post decision-making. Thus, algebraic reasoning is related to the cognitive system in Marzano's taxonomy.

Keywords: algebraic reasoning, cognitive system, Marzano's taxonomy

Corresponding Author: Mochamad Abdul Basir; email: abdulbasir@unissula.ac.id

Published: 28 September 2022

Publishing services provided by Knowledge E

Selection and Peer-review under the responsibility of the ICESRE 2021 Conference Committee.

1. INTRODUCTION

The development of mathematics teaching that focuses on reasoning and sense-making affects students' ability to solve problems non-procedurally and understand the thought process of solving the problems [1]. This understanding can improve the students achieve the learning objectives of mathematics in finding patterns to explain a long process and use it in a real-world context and use them in solving problems or arguments.

The reasoning is also called the process of concluding based on existing evidence or assumptions [2]. The reasoning is an important part of all disciplines, reasoning has a special and fundamental role in mathematics. The mathematical process is a manifestation of the act of understanding mathematics and reasoning. Problem-solving and proof are impossible without reasoning, and they are both processes of developing mathematical reasoning and understanding mathematical ideas. Meanwhile, communication, connection, and representation support reasoning and sense-making in making

How to cite this article: Mochamad Abdul Basir*, S.B. Waluya, Dwijanto, and Isnarto, (2022), "Algebraic Reasoning in Marzano's Taxonomy Cognitive System" in *4th International Conference on Education and Social Science Research (ICESRE)*, KnE Social Sciences, pages 96–105. DOI 10.18502/kss.v7i14.11957

decisions. Reasoning can be applied according to the object of mathematical study, one of which is algebraic material, in this case, we call it algebraic reasoning.

The process of Algebraic thinking means generalizing mathematical ideas from facts, compiling generalizations through statements, and expressing these statements in a formal way and according to the developmental age of students [3]. An algebraic approach to thinking about quantitative situations in general and relational. This kind of thinking is optimized by a fairly broad understanding of algebraic objects, general thinking dispositions, and involvement in higher-order tasks that provide a context for applying and investigating mathematics and the real world.

Cognitive changes are needed by students in studying algebra with the transition from one form to another so that an epistemological barrier appears. Long-term cognitive development about the meaning of symbols in algebra, starting from symbols in arithmetic as procedures, processes and concepts, evaluation, algebraic manipulation, and then axiomatic algebra [4]. In solving numerical and generalization tasks, students experience a transition from arithmetic to algebra and students tend to prefer to explain rhetorically rather than give answers in symbolic form. Although algebraic reasoning has been considered an important factor affecting students' mathematical performance, there are still many students who have difficulty building concrete algebraic reasoning [5]. For this reason, a strategy is needed to facilitate algebraic reasoning through Marzano's taxonomy approach which combines the basics of the cognitive and metacognitive thinking process levels in which it contains the relationship of benefits, motivations, and emotions that support it. Marzano's taxonomy moves from a simple way to a more complete process of information or procedures, then to a less awareness to a more controlled awareness of the process of knowledge and how to structure and use it, and then from a lack of personal involvement or a large commitment of trust in general. centered and reflection of each student [6, 7].

2. Results and discussion

2.1. A Perspective on Marzano's Taxonomy

The combination of several forms of knowledge and the ability to remember and recognize that knowledge with various mental operations that act on that knowledge is one of the main weaknesses of Bloom's taxonomy. This is because it confuses the Action object with the Action itself. Marzano's taxonomy avoids this confusion by postulating three knowledge domains that are operated by three domains of knowledge (information, mental procedures, and psychomotor procedures) and three process categories (cognitive, metacognitive, and self-system) so that it is a thinking system that has a hierarchical structure. that makeup Marzano's taxonomy. Metacognitive processes and self-systems are addressed as an integral part of this type of support curriculum [8].

Marzano's Taxonomy moves from a simple way to a more complete process of information or procedures, from less awareness to more awareness of more control over the process of knowledge and how to structure or use it, and from a lack of personal involvement or commitment. to a great deal of centralized belief and reflection of one's identity [6,7]. Marzano's Taxonomy Model can be used to photograph students' thinking processes [9]. This is because Marzano's taxonomic model not only explains a person's thought process in deciding a new task at a time but also explains the process of obtaining information after making a decision. Marzano's taxonomic model presents three mental systems: the self-system, the metacognitive system, and the cognitive system. The fourth component of the model is knowledge. At the time of starting a new task, the Self-system decides whether to continue the existing habit or do something new, then the metacognitive system sets goals and maintains the competitiveness of those goals, while the cognitive system processes all the necessary information and the knowledge domain provides its content.

Table 1: The Three Systems and Knowledge.

Self-System Thinking			
Examining Importance	Examining Efficacy	Examining Emotional Response	Examining Motivation
Metacognitive System			
Specifying Goals	Process Monitoring	Monitoring Clarity	Monitoring Accuracy
Cognitive System			
Retrieval	Comprehension	Analysis	Knowledge Utilization
Recognizing	Integrating	Matching	Decision Making
Recalling	Symbolizing	Classifying	Problem Solving
Executing		Analyzing Errors	Experimenting
		Generalizing	Investigating
		Specifying	
Domain of Knowledge			
Information		Mental Procedures	Psychomotor Procedures

Mental processes in the cognitive system are carried out from the knowledge domain. This process provides access to a lot of information, procedures, helps manipulate, and use knowledge. Marzano breaks down the cognitive system into four components, namely retrieval, comprehension, analysis, and knowledge utilization. Each process is formed from all the previous processes.

2.2. Algebraic Reasoning Process in Solving Problems

Algebraic reasoning is not just a procedural skill that tends to be mechanistic, but also a skill in connecting problem solutions using reasoning and sense-making [10]. Building algebraic reasoning skills is a long and difficult process but it is very beneficial for students' understanding of mathematical patterns and relationships. Reasoning skills can be honed through solving math problems. Algebraic reasoning ability is a thought process using various representations to solve quantitative situations in a relational way using symbols [11]. In line with this, algebraic reasoning generalizes mathematical ideas from a set of examples, proves these generalizations through argumentative discourse, and expresses them formally according to the level of student age development [12].

Algebraic reasoning encompasses all mathematical thinking because of its use in exploring mathematical structures. Algebraic reasoning also requires students to be able to explore a relationship and build generalizations to support conceptual understanding of the relationship in a formula. Reasoning indicators in solving algebraic problems are formulated as follows: (a) *Knowledge Retrieval*, understanding symbolism, understanding number system structure and patterns and modelling mathematics [13]; (b) *Connecting mathematical representations*, paying attention to quantities that change in context and representing how they are connected, making algebraic connections in solving mathematical problems, using equation models in solving multiple solutions [10,13]; (c) *Pattern Recognition*, Connecting manipulation with arithmetic laws; anticipating the results of manipulation, selecting procedures that are appropriate to the context, mentally imagining calculations [14,15,16]; (d) *Reasoned solving,* Organizing data to represent situations, Investigate, Create and Generalize [14,17].

2.3. The Relationship between Algebraic Reasoning and The Cognitive System of Marzano's Taxonomy

The learning dimension is a metaphor for how the brain works while the person is learning. Learning that uses the learning dimension model approach is learning that uses the learning dimensions as the premise of learning, one of which is learning to use knowledge in a meaningful way. Algebraic reasoning ability is the ability to learn to use knowledge meaningfully which focuses on the regularity of the process of solving algebraic problems. For this reason, the evaluation of learning outcomes must be adjusted to the learning objectives to be achieved. Learning objectives are classified into a taxonomy of learning objectives. One of the learning taxonomies related to algebraic reasoning ability is Marzano's taxonomy.

The problem-solving process on algebraic reasoning ability consists of four aspects, namely: Knowledge Retrieval, Connecting Mathematical Representations, Pattern Recognition, and Reasoned Solving. Furthermore, it will be used as an illustration in developing algebraic reasoning abilities based on the cognitive system in Marzano's taxonomy.

The coordinates of vertices of ΔABC are given as A (1,0), B (3,0), and C (3,3). If the coordinates of the transformation points A, B, and C by matrix $\begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}$ *are A', B', dan C'.*

TABLE 2: The relationship between algebraic reasoning and cognitive systems in Marzano's taxonomy.

Level of Marzano's Taxonomy	Task	Objective	Indicator of Algebraic Reasoning
Retrieval; *Executing*	determine the coordinate of the transformation points A, B, and C by matrix $\begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}$!	The student will be able to discover and execute the coordinates of the transformation by matrix	Knowledge Retrieval
Comprehension; *Symbolizing*	Represent the information so that the area of ΔA'B'C' is related to the area of ΔABC!	The student will be able to represent information about the area	Connecting Mathematical Representations
Analysis: *Matching*	Find a corresponding matrix if the area of ΔA'B'C' becomes 2 times the area of ΔABC, becomes 3 times the area of ΔABC, becomes 10 times the area of ΔABC!	The student will be able to compare the area of the area as desired	Pattern Recognition
Knowledge Utilization; *Decision Making*	Find a corresponding matrix if the area of ΔA'B'C' becomes n times the area of ΔABC! Explain how the mathematical concept underlying the answer!	The student will be able to identify how the mathematical concept apply to generalize rules	Reasoned Solving

DOI 10.18502/kss.v7i14.11957

2.3.1. Knowledge Retrieval

The use of meaningful algebraic symbols requires accuracy in defining problem-solving, including in determining unknown elements as function parameters and using variables. This is because the long-term goal of learning algebra is to find something whose value varies and requires algebraic manipulation to interpret expressions. Initially, the reasons and justifications for shaping and manipulating expressions should be the main emphasis of instruction. When an expression is acquired, it unconsciously takes a gradual effort to construct and interpret it.

Reasoning in algebraic forms depends on being able to read in different ways. As in the problems given, students can choose variables and build an equation model that fits the context; interpret the form of the equation model; manipulate expressions so that interesting interpretations can be made. And in the end, a mathematical model of the problem to be solved can be obtained.

$$\begin{pmatrix} x_{a'} & x_{b'} & x_{c'} \\ y_{a'} & y_{b'} & y_{c'} \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x_a & x_b & x_c \\ y_a & y_b & y_c \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 3 & 3 \\ 0 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 6 & 12 \\ 0 & 0 & 6 \end{pmatrix}$$

2.3.2. Connecting Mathematical Representations

There is an interaction between algebra and geometry. Geometric representations such as graphs can provide expressions in algebraic form and algebraic representations can be used to give conclusions about geometric relationships. Students can draw the problem in a geometric form first to facilitate the problem-solving structure of the problem. Geometry models that can be made by students can resemble the following picture.

This example also illustrates effective representations as to the basis for reasoning and shows how cognitive structures are obtained to solve mathematical problems. The geometric shapes that can be illustrated are not limited to the image above. Students can also illustrate with other geometric patterns that represent the problem in the problem.

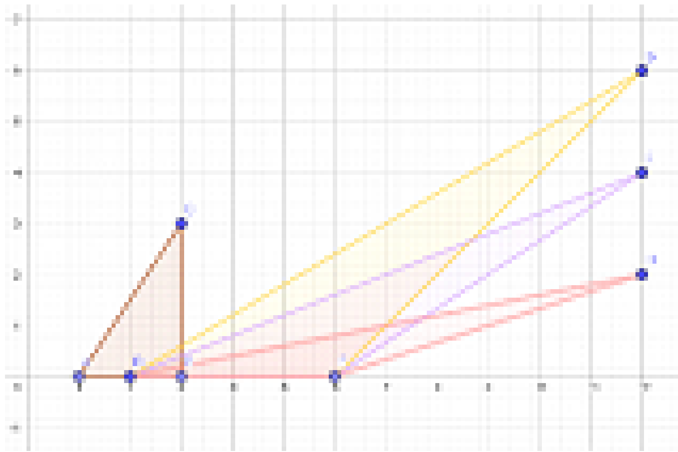

Figure 1: Manipulation of the image representation of the area of a triangle.

2.3.3. Pattern Recognition

Students relate manipulation to the laws of arithmetic; anticipating the results of manipulation; selecting procedures in context; imagine mental calculations. Students perform forms of manipulation that can help solve problems.

The corresponding matrix in order of area $\Delta A'B'C'$ *be 2 times the area of* $\Delta A'B'C'$ *is* $\begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}$

The corresponding matrix in order of area $\Delta A'B'C'$ *be 3 times the area of* $\Delta A'B'C'$ *is* $\begin{pmatrix} 2 & 2 \\ 0 & \frac{3}{2} \end{pmatrix}$

The corresponding matrix in order of area $\Delta A'B'C'$ *be 10 times the area of* $\Delta A'B'C'$ *is* $\begin{pmatrix} 2 & 2 \\ 0 & 5 \end{pmatrix}$

2.3.4. Reasoned solving

See solution steps as logical thinking about equivalence and interpret solutions in context. Students can find patterns and properties of algebraic arithmetic operations from mathematical problems and can conclude the questions given. The process of concluding algebraic reasoning problems through the processes of finding patterns, recognizing patterns, and generalizing the patterns found.

The corresponding matrix in order of area $\Delta A'B'C'$ *be n times the area of* $\Delta A'B'C'$ *is* $\begin{pmatrix} 2 & 2 \\ 0 & \frac{n}{2} \end{pmatrix}$

3. CONCLUSIONS

Regarding students' ability to solve algebraic problems, Marzano's taxonomy model helps to build a test item that can be used to test algebraic reasoning abilities. The ability to design appropriate algebraic reasoning test items is defined at each level in Marzano's taxonomy. Marzano makes the habit of thinking one of the dimensions of learning outcomes. In Marzano's taxonomy, some aspects were developed regarding the components of learning, measurement, and strategies to improve thinking habits

4. AUTHORS' CONTRIBUTIONS

Mochamad Abdul Basir: compile articles and critically revised for important intellectual content. **S.B.Waluya**: make a major contribution to the concept or design of the article. **Dwijanto**: critical revision of manuscript. **Isnarto**: interpretation of result

acknOWLEDGMENTS

This research was generously supported by Universitas Islam Sultan Agung, Semarang State University, and Director General of Higher Education.

References

[1] National Council of Teachers of Mathematics, Reasoning and sense making. The Mathematics Teacher. 2016;110(2);1-6. https://doi.org/10.5951/mathteacher.110.2.0119

[2] Khemlani SS. Stevens' handbook of experimental psychology and cognitive neuroscience. Wiley; New Jersey – USA; 2018. https://doi.org/10.1002/9781119170174.epcn311

[3] Kaput JJ, Blanton ML. Characterizing a classroom practice that promotes algebraic reasoning. Journal for Research in Mathematics Education. 2005;36(5):412-446.

[4] Thomas MOJ, Tall D. The long-term cognitive development of symbolic algebra. Paper presented at: The Future of the Teaching and Learning of Algebra: Proceedings of the 12th ICMI Study Conference; 9-14 December 2001; Melbourne - Australia

[5] Lee Y, Capraro MM, Capraro RM, Bicer A. A meta-analysis: Improvement of students' algebraic reasoning through metacognitive training. International Education Studies. 2018;11(10):42-49. https://doi.org/10.5539/ies.v11 n10p42

[6] Irvine J. Marzano's new taxonomy as a framework for investigating student affect. Journal of Instructional Pedagogies. 2020;24:1–31.

[7] Rasyidi DA, Winarso W. The proportion of cognitive aspects of question in mathematics textbook based on Marzano's taxonomy: An Indonesian case in implementing new curriculum. Eduma: Mathematics Education Learning and Teaching. 2020;9(2):79-89. https://doi.org/10.24235/eduma.v9i2.7374

[8] Marzano RJ, Kendall JS. Praise for the second edition of the new taxonomy of educational objectives. Corwin Press; USA; 2007.

[9] Marzano RJ, Kendall JS. Designing & assessing educational objectives: Applying the new taxonomy. Designing and assessing educational objectives. Corwin Press USA; 2008.

[10] Glassmeyer D, Edwards B. How middle grade teachers think about algebraic reasoning. Mathematics Teacher Education and Development. 2016;18(2):92–106.

[11] Kieran C. Algebraic thinking in the early grades: What is it. The Mathematics Educator. 2004;8(1):139–51.

[12] Cañadas MC, Brizuela BM, Blanton M. Second graders articulating ideas about linear functional relationships. Journal of Mathematical Behavior. 2016;41:87–103. https://doi.org/10.1016/j.jmathb.2015.10.004

[13] Uygun T, Güner P. Representation of algebraic reasoning in sets through argumentation. International Journal of Contemporary Educational Research. 2019;6(2); 215-229. https://doi.org/10.33200/ijcer.557781

[14] Blanton ML, Kaput JJ. Functional thinking as a route into algebra in the elementary grades. Early Algebraization. Advances in Mathematics Education. Springer, Berlin; 2011. https://doi.org/10.1007/978-3-642-17735-4_2

[15] Pertegal-Felices ML. Didactics of mathematics profile of engineering students: A case study in a multimedia engineering degree. Education Sciences. 2020;10(2);1-8. https://doi.org/ 10.3390/educsci10020033

[16] Pourdavood BR, McCarthy K, McCafferty T. The impact of mental computation on children's mathematical communication, problem solving, reasoning, and algebraic thinking. Athens Journal of Education. 2020;7(3):241–54. https://doi.org/10.30958/aje.7-3-1

[17] Otten M, van den Heuvel-Panhuizen M, Veldhuis M, Boom J, Heinze A. Are physical experiences with the balance model beneficial for students' algebraic reasoning? An evaluation of two learning environments for linear equations. Education Sciences. 2020;10(6):1–23. https://doi.org/10.3390/educsci10060163