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Application of Graph Theory to Entrepreneurship Research: New Findings and a New Methodology Framework

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Abstract

This study used Graph Theory to examine relationships within the Greek ICT start-up ecosystem, analyzing founders' views, strategies, and current perceptions regarding their ICT start-ups. The sample consisted of more than 90 founders of Greek IT start ups who participated in exhibitions as start-up companies during 2018 and 2019 and had completed at least one accelerator program. The study analyzed the dataset of a previous research project, using Graph Theory and R as the main tools for statistical analysis, in order to re-examine relationships between the key variables. The study highlighted a set of critical relationships and facts that were not revealed in past research. The dataset was first analyzed with the use of SPSS software for Pearson correlation, but using Graph Theory revealed a different picture. A literature review was also conducted to examine the key issues of entrepreneurship and Graph Theory and their applications to success factors for start-ups. The research findings provided a more precise understanding of the dynamics of Greek ICT start-ups, as well as the importance of Graph Theory for analyzing complex phenomena and business ecosystems. The findings may be useful for entrepreneurs, as they strive to increase the success rates of current and future projects, as well as for academics, since Graph Theory reveals a new, likely more effective and accurate way of analyzing data and estimating relationships in complex environments.

Keywords: innovation management, marketing, start-ups, founders, business models, entrepreneurship, strategy, Greek, Graph Theory

jel CLASSIFICATION codes: L26, M13, O30, O31, O32, O33

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1. Introduction

The Seven Bridges of Königsberg is a famous historically problem in mathematics. In 1736, the famous mathematician Leonhard Euler solved the problem and at the same time founded the theory of graphs as a distinct mathematical field and introduced us to the idea of topology [1]. From that founding time there has been a significant

development with the contribution of great mathematicians. The explosion of graph theory came gradually from the mid-1990s in parallel with the explosion of the internet and what we call «the modern interconnected world within the information society».

The claim that networks are all around us has become self-evident, generally accepted, and easily understood by the average person. The simplest person today is an active member of many networks such as the internet, social networks such as Facebook, forums, political and economic networks, and professional ecosystems. The concept of network and the power of interconnection has evolved from a theoretical mathematical construction to a simple, everyday experience of each person.

As a result, network theory has evolved into a very dynamic scientific field and has penetrated many research areas. This is because it allows a comprehensive understanding and interpretation of the complex relationships between entities as well as the dynamics of their interactions. It is the best scientific tool we have to manage complex systems. 'Network science is broadly employed in many fields: from understanding how friends' bond in a party to how animals interact; from how superheroes appear in the same comic books to how genes can be related to a specific biological process. Network analysis is especially beneficial for understanding complex systems, independent of the research field. Examples of complex biological or medical systems include gene regulatory, ecological and neuropsychology networks. Social networks can include collaborations between scientists or actors, sexual partnerships, or relationships between historic persons, among others' [3].

We will then attempt a brief introduction to the basic terminology required to understand the continuation of this work.

1.1. Graph Theory Basic Terminology

The set of interactions among a set of entities is, in general, called a graph or a network [3, 4]. In graph theory, each entity is called a vertex, while in network notation, it is called a node. Accordingly, the connections between two entities are called edges or links, respectively. In this paper, we will always use the network notation, unless otherwise specified. The total number of nodes in a network is often denoted as N and the number of links in a network is denoted as L [3].

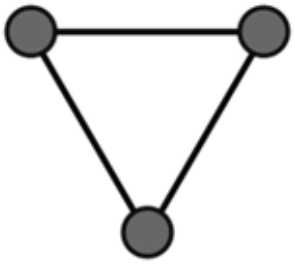

Figure 1: A graph with three vertices and three edges.

While nodes can receive a label, links in general, are not labelled [4] (although, in many cases, weights can also be perceived as a label). A network can be represented

mathematically as an adjacency matrix (usually denoted as A), an edge-list or visually as a graph [3] (see Figure 1).

Links of a network can possess a direction (normally depicted by an arrow), which indicates that the interaction is asymmetric, for example a person follows somebody else in a social network but not the opposite. Networks with directed links are called directed networks, while networks without directed interactions or in which the direction is not known are referred to as undirected networks, that is collaboration in the same project or interactions between companies [3].

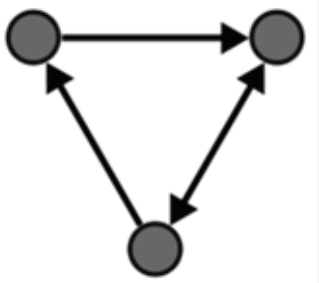

Figure 2: A directed graph with three vertices and four edges directed edges (the double arrow represents an edge in each direction).

The links can also have a weight to express the strength of the interaction, which results in a weighted network [4]. Usually, the weight is graphically displayed as the thickness or the length of the links [3].

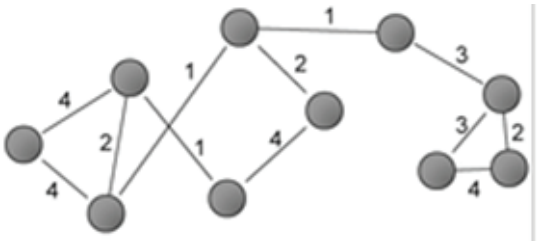

Figure 3: A weighted graph with 10 vertices and 12 edges.

Below, in Table 1, you can see a brief glossary of network terms, most of which will be referenced in this study.

1.2. Networks and Economics

The increasing frequency and scope of financial crises has made global financial stability one of the major concerns of economic policy and decision makers. Under this highly complex environment, supervision of the financial system has to be thought of as a systemic task, focusing not only on the strength of the institutions but also on the interdependent relations among them, unraveling the structure and dynamic of the system as a whole. In recent years, network science has emerged as a leading tool for the investigation of complex systems. Here we review several applications of network science in finance and economics and discuss existing challenges and future

Table 1: A short dictionary of network terms [3].

A **network** is a pair G = (N, L) of a set N of nodes connected by a set L of **links**.
Two nodes are **neighbours** if they are connected.
The **degree** of a node is the number of nodes it interacts with (the neighbours).
The **weight** is a measure of how strong a particular interaction is.
The **direction** of a link specifies the source (starting point) and a target (endpoint) where the interaction occurs.
The **strength** of a node is the sum of the weights attached to links belonging to a node.
Hubs are nodes with a much larger degree compared to the average degree value.
A set of highly interconnected nodes is a **module** or **cluster**.
Two nodes are connected in a network, if a sequence of adjacent nodes, a **path**, connects them.
The **shortest path length** is the number of links along the shortest path connecting two nodes.
The **average path length** is the average of the shortest paths between all pairs of nodes.
The **diameter** is the maximum distance between two nodes.
The **modularity** index is a measure of the strength of the network division into modules when this measure is maximized; it can be used for identifying **nodes communities**.
Preferential attachment is the tendency of nodes to form new links preferentially to nodes with a high number of links.
The probability that a random node in the network has a particular degree is given by the **degree distribution**.
The **clustering coefficient** describes the degree with which a node is connected to all its neighbours.
The **global clustering coefficient** measures the total number of triangles in a network.
The **average clustering coefficient** is the average of the clustering coefficient of all nodes in a network.
Centrality is a set of measures that have been proposed to help to define the most central nodes. It has many interpretations for autonomy, control, risk, exposure, influence, and power.
Closeness centrality is defined as the average distance from a single vertex to all other vertices.
Betweenness centrality is defined as the total number of shortest paths between pairs of nodes that pass through a particular node.
Global measures are measures that describe the whole network, for example, degree distribution; average clustering coefficient; path length; modularity index.
Local measures are characteristics of individual nodes of a network, such as their degree and centrality.

directions which will substantiate network science as a key tool for financial academics, practitioners, and policy and decision makers.

Network based research in entrepreneurship endures to grow in coming years [1]. Social Network Analysis (SNA) is not known as traditional data mining technique, but it can be used for mining interested patterns in large data. Entrepreneurs are usually connected with the agents known as brokers (in SNA perspective) in order to advance their strategic position in networked space [2]. The aim of this article is twofold. First, introduce concepts of network science and present a model of dependence centrality with an application of hypothetical small network of entrepreneurs; second, differentiate the model with betweenness centrality and pair dependency with theoretical and practical implications in the area.

2. Methodology

In order to evaluate the value of Graph Theory on Entrepreneurship Research, it was decided to apply key elements of Graph Theory on datasets already used in Entrepreneurial Research publications, in order to be in position to compare results and key findings based on the exact same dataset. More specifically the dataset of research paper [11] was used.

More specifically, past research [11] used Pearson Coefficient and identified the following correlations between variables.

TABLE 2: Past Research Results [11].

Variable 1	Variable 2	Relationship	Correlation	Value Pearson	Statistical Significance
Success	B2B	Reverse Analogous	Weak	-0.206	*Correlation is significant at the 0.05 level (two-tailed).
Success	Sales 100K	Analogous	Weak	0.218	*Correlation is significant at the 0.05 level (two-tailed).
Education	Disruption	Analogous	Weak	0.293	**Correlation is significant at the 0.01 level (two-tailed)
Education	Get Funding as Challenge	Analogous	Weak	0.282	** Correlation is significant at the 0.01 level (two-tailed
Education	Improve Product as Challenge	Analogous	Weak	0.204	* Correlation is significant at the 0.05 level (two-tailed).
Number of Founders	IP as Competitive Advantage	Reverse Analogous	Weak	-0.241	* Correlation is significant at the 0.05 level (two-tailed).
Number of Founders	New Market Creation	Reverse Analogous	Weak	-0.238	* Correlation is significant at the 0.05 level (two-tailed
Experience	Get Funding as Challenge	Analogous	Weak	0.296	** Correlation is significant at the 0.01 level (two-tailed
B2B	Technology as Competitive Advantage	Analogous	Weak	0.222	* Correlation is significant at the 0.05 level (two-tailed).
Opportunity based	Find new customers challenge	Analogous	Weak	0.274	** Correlation is significant at the 0.01 level (two-tailed).

- Use of Graph Theory for Data Analysis
- Conceptualization of the 4 Networks – Based on the Perception of Success
- Use of R language, R – Studio for Analysis of the Data based on Graph Theory, using i-graph library
- Use the exact same dataset of a Previous Published research (Peer Reviewed) [11]

- Considering previous academic research on Entrepreneurship Research
- Formation of three Levels of Analysis

  A) A Network of two Levels – Start-Up Founders and Variables

  B) A Network of three Variables – Start Up Founders, Categories of Variables and Variables.

  C) A Network Relationship between Final Choices (Weighted)
- In each one of the cases, (A, B, C), four different networks were formed based on the founder's perception of success.
- The four networks in each one of the cases were compared (weighted) in order to reach safe conclusions.

The dataset consists of the following:

- 94 Start-Up Founders who filled in the Research Questionnaire
- 23 Variables (Such as Age, Education, Experience, No. of Founders, Competitive Advantage, Previous Ventures, Strategic Alliances, Reasons for Establishing a Start-Up, Success, etc) deriving from relevant Entrepreneurial Literature as summarized in past research [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23]
- 87 Different Options (Options which are the exact values deriving from the 23 Variables)

First of all, four different networks were created, based on the founder's perception of Success. More specifically Levels 1-4 Networks were created. Table 3 provides the Success feeling of the founders, and the number of founders who belong to each network, thus sharing the same level of success.

Table 3: Definition of the Networks.

Networks – Level of Success	Success Estimation	No of Founders per Network
Level 1	Not Yet	15
Level 2	We will know in 1 Year	24
Level 3	We are on the right path	43
Level 4	Definitely Successful	12

Three different approaches were used in order to create relevant Graphs and analyze the networks using Graph Theory to draw conclusions. All rations are scaled in order to be consistent.

3. Key Findings and Discussion

The First Approach is the creation of Networks consisting and connecting the 94 Start-Up Founders with the 87 Final Options, and creating four Tree Networks (Levels 1, 2, 3 and 4). Both the Start-Up founders and the Final Options are represented as Nodes of the Network, and their connections are analyzed with the use of Graph Theory.

Table 4: Approach 1 – 2 Level Network between the Founders and End Options.

Key Rations	Network Level 1	Network Level 2	Network Level 3	Network Level 4
No of Founders	15	24	43	12
Density	88	101	123	82
Number of Connections	346	586	1070	286

The key findings of this approach are described on the following Table 4.

The first conclusion is that Network 4 (the most successful founders) appears to be the one with the lowest density and number of connections; this is interpreted that the most successful founders of Start-Up are more consistent – their paths to success is much more focused on specific patterns. Less successful founders, or founders who are seeking their way to entrepreneurial success have lower density scores, which means that they are less consistent in their choices; still experimenting across several business choices.

Top 10 nodes and respective values for Degree, Eigenvector, Authority and Pagerank Ratios for Level 1 Network are presented below (Table 5.).

Table 5: Top 10 nodes and respective values for Degree, Eigenvector, Authority and Pagerank Ratios for Level 1 Network are presented below

Degree	Scaled	Eigenvector	Value	Authority	Value	PageRank	Value
V-BB	0.93	V-BB	0.93	V-BB	1.00	V-BB	0.0150
V-DISY	0.87	V-DISY	0.85	V-DISY	0.92	V-DISY	0.0147
V-G1	0.80	V-G1	0.80	V-G1	0.86	V-G1	0.0143
V-PRS1	0.80	V-PRS1	0.80	V-PRS1	0.86	V-PRS1	0.0143
V-PSU1	0.60	V-CGC	0.61	V-CGC	0.65	V-PSU1	0.0132
V-PR1	0.60	V-PR1	0.60	V-PR1	0.64	V-PR1	0.0132
V-CGC	0.60	V-PSU1	0.59	V-PSU1	0,63	V-CGC	0.0132
V-APOC	0.53	V-APOC	0.55	V-APOC	0.59	V-STRAT1	0.0129
V-Exp4	0.53	V-Exp4	0.54	V-Exp4	0.59	V-APOC	0.0128

Table 6: Top 10 nodes and respective values for Degree, Eigenvector, Authority and Pagerank Ratios for Level 2 Network are presented below.

Degree	Scaled	Eigenvector	Value	Authority	Value	PageRank	Value
V-BB	0.88	V-BB	0.95	V-BB	1.00	V-BB	0.0150
V-G1	0.75	V-PR1	0.84	V-PR1	0.89	V-DISY	0.0147
V-PR1	0.75	V-G1	0.80	V-G1	0.85	V-G1	0.0143
V-CompTr	0.71	V-CompTr	0.78	V-CompTr	0.82	V-PRS1	0.0143
V-CGC	0.67	V-CGC	0.73	V-CGC	0.77	V-PSU1	0.0132
V-PSU1	0.67	V-PSU1	0.72	V-PSU1	0.76	V-PR1	0.0132
V-PRS1	0.67	V-PRS1	0.72	V-DISY	0.76	V-CGC	0.0132
V-DISY	0.67	V-DISY	0.72	V-PRS1	0.75	V-STRAT1	0.0129
V-APOC	0.62	V-STRAT1	0.70	V-STRAT1	0.73	V-APOC	0.0128

Table 7: Top 10 nodes and respective values for Degree, Eigenvector, Authority and Pagerank Ratios for Level 3 Network are presented below.

Degree	Scaled	Eigenvector	Value	Authority	Value	PageRank	Value
V-BB	0.88	V-BB	1.00	V-BB	1.00	V-CompTr	0.0149
V-PRS1	0.77	V-PRS1	0.90	V-PRS1	0.90	V-DISY	0.0148
V-DISY	0.74	V-DISY	0.87	V-DISY	0.87	V-PSU1	0.0147
V-PR1	0.72	V-PR1	0.84	V-PR1	0.84	V-PRS1	0.0147
V-G1	0.70	V-CGC	0.80	V-CGC	0.80	V-G1	0.0144
V-CGC	0.67	V-G1	0.78	V-G1	0.78	V-O3	0.0141
V-CAT	0.67	V-CAT	0.78	V-CAT	0.78	V-Ed2	0.0140
V-AP	0.65	V-AP	0.76	V-AP	0.76	V-PR1	0.0140
V-APOC	0.65	V-APOC	0.76	V-APOC	0.76	V-CVMa	0.0139

Table 8: Top 10 nodes and respective values for Degree, Eigenvector, Authority and Pagerank Ratios for Level 4 Network are presented below.

Degree	Scaled	Eigenvector	Value	Authority	Value	PageRank	Value
V-PSU1	0.83	V-PSU1	0.66	V-PSU1	1.00	V-CompTr	0.0149
V-PRS1	0.83	V-PRS1	0.66	V-PRS1	1.00	V-DISY	0.0148
V-DISY	0.83	V-DISY	0.64	V-DISY	0.97	V-PSU1	0.0147
V-CompTr	0.83	V-CompTr	0.63	V-CompTr	0.95	V-PRS1	0.0147
V-G1	0.75	V-G1	0.58	V-G1	0.88	V-G1	0.0144
V-Ed2	0.67	V-CAT	0.54	V-CAT	0.81	V-O3	0.0141
V-PR1	0.67	V-PR1	0.53	V-PR1	0.80	V-Ed2	0.0140
V-CVMa	0.67	V-Ed2	0.52	V-CVMa	0.79	V-PR1	0.0140
V-CAT	0.67	V-CVMa	0.52	V-Ed2	0.78	V-CVMa	0.0139

4. Comments

4.1. Comments at variable Level

The Second Approach is the creation of a 3-Level Network, with the Nodes consisting of the 94 Start-Up Founders (Level 1), the 23 Variables and the 87 end choices. Again, four networks were examined based on the Level of success (as defined in Table 1).

Table 9: Approach 2 – 3 Level Network between the Founders and End Options.

Key Rations	Network Level 1	Network Level 2	Network Level 3	Network Level 4
No of Founders	15	24	43	12
Density	110	123	145	104
Number of Connections	680	1146	2100	562

The first finding is that again, Level 4 Network appears to be that the most successful founders Level 4 tend to be more consistent in their choices.

However, the fact that we introduced a third level of analysis, with the definition of the Variables as Nodes, provides highlights of the importance of each variable, and more

specifically how the importance of each variable changes across the success based Networks.

In this case the following results are provided:

Table 10: Group (Variable) Results from Network Level 1 (top Variables)

Degree	Scaled	Eigenvector	Value	PageRank	Value
G-Achievement	2.93	G-Achievement	1.00	G-Achievement	0.0119
G-Disruption	2.53	G-Disruption	1.00	G-Disruption	0.0112
G-Comp_Advantage	2.27	G-B2B	0.76	G-Comp_Advantage	0.0107
G-Current_Reasons	2.27	G-Gender	0.70	G-Current_Reasons	0.0107
G-Key_Challenge	2.27	G-Pr_Surviving	0.70	G-Key_Challenge	0.0107
G-Age	2.00	G-Key_Challenge	0.69	G-Age	0.0102
G-Degree_Innovation	2.00	G-Comp_Advantage	0.67	G-Degree_Innovation	0.0102
G-Education	2.00	G-Current_Reasons	0.67	G-Education	0.0102
G-Experience	2.00	G-Previous_SU	0.62	G-Experience	0.0102

Table 11: Group (Variable) Results from Network Level 2 (top Variables)

Degree	Scaled	Eigenvector	Value	PageRank	Value
G-Comp_Advantage	3.17	G-Key_Challenge	1.00	G-Comp_Advantage	0.0129
G-Achievement	3.00	G-Achievement	0.99	G-Achievement	0.0126
G-Key_Challenge	3.00	G-Comp_Advantage	0.98	G-Key_Challenge	0.0125
G-Current_Reasons	2.75	G-Competition	0.77	G-Current_Reasons	0.0119
G-Competition	2.58	G-Current_Reasons	0.73	G-Competition	0.0116
G-Degree_Innovation	2.58	G-Degree_Innovation	0.65	G-Degree_Innovation	0.0116
G-Disruption	2.25	G-Disruption	0.59	G-Disruption	0.0110
G-Age	2.00	G-B2B	0.56	G-Age	0.0104
G-Education	2.00	G-Gender	0.55	G-Education	0.0104

A key finding is the fact that in Level 4 Network the top variables are the Competitive Advantage, the Reasons for Start-Up Formation while Achievement and Competition are ranked third and fourth, depended on the Ratio used.

However less successful founders tend to focus more on other variables; in fact it is highlighted that different Variables are more important for different Networks, defined by the levels of success. Achievement is the top variable for Network Level 3, while Network Level 2 provides a more complex situation; the top variables for Network Level 2 are Competitive Advantage, Achievements, Key Challenges, and Reasons for Start-Up formation. Level 1 Network presents an even more complex image, with the top variables been Achievement, Disruption, Competitive Advantage, Gender, B2B, Reasons for Start-Up Formation, and some differences between the Graph Theory Ratios.

Table 12: Group (Variable) Results from Network Level 3 (top Variables)

Degree	Scaled	Eigenvector	Value	PageRank	Value
G-Achievement	3.53	G-Achievement	1.00	G-Achievement	0.0156
G-Comp_Advantage	3.07	G-Comp_Advantage	0.49	G-Key_Challenge	0.0140
G-Key_Challenge	3.02	G-Key_Challenge	0.48	G-Comp_Advantage	0.0138
G-Current_Reasons	2.60	G-Current_Reasons	0.31	G-Degree_Innovation	0.0125
G-Degree_Innovation	2.60	G-B2B	0.30	G-Current_Reasons	0.0124
G-Disruption	2.23	G-Disruption	0.29	G-Disruption	0.0116
G-Competition	2.05	G-Degree_Innovation	0.28	G-Strategic	0.0110
G-Strategic	2.05	G-Pr_Surviving	0.26	G-Competition	0.0109
G-Age	2.00	G-Gender	0.24	G-Age	0.0108

Table 13: Group (Variable) Results from Network Level 4 (top Variables)

Degree	Scaled	Eigenvector	Value	PageRank	Value
G-Comp_Advantage	3.33	G-Comp_Advantage	1.00	G-Comp_Advantage	0.0119
G-Current_Reasons	3.00	G-Current_Reasons	0.80	G-Current_Reasons	0.0114
G-Achievement	2.67	G-Competition	0.74	G-Competition	0.0112
G-Competition	2.67	G-Achievement	0.70	G-Achievement	0.0111
G-Degree_Innovation	2.50	G-Degree_Innovation	0.65	G-Degree_Innovation	0.0108
G-Key_Challenge	2.50	G-Key_Challenge	0.64	G-Key_Challenge	0.0108
G-Disruption	2.17	G-Disruption	0.59	G-Disruption	0.0103
G-Age	2.00	G-Pr_Surviving	0.53	G-Age	0.0101
G-Education	2.00	G-Previous_SU	0.53	G-Education	0.0101

At Final Option Level (final outcome of the variable selected by founders) the top results for the Second Approach are as follows:

Table 14: Level 1 Network

Degree	Scaled	Eigenvector	Value	PageRank	Value
V-BB	0.93	V-DISY	0.60	V-BB	0.0150
V-DISY	0.87	V-BB	0.49	V-BC	0.0135
V-G1	0.80	V-G1	0.39	V-G1	0.0135
V-PRS1	0.80	V-PRS1	0.39	V-PRS1	0.0135
V-CGC	0.60	V-APOC	0.37	V-BBC	0.0133
V-PR1	0.60	V-AP	0.33	V-DISY	0.0131
V-PSU1	0.60	V-CGC	0.29	V-PR1	0.0117
V-APOC	0.53	V-PSU1	0.26	V-PSU1	0.0117
V-Exp4	0.53	V-PR1	0.25	V-CGC	0.0114

Table 15: Level 2 Network

Degree	Scaled	Eigenvector	Value	PageRank	Value
V-BB	0.88	V-CGC	0.51	V-BB	0.0138
V-G1	0.75	V-APOC	0.47	V-G1	0.0122
V-PR1	0.75	V-AP	0.44	V-PR1	0.0122
V-CompTr	0.71	V-CompTr	0.42	V-BC	0.0119
V-CGC	0.67	V-CABM	0.40	V-PRS1	0.0114
V-DISY	0.67	V-BB	0.37	V-PSU1	0.0114
V-PRS1	0.67	V-CAT	0.37	V-BBC	0.0113
V-PSU1	0.67	V-CGF	0.35	V-DISY	0.0111
V-APOC	0.62	V-G1	0.31	V-STRAT1	0.0111

Table 16: Level 3 Network

Degree	Scaled	Eigenvector	Value	PageRank	Value
V-BB	0.88	V-AP	0.57	V-BB	0.0129
V-PRS1	0.77	V-APOC	0.57	V-PRS1	0.0114
V-DISY	0.74	V-CAT	0.29	V-BC	0.0113
V-PR1	0.72	V-CGC	0.28	V-PR1	0.011
V-G1	0.70	V-AS100	0.27	V-DISY	0.0109
V-CAT	0.67	V-BB	0.23	V-G1	0.0107
V-CGC	0.67	V-DISY	0.19	V-BBC	0.0106
V-AP	0.65	V-PRS1	0.17	V-CVMa	0.0101
V-APOC	0.65	V-CIMP	0.16	V-PSU1	0.0101

Table 17: Level 4 Network

Degree	Scaled	Eigenvector	Value	PageRank	Value
V-CompTr	0.83	V-CAT	0.4	V-CVMa	0.0147
V-DISY	0.83	V-CompTr	0.37	V-BB	0.0145
V-PRS1	0.83	V-DISY	0.29	V-BC	0.0145
V-PSU1	0.83	V-PRS1	0.26	V-PRS1	0.0142
V-G1	0.75	V-PSU1	0.26	V-PSU1	0.0142
V-CAT	0.67	V-CABM	0.25	V-DISY	0.0138
V-CVMa	0.67	V-CAM	0.25	V-G1	0.0135
V-Ed2	0.67	V-APOC	0.24	V-BBC	0.0134
V-O3	0.67	V-CRO	0.24	V-CompTr	0.0130

The Third Approach provides an In-depth analysis of the final choices; in this case the Nodes of each Network represent the 87 end choices of the founders, and the connections between them are examined. Again, four networks were examined based on the Level of success (as defined in Table 1).

Table 18: Approach 3 – 2 Level Network between the Founders and End Options

Key Rations	Network Level 1	Network Level 2	Network Level 3	Network Level 4
No of Founders	15	24	43	12
Density	87	87	87	87
Number of Connections	1760	2085	2454	1487
Modularity	0.052	0.024	0.023	0.078
Global Transitivity	0.783	0.083	0.860	0.770
Edge Density	0.457	0.544	0.641	0.388

4.2. Comments at Network Level

By comparing the Global Transitivity Index – Network 4 has lowest Transitivity Index Value, confirming the more focused scenario of behavior and choices of the founders.

Regarding the Modularity Index – Network 3 has the highest Density and Transitivity Values (less focused scenario regarding the behavior and choices of Founders), and lowest Modularity (less connections between nodes).

Even though Networks 1 and 4 have similar values of Density and Transitivity Indexes they have different Modularity index values, which confirms that Network 4 represents a scenario of more focused behavior and options of the founders.

4.3. Comments regarding the absence of Variables

This section includes information that can be studied with the use of Graph and Network theory, since other methods of statistical analysis (Pearson and Spearman Coefficients) did not reveal the specific type of information. More specifically this section examines with the use of Graph Theory the impact of variables and parameters missing when a specific founder reaches a certain, pre-defined degree of perceived entrepreneurial success.

Key Findings at Variable Values Analysis: In order to analyze the indexes Degree, Eigenvector, Pagerank and Local Transitivity are used (weighted).

Eg. V-BB is first when measuring degree & eigenvector but has low local transitivity (0.68), while V-G2 ranks 51st at indexes degree & eigenvector (2.8 & 2.14) and highest transitivity (0.887).

This means that while most founders choose **V-BB, and the fact that V-BB is part of many scenarios (in combination with a number of other variable values). Therefore V-BB connects with a wider range of other nodes (variables values) -- as a part of less focused combinations**.

Therefore, even through Pearson Coefficient provides a value for B2B (and a reverse analogous correlation), in fact impact of B2B Value as a founder's choice greatly depends on its association with other nodes (variables values selected) and the way B2B variable connects with other variables.

Result 1 Network

Based on **Degree Index**

Variable Values V

V2014, V2013, V2012, V-2011, V2010, V-2009, V-2008, V-2007, V-2006

V-PSU3, V-PSU4, VPRS3, V-PRS4, VPR4, V-CO

Are not Connected to Result 1

Interpretation Founders of Start -- Ups founded in (2014, 2013, 2011, 2010, 2009, 2008, 2007, 2007), Founders who had previous business successes (PSU3 & PSU4), and companies surviving (PRS3, PRS4), as well as Other Challenges than Product Improvement, Financing or Getting Customers

Degree Index - Result 2 findings

Not Connected Nodes

A4, 2015, 2012, 2011, 2009, 2008, 2007, PRS4, PR2, STRAT2, CVUn

Founders who made these choices had not reached Result 2

Year of Company Formation => 2015, 2012, 2011, 2009, 2008, 2007,

3 Active Surviving Companies

Business Angels

Unclear Value to Customer

Based on **Degree Index -- Network Result 3**

Nodes PSU3, 2011, 2010, 2009, 2008, 2007, 2006, PRS3 do not connect with Result 3.

Founders with the following Choices not reached Level 3

Year of Company Formation 2011, 2010, 2009, 2008, 2007, 2006)

2 Previous Companies Formation (PSU3)

Both Companies are active (PRS3)

Based on **Degree Index -- Network Result 4** (Highest rate of Perceived success) the following apply

No Connection for Nodes A4: Ed1, F4, Exp1, PSU2, 2018, 2016, 2011, 2010, 2009, 2008, 2007, 2006, PRS4, STRAT4, CVUn, CVMin, NP

Founder who Reached Result 4 have not made the following

Year of Company Formation (2018, 2016, 2011, 2010, 2009, 2008, 2007, 2006)

Age over 46 years old

High School Graduates

4 Founders, 2 Active Companies, $ Previous Companies Established

Are not spin-off, They have understood the value offered to customer and not offer Low Value, not just an improved product version

5. Key Conclusions

Graph Theory offers a new tool for analysis of Entrepreneurship, and most specifically for identifying paths to enter (or avoid). Graph Theory offers the ability for multiple

Table 19: Top Variables Values – Network Level 1

Degree	Scaled	Eigenvector	Value	Hub	Value	PageRank	Value
V-BB	4.73	V-BB	1.000	V-BB	1.000	V-BB	0.0360
V-G1	4.67	V-DISY	0.928	V-DISY	0.928	V-DISY	0.0336
V-DISY	4.67	V-PRS1	0.878	V-PRS1	0.878	V-PRS1	0.0315
V-PRS1	4.53	V-G1	0.873	V-G1	0.873	V-G1	0.0314
V-AP	4.47	V-CGC	0.673	V-CGC	0.673	V-PR1	0.0239
V-PR1	4.40	V-PR1	0.660	V-PR1	0.660	V-PSU1	0.0237
V-CGC	4.40	V-PSU1	0.651	V-PSU1	0.651	V-CGC	0.0237
V-PSU1	4.33	V-APOC	0.610	V-APOC	0.610	V-STRAT1	0.0215
V-CVMa	4.27	V-Exp4	0.608	V-Exp4	0.608	V-Exp4	0.0213

Table 20: Variables Values not Used – Network Level 1

V-PSU3	0
V-PSU4	0
V-2014	0
V-2013	0
V-2012	0
V-2011	0
V-2010	0
V-2009	0
V-2008	0
V-2007	0
V-2006	0
V-PRS3	0
V-PRS4	0
V-Pr4	0
V-CO	0

Table 21: Top 10 Variable Values Network 2

Degree	Scaled	Eigenvector	Value	Hub	Value	PageRank	Value
V-BB	3.12	V-BB	1.000	V-BB	1.000	V-BB	0,0326
V-G1	3.08	V-PR1	0.897	V-PR1	0.897	V-G1	0,0278
V-APOC	3.08	V-G1	0.855	V-G1	0.855	V-PR1	0,0277
V-AP	3.04	V-CompTr	0.832	V-CompTr	0.832	V-CompTr	0,0261
V-CABM	3.04	V-CGC	0.783	V-CGC	0.783	V-CGC	0,0255
V-DISY	2.96	V-DISY	0.770	V-DISY	0.770	V-DISY	0,0252
V-CGC	2.92	V-PSU1	0.768	V-PSU1	0.768	V-APOC	0,0244
V-CompTr	2.92	V-PRS1	0.768	V-PRS1	0.768	V-PSU1	0,0243
V-CAT	2.92	V-STRAT1	0.749	V-STRAT1	0.749	V-PRS1	0,0243

analysis of combinations of factors / values compared to the end result an analysis far beyond Correlation of Variables.

Table 22: Variable Values Not Used – Network 2

V-A4	0
V-2015	0
V-2012	0
V-2011	0
V-2009	0
V-2008	0
V-2007	0
V-PRS4	0
V-PR2	0
V-STRAT2	0
V-CVUn	0

Table 23: Variable Values for Network 3

Degree	Scaled	Eigenvector	Value	Hub	Value	PageRank	Value
V-BB	1.79	V-BB	1.000	V-BB	1.000	V-BB	0.0320
V-APOC	1.79	V-PRS1	0.904	V-PRS1	0.904	V-PRS1	0.0280
V-G1	1.77	V-DISY	0.878	V-DISY	0.878	V-DISY	0.0274
V-CAT	1.77	V-PR1	0.850	V-PR1	0.850	V-PR1	0.0263
V-BC	1.74	V-CGC	0.814	V-CGC	0.814	V-CGC	0.0254
V-CRO	1.74	V-CAT	0.793	V-CAT	0.793	V-G1	0.0253
V-DISY	1.74	V-G1	0.790	V-G1	0.790	V-CAT	0.0252
V-NPAW	1.74	V-AP	0.775	V-AP	0.775	V-APOC	0.0244
V-PRS1	1.72	V-APOC	0.772	V-APOC	0.772	V-AP	0.0241

Table 24: Variable Values Not Used for Level 3

V-PSU3	0
V-2011	0
V-2010	0
V-2009	0
V-2008	0
V-2007	0
V-2006	0
V-PRS3	0

- At Network Level, there are differences (qualitative/quantitative) between the Networks

  (= levels of Perceived Success)

- At Variable Level, there is differentiation at the impact of each variable across Networks

  (= levels of Perceived Success)

Table 25: Variable Values for Network 4

Degree	Scaled	Eigenvector	Value	Hub	Value	PageRank	Value
V-DISY	5.50	V-PSU1	1.000	V-PSU1	1.000	V-PSU1	0.0313
V-CompTr	5.50	V-PRS1	0.997	V-PRS1	0.997	V-PRS1	0.0312
V-PRS1	5.33	V-DISY	0.964	V-DISY	0.964	V-DISY	0.0310
V-PSU1	5.25	V-CompTr	0.945	V-CompTr	0.945	V-CompTr	0.0299
V-Ed2	5.17	V-G1	0.887	V-G1	0.887	V-G1	0.0277
V-G1	5.08	V-CAT	0.815	V-CAT	0.815	V-CAT	0.0257
V-APOC	5.08	V-PR1	0.814	V-PR1	0.814	V-CVMa	0.0256
V-CAT	5.08	V-CVMa	0.809	V-CVMa	0.809	V-PR1	0.0252
V-CVMa	5.00	V-Ed2	0.787	V-Ed2	0.787	V-Ed2	0.0247

Table 26: Variable Values Not Used for Level 4

V-A4	0
V-Ed1	0
V-F4	0
V-Exp1	0
V-PSU2	0
V-2018	0
V-2016	0
V-2011	0
V-2010	0
V-2009	0
V-2008	0
V-2007	0
V-2006	0
V-PRS4	0
V-STRAT4	0
V-CVUn	0
V-CVMin	0
V-NP	0

- At Variable Value (End Option), Level Graph Theory offers a new level of analysis (Negative & Positive information) for each Network, examining combinations of values that contributed to a result.

Graph Theory offers a new tool for analysis of Entrepreneurship, and most specifically for identifying paths to enter (or avoid). Graph Theory offers the ability for multiple analysis of combinations of factors / values compared to the end result an analysis far beyond Correlation of Variables

- At Network Level, there are differences (qualitative/quantitative) between the Networks

  (= levels of Perceived Success)

- At Variable Level – there is differentiation at the impact of each variable across Networks

  (= levels of Perceived Success)

- At Variable Value (End Option) Level Graph Theory offers a new level of analysis (Negative & Positive information) for each Network, examining combinations of values that contributed to a result.

A key conclusions based on the findings, and more specifically by a closer examination of the V-BB value across the network was the following: as suggested earlier, while most founders choose V-BB (engage in Business to Business activities), and the fact that V-BB is part of many scenarios (in combination with a number of other variable values). Therefore V-BB as a variable value connects with a wider range of other nodes (variables values) – as a part of less focused combinations.

Therefore, even through Pearson Coefficient provides a value for B2B (and a reverse analogous correlation), in fact impact of B2B Value as a founder's choice (and as a specific variable value), as a measurement correlated to perceived success, greatly depends on its association with other network nodes (variables values selected) and the way B2B variable connects with other variables.

This is actually a suggestion that in fact it is hard to create steady models when studying entrepreneurship, which suggest that Variable X has a positive or negative correlation – or impact on another Variable Y (in the case studies, Success). Our findings suggest that rather the opposite may be true; the way that any Variable X (and more specifically the selected Values of Variable X) is connected (or correlated) with other Variables (and selected values of these Variables) provides a much clearer understanding of the phenomenon examined, in our case entrepreneurial success.

To this respect, in order to achieve a better understanding of entrepreneurial success, it is recommended that combination of variables (and specific values of these variables) can lead to safer conclusions; or – if Graph Theory is applied combined with more traditional analysis (such as Person Coefficient) researchers may reach safer conclusions.

6. Limitations and Further Research Considerations

The original contribution of Graph Theory is the fact that it focuses on the connections between nodes and – in our case – provides us a method to analyze the combination of (specific values of) the variables which form an entrepreneurial model, instead of focusing on the variables itself. In fact, Graph Theory examines the way that start-up entrepreneurs actually make their choices or the key elements of the specific business environment in which they operate.

Therefore instead of focusing to an individual variable focus shifts to the way variables (and more specifically the specific values of the variables) are combined, using a single criterion (variable) to define networks.

There is very limited published literature review and supporting evidence regarding the use of Graph Theory in Entrepreneurship research; as a direct result further research

is recommended to take place in the specific field by more researchers in order to provide further evidence.

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