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Conference Paper

Mathematical Model of Avian Influenza Epidemics with Vaccinating Poultry and Giving Treatment to Quarantined Infected Human

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Abstract

Avian Influenza epidemics have an impact on human life both in the health and economic fields. This epidemic is one of major problem that causes the infected human get hospitalization. Some action are needed to prevent and reduce the impact of this outbreak. The actions which were done are vaccination in poultry, burning infected poultry, quarantining and giving treatment infected humans.

Keywords: Avian Influenza; vaccination; Mathematical Model; Equilibrium Point; Stability

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1. Introduction

Avian influenza is one of epidemics that have a major impact on human life both in health and economic fields. Avian Influenza (AI) virus can be transmitted to humans (see [1], [2], [3]). Because of that, This epidemic causes the infected human get hospitalization. AI viruses is known as the flu that attacks poultry and mammals (see [4]). Aulia et al ([5]) stated that AI virus was transmitted through the air by coughing or sneezing, which will lead to aerosol containing the virus. This epidemic had a great influence like as profit loss on the economic aspects of poultry-related issues (see [6]).

Gooskens et al ([8]) stated that there was a mutation of influenza A virus that immune to oseltamivir. de Jong et al in [7] mentioned that the H5N1 subtype virus has immunity from drug. The mutation virus is contagious pathogenic and lethal for high-risk patients. H5N1 virus has high mutation ability. Because of that, this virus needs more attention to prevent becoming an outbreak in poultry and human. Kharis and Amidi in [9] developed mathematical model of avian influenza epidemics poultry vaccination in constant population. Model in [9] used assumption that success ratio of vaccination is 100%. In this paper, we will develop the model using vactination in poultry and treatment of infected human.

2. Methods

The method of this research is analysis method of deterministic mathematic model. Analysis method has some step. The first step is analysis of the equilibrium points existence. The next step is stability analysis of the equilibrium point. The next step is making simulation to clarify the result of the analysis. We do literacy study before developing model. In this activity, we determine facts and assumption which will be used to develop the model.

3. Mathematical Model

In this paper, we bounded the model by assuming human population is constant. In human, the birth rate has same value with natural death rate. We also assuming that the probability of success of vaccination has value p which is $0 \leq p \leq 1$ and the vaccinated poultry will be immune toward infection within the period of epidemics. We also assume that the death of infected poultry was only caused of infection. Transfer diagram of AI epidemic was given at Fig. 1.
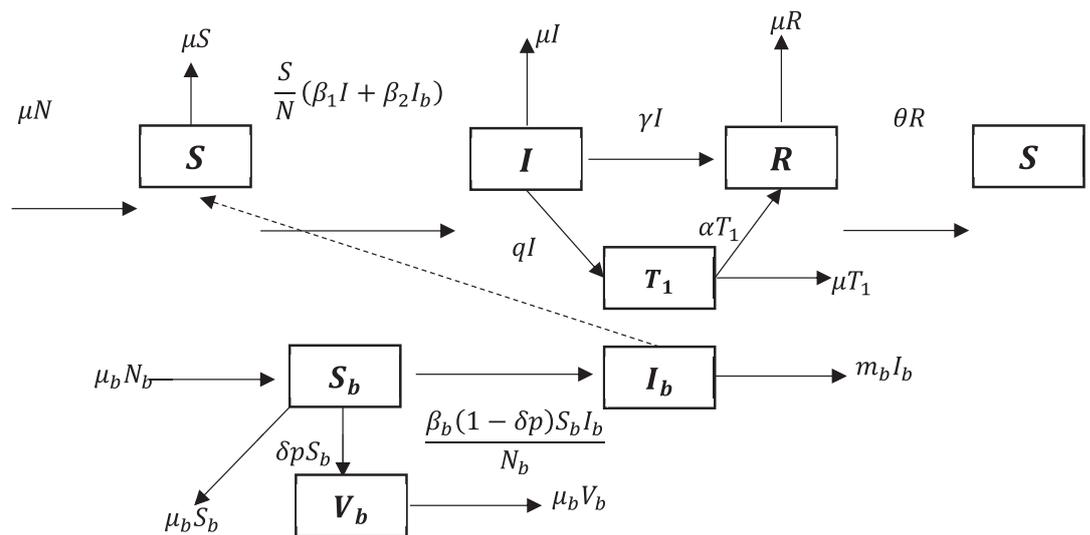
Figure 1: Transfer diagram of AI epidemic with vaccination and treatment.

Where N is the total number of human, S is total number of susceptible human, I is total number of infected human, R is total number of recovered human, T is total number of recovered human, N_b is the total number of poultry, S_b is total number of susceptible poultry, and I_b is total number of infected poultry, and V_b is total number of vaccinated poultry. The meaning of parameters in human population: μ means birth rate in human is assumed same with natural death rate, β_1 means probability of infectious contact

among human, β_2 means probability of infectious contacts between susceptible human and infected poultry, γ means recovery rate of infected human, q means proportion of infected human that are given treatment, α means recovery rate by treatment, and θ means immunity loss rate. The meaning of parameters in poultry population: μ_b means birth rate in poultry is assumed same with natural death rate, β_b means probability of infectious contact among poultry, μ_b means natural death rate in poultry, m_b means rate of death by infection in poultry, δ means the proportion of susceptible bird to be vaccinated every unit time, and p is success ratio of vaccination. From Fig. 1 we construct System (1).

$$\frac{dS}{dt} = \mu N + \theta R - \frac{S}{N} \left(\beta_1 I + \beta_2 I_b \right) - \mu S$$

$$\frac{dI}{dt} = \frac{S}{N} \left(\beta_1 I + \beta_2 I_b \right) - (\mu + \gamma + q) I$$

$$\frac{dT_1}{dt} = qI - (\mu + \alpha) T_1$$

$$\frac{dR}{dt} = \alpha T_1 + \gamma I - (\theta + \mu) R$$

$$\frac{dS_b}{dt} = \mu_b N_b - \left[\beta_b (1 - \delta p) \frac{I_b}{N_b} + \delta p + \mu_b \right] S_b \qquad (1)$$

$$\frac{dI_b}{dt} = \beta_b (1 - \delta p) \frac{S_b}{N_b} I_b - m_b I_b$$

$$\frac{dV_b}{dt} = \delta p S_b - \mu_b V_b$$

$$S + I + T + R = N$$

$$S_b + I_b + V_b = N_b$$

We assumed that $\beta_1 = \beta_2 = \beta$ and $m_b = \mu_b$.

Clear that $\frac{dN}{dt} = 0 \iff N = K > 0, K \in R$ and $R = N - I - S - T_1 = K - I - S - T_1$.

Clear that $\frac{dN_b}{dt} = 0 \iff N_b = K_b > 0, K_b \in R$ and $V_b = N_b - I_b - S_b = K_b - I_b - S_b$.

Hence, we get System (2).

$$\frac{dS}{dt} = \mu K + \theta \left(K - I - S - T_1 \right) - \beta \frac{S}{K} \left(I + I_b \right) - \mu S$$

$$\frac{dI}{dt} = \beta \frac{S}{K} \left(I + I_b \right) - \left(\mu + \gamma + q \right) I$$

$$\frac{dT_1}{dt} = qI - \left(\mu + \alpha \right) T_1$$

$$\frac{dS_b}{dt} = \mu_b K_b - \left[\beta_b \left(1 - \delta p \right) \frac{I_b}{K_b} + \delta p + \mu_b \right] S_b \tag{2}$$

$$\frac{dI_b}{dt} = \beta_b \left(1 - \delta p \right) \frac{S_b}{K_b} I_b - \mu_b I_b$$

Domain of System (2) is defined

$$\Gamma = \left\{ P \in R_5^+ \; P = \left(S, I, T_1, S_b, I_b \right) \text{ where } 0 \leq S + I + T_1 \leq K \text{ and } 0 \leq \; S_b + I_b < K_b \right\}$$

The existence of equilibrium points of System (2) is given in Theorem 1.

Theorem 1. *Let*

$$r_0 = \frac{\beta_b \left(1 - \delta p \right)}{\left(\mu_b + \delta p \right)}$$

and

$$R_0 = \frac{\beta}{\mu + \gamma + q}.$$

1. If $r_0 < 1$ and $R_0 < 1$ then System (2) has only one equilibrium point i.e. non endemic
   equilibrium point

$$P_0 = \left(S, I, T_1, S_b, I_b \right) = \left(K, \; 0, 0, \; \frac{\mu_b K_b}{\delta p + \mu_b}, 0 \right).$$

2. If $r_0 < 1$ and $R_0 > 1$ then System (2) has two equilibrium i.e P_0 and

$$P_1 = \left(S, I, T_1, S_b, I_b \right)$$

$$= \left(\frac{K \left(\mu + \gamma + q \right)}{\beta}, \; \frac{K \left(\mu + \theta \right) \left(\mu + \alpha \right) \left[\beta - \left(\mu + \gamma + q \right) \right]}{\beta \left[\left(\mu + \alpha \right) \left(\mu + \gamma + q + \theta \right) + q \right]}, \; \frac{K \left(\mu + \theta \right) q \left[\beta - \left(\mu + \gamma + q \right) \right]}{\beta \left[\left(\mu + \alpha \right) \left(\mu + \gamma + q + \theta \right) + q \right]}, \frac{\mu_b K_b}{\delta p + \mu_b}, 0 \right)$$

3. If $r_0 > 1$ then System (2) has Three equilibrium i.e P_0, P_1, and

$$P_2 = \left(S, I, T_1, S_b, I_b\right) = \left(\frac{K\left(\mu + \gamma + q\right)I_1}{\beta\left(I_1 + I_b\right)}, I_1, \frac{qI_1}{\mu + \alpha}, \frac{\mu_b\,K_b}{\beta_b\left(1 - \delta p\right)}, I_b\right)$$

where

$$I_b = \frac{K_b\left[\beta_b\left(1 - \delta p\right) - \left(\delta p + \mu_b\right)\right]}{\beta_b\left(1 - \delta p\right)}$$

and

$$I_2 = \frac{-A_1 - \sqrt{A_1^2 - 4.A_2.A_0}}{2A_2}$$

where

$$A_2 = -\beta\left[\left(\mu + \alpha\right) + q + \left(\mu + \alpha\right)\left(\mu + \gamma + q\right)\right]$$

$$A_1 = K\left(\mu + \alpha\right)\left[\beta\left(\mu + \theta\right) - \left(\mu + 1\right)\left(\mu + \gamma + q\right)\right] - \beta I_b\left[\left(\mu + \alpha + q\right) + \left(\mu + \alpha\right)\left(\mu + \gamma + q\right)\right]$$

$$A_0 = K\beta I_b\left(\mu + \alpha\right)\left(\mu + \theta\right).$$

Proof:

The equilibrium points were solution of System (3).

$$\mu K + \theta\left(K - I - S - T_1\right) - \beta\frac{S}{K}\left(I + I_b\right) - \mu S = 0$$

$$\beta\frac{S}{K}\left(I + I_b\right) - \left(\mu + \gamma + q\right)I = 0$$

$$qI - \left(\mu + \alpha\right)T_1 = 0$$

$$\mu_b K_b - \left[\beta_b\left(1 - \delta p\right)\frac{I_b}{K_b} + \delta p + \mu_b\right]S_b = 0 \qquad (3)$$

$$\beta_b\left(1 - \delta p\right)\frac{S_b}{K_b}I_b - \mu_b I_b = 0$$

From the fifth equation of System (3), we get

$$I_b = 0 \vee S_b = \frac{\mu_b\, K_b}{\beta_b\, (1 - \delta p)}.$$

Case of $I_b = 0$:

Substitute the value of I_b to the fourth equation, we get

$$S_b = \frac{\mu_b K_b}{\delta p + \mu_b}.$$

Substitute the value of I_b to the second equation, we get

$$I = 0 \vee S = \frac{K\,(\mu + \gamma + q)}{\beta}.$$

Case of $I = 0$:

Substitute the value of I to the third equation, we get $T_1 = 0$

Substitute the value of I_b, I, and T_1 to the first equation, we get $S = K$.

Hence, we get

$$P_0 = \left(S, I, T_1, S_b, I_b\right) = \left(K,\ 0, 0,\ \frac{\mu_b K_b}{\delta p + \mu_b}, 0\right).$$

Case of $I \neq 0$:

Clear that

$$S = \frac{K\,(\mu + \gamma + q)}{\beta}.$$

From the third equation, we get

$$T_1 = \frac{qI}{\mu + \alpha}.$$

Substitute to the first equation then we get

$$I = \frac{K\,(\mu + \theta)\,(\mu + \alpha)\,[\beta - (\mu + \gamma + q)]}{\beta\,[(\mu + \alpha)\,(\mu + \gamma + q + \theta) + q]}.$$

Clear that if $R_0 = \frac{\beta}{\mu + \gamma + q} > 1$ then $I > 0$.

Hence, we get if $R_0 > 1$ then

$$P_1 = (S, I, T_1, S_b, I_b)$$

$$= \left(\frac{K(\mu + \gamma + q)}{\beta}, \frac{K(\mu + \theta)(\mu + \alpha)[\beta - (\mu + \gamma + q)]}{\beta[(\mu + \alpha)(\mu + \gamma + q + \theta) + q]}, \frac{K(\mu + \theta)q[\beta - (\mu + \gamma + q)]}{\beta[(\mu + \alpha)(\mu + \gamma + q + \theta) + q]}, \frac{\mu_b K_b}{\delta p + \mu_b}, 0 \right)$$

The case of $I_b^* \neq 0$:

Clear that

$$S_b = \frac{\mu_b \, K_b}{\beta_b \, (1 - \delta p)}.$$

From the fourth equation, we obtain

$$I_b = \frac{K_b \left[\beta_b (1 - \delta p) - (\delta p + \mu_b) \right]}{\beta_b (1 - \delta p)}.$$

Let

$$r_0 = \frac{\beta_b (1 - \delta p)}{(\mu_b + \delta p)}.$$

Clear that for $r_0 > 1$, we got

$$\frac{\beta_b (1 - p)}{(\mu_b + \delta p)} > 1 \iff \beta_b (1 - \delta p) - (\delta p + \mu_b) > 0.$$

Hence, if $r_0 > 1$ then $I_b > 0$.

From the third equation of System (3), we obtained

$$T_1 = \frac{qI}{\mu + \alpha}.$$

From the second equation of System (3), we got

$$\beta \frac{S}{K} (I + I_b) = (\mu + \gamma + q) I \Leftrightarrow S = \frac{K(\mu + \gamma + q) I}{\beta (I + I_b)}.$$

From the first equation, we got

$$\frac{A_2 I^2 + A_1 I + A_0}{\beta (I + I_b)(\mu + \alpha)} = 0 \Leftrightarrow A_2 I^2 + A_1 I + A_0 = 0$$

Where

$$A_2 = -\beta \left[(\mu + \alpha) + q + (\mu + \alpha)(\mu + \gamma + q) \right]$$

$$A_1 = K(\mu + \alpha)[\beta(\mu + \theta) - (\mu + 1)(\mu + \gamma + q)] - \beta I_b[(\mu + \alpha + q) + (\mu + \alpha)(\mu + \gamma + q)]$$

$$A_0 = K\beta I_b(\mu + \alpha)(\mu + \theta)$$

Clear that $A_2 < 0$ and $A_0 < 0$.

Clear that $A_1^2 - 4.A_2.A_0 > A_1^2 > 0$ for every sign of A_2, so we got

$$I_1 = \frac{-A_1 - \sqrt{A_1^2 - 4.A_2.A_0}}{2A_2} > 0 \ and \ I_2 = \frac{-A_1 + \sqrt{A_1^2 - 4.A_2.A_0}}{2A_2} < 0.$$

Hence, we got

$$P_2 = (S, I, T_1, S_b, I_b) = \left(\frac{K(\mu + \gamma + q)I_1}{\beta(I_1 + I_b)}, I_1, \frac{qI_1}{\mu + \alpha}, \frac{\mu_b K_b}{\beta_b(1 - \delta p)}, I_b \right)$$

where

$$I_b = \frac{K_b[\beta_b(1 - \delta p) - (\delta p + \mu_b)]}{\beta_b(1 - \delta p)}$$

and

$$I_2 = \frac{-A_1 - \sqrt{A_1^2 - 4.A_2.A_0}}{2A_2}$$

where

$$A_2 = -\beta[(\mu + \alpha) + q + (\mu + \alpha)(\mu + \gamma + q)]$$

$$A_1 = K(\mu + \alpha)[\beta(\mu + \theta) - (\mu + 1)(\mu + \gamma + q)] - \beta I_b[(\mu + \alpha + q) + (\mu + \alpha)(\mu + \gamma + q)]$$

$$A_0 = K\beta I_b(\mu + \alpha)(\mu + \theta).$$

The Stability of equilibrium points of System (2) is given in Theorem 2.

Theorem 2. Let

$$r_0 = \frac{\beta_b(1 - \delta p)}{(\mu_b + \delta p)}$$

and

$$R_0 = \frac{\beta}{\mu + \gamma + q}.$$

1. If $r_0 < 1$ and $R_0 < 1$ then P_0 is locally asymptotically stable

2. If $r_0 < 1$ *and* $R_0 > 1$ then P_0 is unstable and P_1 is locally asymptotically stable.

3. if $r_0 > 1$ then P_0 and P_1 are unstable.

Proof:
The Jacobian matrix of System (2) was given below

$$J(P) = \begin{bmatrix} -\theta - \dfrac{\beta(I+I_b)}{K} - \mu & -\theta - \dfrac{\beta S}{K} & -\theta & 0 & -\dfrac{\beta S}{K} \\ \dfrac{\beta(I+I_b)}{K} & \beta\dfrac{S}{K} - (\mu+\gamma+q) & 0 & 0 & \dfrac{\beta S}{K} \\ 0 & q & -(\mu+\alpha) & 0 & 0 \\ 0 & 0 & 0 & -\left[\beta_b(1-\delta p)\dfrac{I_b}{K_b} + \delta p + \mu_b\right] & \dfrac{-\beta_b(1-\delta p)S_b}{K_b} \\ 0 & 0 & 0 & \beta_b(1-\delta p)\dfrac{I_b}{K_b} & \beta_b(1-\delta p)\dfrac{S_b}{K_b} - \mu_b \end{bmatrix}$$

For P_0, we got eigen values of $J(P_0)$:

$$\lambda_1 = -(\theta+\mu),\ \lambda_2 = \beta - (\mu+\gamma+q) = (\mu+\gamma)(R_0-1),\ \lambda_3 = -(\mu+\alpha),\ \lambda_4 = -(\delta p + \mu_b),$$

and

$$\lambda_5 = \frac{\mu_b}{\delta p + \mu_b}\left[\beta_b(1-\delta p) - (\delta p + \mu_b)\right] = \mu_b(r_0-1).$$

Hence, λ_1, λ_3, and λ_4 are negative, $\lambda_2 < 0$ if $R_0 < 1$ and $\lambda_2 > 0$ if $R_0 > 1$.

Clear that $\lambda_5 < 0$ if $r_0 < 1$ and $\lambda_5 > 0$ if $r_0 > 1$.

For P_1, we got characteristics equation of Matrix $J(P_1)$:

$$\frac{1}{(\mu+\gamma+\theta)}\left[(\lambda+\mu_b)(\lambda+\delta p+\mu_b)\left(\lambda - \frac{\beta_b(1-\delta p)\mu_b - (\mu_b+m_b+M)(\delta p+\mu_b)}{(\delta p+\mu_b)}\right)(A\lambda^2 + B\lambda + C)\right] = 0$$

where $A = \mu+\gamma+\theta,\ B = (\mu+\theta)(\mu+\theta+\gamma)+(\mu+\gamma)[\beta-(\mu+\gamma)]$, and

$$C = (\mu+\theta)(\mu+\theta+\gamma)[\beta-(\mu+\gamma)]$$

Clear that $A > 0$ for every R_0, $B > 0$ and $C > 0$ if $R_0 > 1$.

$$From\ (\lambda+\mu_b)(\lambda+\delta p+\mu_b)\left(\lambda - \frac{\beta_b(1-\delta p)\mu_b - (\mu_b+m_b+M)(\delta p+\mu_b)}{(\delta p+\mu_b)}\right) = 0$$

We got

$$\lambda_1 = -\mu_b,\ \lambda_2 = -(\delta p+\mu_b),$$

and

$$\lambda_3 = \frac{\beta_b \left(1 - \delta p\right) \mu_b - \left(\mu_b + m_b + M\right) \left(\delta p + \mu_b\right)}{\left(\delta p + \mu_b\right)}.$$

Hence, $\lambda_1 < 0$ and $\lambda_2 < 0$, $\lambda_3 < 0$ if $r_0 < 1$ and $\lambda_3 > 0$ if $r_0 > 1$.

From $A\lambda^2 + B\lambda + C = 0$ where $A > 0$ for every R_0, $B > 0$ and $C > 0$ if $R_0 > 1$, we got

$$\lambda_4 = \frac{-B - \sqrt{B^2 - 4.A.C}}{2A} \; and \; \lambda_5 = \frac{-B + \sqrt{B^2 - 4.A.C}}{2A}.$$

Because $A > 0$ and $C > 0$ then $B^2 - 4AC < B^2$ and because $B > 0$ then $Re\left(\lambda_4\right) < 0$ and $Re\left(\lambda_5\right) < 0$ for every sign of $B^2 - 4AC$.

For P_2, it was complicated to determine the eigen values of jacobian matrix $J\left(P_2\right)$ so we suspended it.

4. Conclusion and Discussion

From analysis above, we get the dynamic of mathematics model of AI epidemic with vaccination where this activity has succes ratio. We also got the reproduction number which can be used to determine whether the epidemic spread widely or vanish. For the next research, we propose to make the mathematics model for non constant population in both population.

Acknowledgments

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