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The Effectiveness of Concrete Representational Abstract Approach (CRA) Approach and Problem Solving Approach on Mathematical Representation Ability at Elementary School

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Abstract

The selection of the wrong approach will be able to make the effectiveness of the learning decrease, so the need for attention to the approach used by the teacher in his learning. If the approach adopted is not appropriate, there will be a form of boredom from students and tend to ignore the lessons given that ultimately the results obtained are less in line with expectations. The Concrete Representational Abstract we approach systematically and explicitly teaches students through three stages of learning: 1) concrete, 2) representation and 3) abstract. Teaching with CRA is a three-stage learning process in which students solve problems through the through concrete object manipulation followed by learning through pictorial representation of concrete object manipulations, ending with solving mathematical problems through abstract notation. Problem-solving approach, which is one of the learning approaches that can be applied in the learning process of mathematics. Many authors have attempted to explain what is the problem- solving approach for teaching mathematics. Ability of mathematical representation of students who get learning with CRA approach better than students who get learning conventional approach and to know the ability of mathematical representation of students who obtained learning with problem -solving approach better than students who obtained learning of conventional approach can be shown from the calculation of post-ANOVA test with Scheffe 'method and t-test.

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1. Introduction

Mathematics learning in each educational unit is expected to equip learners with the skills and abilities to face various problems of math and daily life. This ability is referred to as mathematical power. According to National Council of Teachers Mathematics (NCTM, 2000: 7) that includes; 1) Problem -solving ability, 2) reasoning ability, 3) communication ability, 3) ability to make connection, and 5) Representation ability.

One of the important roles of learning mathematics is to understand the abstract object of mathematics directly. At the age of elementary school children according to Piaget about 7 to 11 years, is a stage of concrete operational development, in this phase, the child can perform operations, and logical reasoning as long as reasoning can be applied to specific and concrete examples (Santrock, 2007: 50). To achieve student abstraction ability, it takes something bridging from concrete to the abstract. In its application, students can present ideas in the form of mathematical representations in the form of concrete models in the form of images or other forms.

Mathematics teaching does not merely convey information such as rules, definitions, and procedures to be memorized by students but teachers must actively involve students in the learning process. Active participation of students will strengthen their understanding of mathematical concepts. This is in accordance with the principles of constructivism is knowledge built by students themselves, both personally and socially, knowledge can not be transferred from teacher to student. Each student has a different way to construct his knowledge.

Ability representation is one of the standard mathematics learning processes that needs to be grown and owned by students. The standard of this process should be presented not separately with mathematical material. Unfortunately, representations are often taught and studied as if standing alone without any connection in mathematics. In fact, with the representation is expected to support students' understanding of mathematical concepts and their relationship in communicating mathematics, arguments, and understanding of one another, in recognizing the relationship between mathematical concepts (NCTM 2000: 206).

Learning is no longer viewed as a process of receiving information to be stored in students' memory obtained through practice repetition (practice) and reinforcement. However, students learn by approaching each new problem/task with prior knowledge, assimilating new information, and building their own understanding. Mathematics learning should be linked to the problems that arise in the real world of learners. Mathematics is not a lesson that only gives students knowledge about how to count and teach formulas, more than that math is a lesson that can train students to think critically, logically, carefully, and objectively and openly in everyday life. With these skills, students are expected to solve various problems both current and future. Thus learning in mathematics must be based on problems.

Problem solving approach is one part of mathematical curriculum. This approach allows students to gain experience to use the knowledge and skills they have gained in solving non-routine problems. In problem solving, teachers present problems that are

not routinely to be solved by the students. In this case, students are required to have the ability to synthesize knowledge, skills, and understanding so that in the end can solve the problems faced well. However, teachers face difficulties in teaching how to solve problems well, on the other hand students face difficulties how to solve problems given by teachers. These difficulties arise partly because searching for answers is seen as the only goal to be achieved.

Therefore, the selection of the wrong approach will be able to make the effectiveness of the learning decrease, so the need for attention to the approach used by the teacher in his learning. If the approach adopted is not appropriate, there will be a form of boredom from students and tend to ignore the lessons given that ultimately the results obtained are less in line with expectations. The CRA (Concrete Representational Abstract) approach systematically and explicitly teaches students through three stages of learning: 1) concrete, 2) representation, and 3) abstract. Teaching with CRA is a three-stage learning process in which students solve problems manipulation of concrete objects followed by learning through pictorial representation of concrete object manipulations, ending with solving mathematical problems through abstract notation (Witzel, 2005).

2. Literature Review

Approach mathematics learning by Erman Suherman, et al (2003: 74) is a way in which the teacher in the implementation of learning so that the concept to be achieved students can adapt to students. Each of the learning approaches has certain characteristics that are typical compared to other approaches.

2.1. Approach concrete representational abstract (CRA)

Concrete Representational Abstract Approach (CRA) systematically and explicitly teaches students through three stages of learning, namely (1) concrete, (2) representation, and (3) abstract. According to Kamii, Kirkland, & Lewis (2001) ”another term that has been used to describe this series of teachings is a concrete teaching sequence to semiconscious to the abstract. This approach is effective in teaching concepts and skills in primary schools to higher levels. Students use the ability of manipulation to construct their minds and understandings, from the manipulation of concrete objects and the experiences of students are directed toward achieving abstract mathematics. Jailani (2011) reveals that ”Mathematics is a study that starts with he assessment of

the parts that are very familiar (simple) in the direction of the unknown." To get to this unknown thing required a bridge from a real problem into formal mathematics.

The thinking ability of elementary school students still use the ability to think concretely, so that abstract mathematical concepts can be understood naturally needed an approach that can help their thought stage from concrete to abstract. Mathematics lessons are taught gradually starting from simple concepts to a more difficult concept. Witzel (2005) reveals that teaching with Concrete Representational Abstract (CRA) approach is a three-stage learning process in which students solve mathematical problems through the physical manipulation of concrete objects, followed by learning through pictorial representation of concrete object manipulations and ends with solving mathematical problems through abstract notation.

The process of solving a mathematical problem in principle can start from informal mathematics to formal mathematics. Sugiman & Kusumah (2010: 42) "The level of formal mathematics is placed as the ultimate goal of learning, not as the beginning of learning". Informal mathematics presents a variety of freedom of thought in acquiring ideas and pouring ideas into a mathematical solution. The Concrete Representation Abstract instructional sequence consists of three stages: concrete, representation, and abstract. In the concrete stage, the teacher begins by modeling each mathematical concept with concrete materials; Representational. In this stage, the teacher transforms the concrete model into a representational (semi concrete) level, which may involve drawing pictures; using circles, dots, and tallies; or using stamps to imprint pictures for counting; Abstract. At this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notations, and mathematical symbols to represent the number of circles or groups of circles.

2.2. Problem solving approach

Problem solving is the center of mathematics learning. This matter involves the acquisition and application of mathematical concepts and skills in a wide variety of situations, including non-routine, open and issues real-world problems. Problem solving in learning mathematics depends on five interrelated components, concepts, skills, processes, attitudes and metacognitions

Problem solving approach, which is one of the learning approaches that can be applied in the learning process of mathematics, also has certain characteristics. Many authors have attempted to explain what is the problem solving approach for teaching mathematics. The focus is on teaching mathematical topics through problem solving

and the investigation of an environmentally-oriented context characterized by teachers helping students build a deep understanding of mathematical ideas and engaging students in math problems: creating, conjecturing, exploring, testing, and verifying. NCTM (2000: 52) states that "solving problems is not only a goal of learning mathematics but also a major mean of doing so.... Problem solving is an integral part of mathematics learning, and so should not an isolated part of the mathematics program ". That is, problem solving is not only the goal of mathematics learning, but also a major tool for learning mathematics. Problem solving is an integral part of all the learning process of mathematics, so it should not be taken as a separate part of the mathematics teaching program.

In the process of mathematics, there is a skill of thinking and heuristics which one example is giving representation in the form of diagrams, tables, mathematical equations, and others. In addition, in problem solving approach, there are several strategies might be introduced to the students, one of which is the strategy of making a diagram or picture. This helps students to disclose the information contained in the problem so that the relationships between components in the problem can be seen clearly. Creating an image or diagram is one example of visual representation. Thus it can be concluded that in problem solving requires the ability of representation in the process.

2.3. Ability of mathematical representation

The purpose of learning mathematics has changed, not only emphasize on improving learning outcomes, but also expected to increase various capabilities. The inclusion of NCTM's representation capabilities on the competencies that students must possess, indicates that the ability of mathematical representation is seen as a fundamental process for developing students' thinking skills and parallel to other process competencies. Representative ability provides a very important role in mathematics learning because representation is the way students use to communicate ideas, ideas, or answers to a problem. Rowland (2009: 42) reveals that "Physical and pictorial representations are widely used in order to support the teaching and learning of mathematics, acting as intermediaries between the concrete to abstract". The use of mathematical representation capabilities helps teachers in learning mathematics as the intermediate from the concrete to the abstract, where mathematics is abstract so that necessary understanding of symbols and manipulations is something that is important in mathematics.

A complicated problem would be simpler if using the appropriate representation of the problem. This is reinforced by Gursel (2011: 1) states that the importance of using representations in mathematics education can be explained by the contribution it makes to the development of understanding and intitutional prespective. With representational skills it helps in understanding and perception of students in solving math problems.

The mathematical representation of students is necessary in understanding the concept as well as solving math problems. Abstract mathematics makes it difficult for students to understand mathematics. This is in line with Schnotz's opinion, et al, (2010: 13) "If the right representation of the problem has been found, the solution has been immaturely evident" meaning that if it has found the correct representation of the problem, the solution becomes clearer. A problem that is considered complicated and complex, can be simple if the strategy and utilization of mathematical representation used in accordance with the problem. Along with that Godin (2002: 209) representation is a configuration (form or arrangement) that can describe, represent, or represent something in a way. Through the ability of representation will trigger the emergence of the ability to link mathematical ideas in topics or with everyday situations, or bring the ability of students to reason and communicate.

The use of representations helps students make concrete math ideas more concrete. Lesh, Post, and Behr (Hwang, et al., 2007: 192) divide the representations used in mathematics education in five types, including representations of real-world objects, concrete representations, representations of arithmetic symbols, oral or verbal language representations and image or graph representations.

3. Material & Methodology

This type of research is a quasi-experimental research, because some variables can not be controlled like full control of pure experimental research. The main feature of experimental research is the existence of manipulated treatment variables. In this study not all variables can be controlled considering the ability of mathematical representation can be influenced by many factors, such as the influence of the family environment and school environment. The design used in this study is the pretest-post test control group design. Type of sampling technique used is Cluster Random sampling. The design of this study was Nonequivalent Control Group pretest-posttest Design. In this case there are three groups are one group control and two experimental groups. The chart of the research design is as follows. Nonequivalent Control Group pretest-postest Design

Group	*Pretest*	Treatment	*Postest*
KE1	O1	X1	O2
KE2	O3	X2	O4
KK	O5		O6

where:

KE1: First Experimental Group

KE2: Second Experimental Group

KK: Control Group

O1: *Pretest* First Experimental Group

O2: *Pos-test* First Experimental Group

O3: *Pretest* Second Experimental Group

O4: *Postest* Second Experimental Group

O5: *Pretest* Control Group

O6: *Pos-test* Control Group

X1: Treatment at First Experimental Group with Concrete Representational Abstract approach

X2: Treatment at First Experimental Group with Problem Solving approach

Muhammadiyah 1 Magelang in the second semester of academic year of 2017/2018. The students of the three classes of the four grades were drawn using simple cluster random sampling technique to select the members of the sample. Among the three classes of the sample, two classess were determined as experimental groups and a class as control group. There were 30 students as the xperimental group 1 and 2 and 33 students for the control group.

Research data obtained from the data of students' mathematical representation ability. Data collection techniques are tests, observations, and documentation. The tests were given to all three classes before and after treatment, student activity observation was conducted during the learning process, and documentation was used to obtain evidence during the experiment and control class. Instrument of data collection in this research is test in the form of description problem to measure student mathematical representation ability. Problems are made and adapted to indicators of students' mathematical representation abilities.

4. Results and Discussion

After normality test and homogeneity test of variance at pretest data, it is concluded that the data come from normal and homogenous distribution. Furthermore, the one-way ANOVA test was conducted with unequal cell with 5% significance level and obtained the conclusion that the three samples used had the same mean. Then for the data posttest also conducted normality test and homogeneity test of variance which result

same, that is data come from normal and homogeneous distribution. Furthermore, the ANOVA test is done to test the first hypothesis is to know whether the learning model used to give impact on the ability of representation mathematically or not students. For the calculation of statistical test of ANOVA using SPSS, it can be seen in Table 1.

Table 1: ANOVA Test Result.

ANOVA					
Value					
	Sum of Square	Df	Mean Square	F	Sig.
Between Groups	4910.61	2	2455.30	6.39	.003
Within Groups	24976.61	65	385.225		

Based on Table 1 above it is known that the significance value is 0.003. From the basis of decision making, rejected if the value So it can be concluded that the three samples used have unequal mean or also can be concluded that the use of CRA approach, problem solving approach and conventional learning affect the ability of students' mathematical representation.

While the second and third hypothesis test is to know the ability of mathematical representation of students who get learning with CRA approach better than students who get learning conventional approach. To know the ability of mathematical representation of students who obtained learning with problem solving approach better than students who obtained learning of conventional approach, can be shown from the calculation of post-ANOVA test with Scheffe 'method and t-test. For the results of calculation post ANOVA test using SPSS can be seen in table 2.

Table 2

Multiple Comparisons						
Dependent Variable: Value						
Scheffe						
(I) Kode	(J) Kode	Mean Diff (I-J)	SE	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
E1	E2	4.962	5.786	.694	-9.53	19.46
	K	20.008*	5.786	.004	5.51	34.50
E2	E1	-4.962	5.786	.694	-19.46	9.53
	K	15.045*	5.910	0.046	.21	29.85
K	E1	-20.008*	5.786	.004	-34,50	-5.51
	E2	-15.045*	5.910	.046	-29.85	-24

*The mean difference is significant at the 0.05 level

Based on Table 2, it is known that the significance value of the comparison of experimental class one and control class is 0,004 and the significance value of comparison of experiment class two with control class is 0,046. So it can be concluded that, the ability of mathematical representation of students who obtained learning with CRA approach better than students who obtained learning mathematics using conventional learning approach and students who obtained learning with problem solving approach, the ability of representation is also better than students who received learning conventional approach. While the students who obtained learning with CRA approach compared with students who obtained learning with problem solving approach, have the same representation ability, it can be seen from the value of the comparative significance of the experimental class one and the experimental class two of 0.694. This result is similar to the calculation analysis using Ms.Excel.

Furthermore, for Next for t-test calculations using SPSS, the second hypothesis obtained values. From the basis of decision making, rejected if the value. So it can be concluded that the ability of mathematical representation of students who obtain learning with CRA approach better compared with students who obtained learning mathematics using conventional learning. While the t-test statistic using SPSS for the third hypothesis, the results obtained value. From the base of decision making, rejected if. Then it can be concluded that the ability of mathematical representation of students who get a problem solving approach is better than students who obtain learning mathematics using conventional learning. This result is similar to the calculation analysis using Ms.Excel.

5. Conclusion

The use of learning with CRA approach, problem solving approach and conventional learning have an effect on the ability of mathematical representation of fourth grade students of SD Muhammadiyah 1 Magelang. The ability of mathematical representation of fourth grade students of SD Muhammadiyah 1 Magelang who get learning with CRA approach better than student gain mathematics learning using conventional learning. Ability of representation of elementary school students who gain learning with problem solving better than students who gain learning mathematics using conventional learning.

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