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Valuation of Actuarial Liability Using Markov

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Abstract

In retirement planning, normal retirement benefits are the main benefit and also usually contain additional benefits that are paid when the participants experience disability before retirement or death before or after retirement. The possibility of that situation makes more and more the need to calculate the Pension Fund. Therefore, a calculation model is required that can describe all of those possibilities. A Retirement planning is a life-term insurance contract with the normal costs as premiums and the actuarial liabilities as reserves in the pension fund. This article will develop a calculation of prospective actuarial liability using a calculation of life insurance reserves through Markov chain. The calculation of actuarial liability is only for individuals with same age and situation. The rates and transition probabilities are also required. The amount of benefits in retirement planning usually depend on the time of an event so the equation model for these benefits are also required. Moreover, the pension fund can calculate the actuarial liability of participants with a disability pension condition. In this study, the actuarial liability is calculated using two conditions, namely, a defined cost and defined benefit. In the calculation of actuarial liability with the defined costs, pension fund requires initial funding. Furthermore, the rate of increasing in actuarial liability with the defined benefits will be greater than the defined costs. Thus, the results of this calculation can contribute to the Pension Funding determining the company policies.

Keywords: pension fund, actuarial liability, Markov chains, disability pension, defined costs, defined benefit

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1. Introduction

Everyone who works, particularly as the head of the family is expecting to meet the needs of a decent life for his family. However, it is inevitable that each individual will experience old age and according to the rules causing these individuals to stop working. One of the effects will arise, namely the lack of income earned by the individual. However, in 1992 the government has issued Law of the Republic of Indonesia Number 11 of the Pension Fund to ensure continuity of individual income after retirement. In

the pension plans, the normal retirement benefits as the main benefit and usually also contain additional benefits paid during a total disability prior to retirement, death before retirement, or death after retirement [1]. Additional benefits the more possible states experienced by participants who need to be taken into account by the Pension Fund. Thus, it is required that the model can reflect a variety of possible circumstances experienced by the participants. This model is known as multiple state model [4]. In this study, will be studied three states, namely the state of active work, disability pensions, and death. In funding pensions, benefits participants who died before and after retirement have a different major. Therefore, we need more specialized models, the Markov chain model [15]. This model has the assumption that the next state depends only on the present situation [4].

How to tell the Pension Fund met in a state fund that is a wealth of funding, not less than actuarial liabilities. Therefore, the calculation required actuarial liability. Actuarial liabilities can be calculated with the two approaches, namely prospective and retrospective approach. Actuarial liabilities with a prospective approach is defined as the difference between the present value of the benefits to the present value of the future payment of dues [1]. Penelitian will discuss about the calculation of actuarial liabilities with a prospective approach for pension funds need to calculate the expected obligations of individuals reach retirement age in order to determine the funds that need to be prepared pension fund. This study was to identify the process of calculating the actuarial liabilities of the Pension Fund as a stochastic process, in particular through the Markov chain.The process of calculating actuarial liabilities through Markov chain prior to solve differential equations Thiele. In the equation, will be estimated rate and transition opportunities with duration method [5]. Furthermore, it can determine the general equation of benefits and contributions to the Pension Fund. Thus, it can calculate the actuarial liabilities using differential equations Thiele.

2. Model Actuarial Liabilities Through Markov Chain

Here is an illustration of the system of payment of contribution employer pension funds.

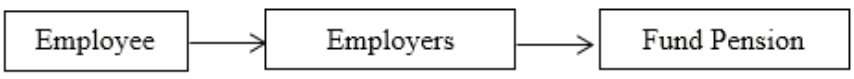

Figure 1: Payment of employee contributions through the withholding of employer. Employers depositing the contribution to the Fund Pension.

Retirement benefits are periodic payments paid to participants at the time and in the manner specified in the regulations of the Pension Fund (Law Decree No. 11 of 1992). Some types of retirement benefits, namely:

1. Normal Retirement benefits, that is, pension benefits for participants who begin to be paid to the participants retire upon reaching normal retirement age
2. Retirement benefits, pension benefits for participants are paid when participants retire at a certain age before the normal retirement age. Usually ten years before normal retirement age.
3. Disability Retirement benefits, that is, pension benefits for participants who paid if the participant becomes disabled.

For example, a stated age of individuals included in the funding and r denote the normal retirement age of the individual. The following will be given an illustration of the possibility of state benefits that occur in pension funding.

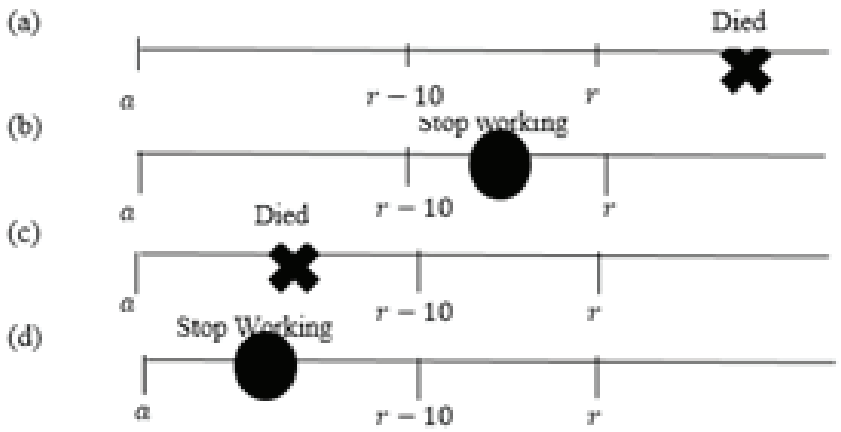

Figure 2: Individuals (a) may benefit pension is normal because it is still active until normal retirement age, the individual (b) can benefit early retirement since decided to stop working 10 years before normal retirement age, and individuals (c) may benefit before retirement, as well as individuals (d) can benefit disability pension due to cease working by reason of disability.

Definition II.1

Actuarial liabilities (Actuarial Liability) is defined as the difference between the present value of benefits (Benefit) to the present value of the payment of dues (Normal Cost) in the future [1].

Definition II.2

Let Z = {Z (*t*), *t* ≥ o} is a stochastic process. Z (*t*) is a random variable that is declared as a state that is occupied by an individual or group of individuals at time *t*. The set of events Z (*t*) that may occur is called the state space of the model Z. Markov chain is a stochastic process with discrete state space. In a Markov chain there are two parameter space of time, that is, the time parameters of discrete and continuous time parameter. In the pension funding, will use continuous time or age.In this pension funding model consists of three circumstances, namely the active state (o), state disability pension (1), and a state of death (2). The following illustration of retirement funding model as follows.

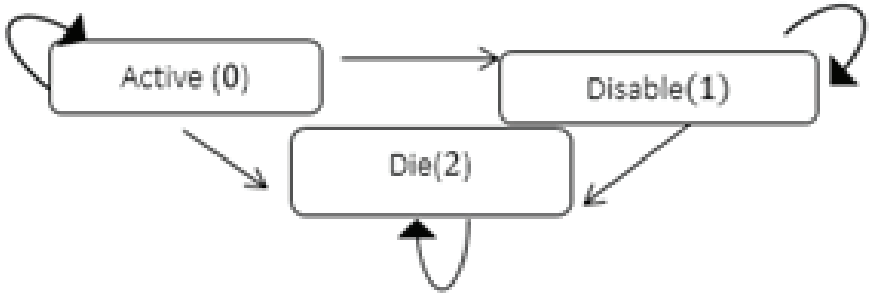

Figure 3: In the Pension Fund, the contributions paid in the active state. Benefits will be paid when the participants changed circumstances, that is, disability or death [6].

Active state is a state of a person is still healthy and still actively working. Up to this condition, dues paid during a state disability pension is a state of a person who is unable to work because of a disability back.

Definition II.3

Differential Thiele. Differential Equations Thiele stated as follows:

$$\frac{d}{dt}V_i(t) = V_i(t)\,r(t) - b_i(t) - \sum_{j=0, j\neq i}^{n} \mu_{ij}(x+t)\left[b_{ij}(t) + V_j(t) - V_i(t)\right], \qquad (1)$$

for $i = 0, 1, 2$ and $t \geq 0$ with

$V_i(t)$ states actuarial liability of participants in a state i when t

$r(t)$ expressed the interest rate each year

$b_i(t)$ state benefits are paid to the state i when t

$\mu_{ij}(t)$ through the rate of displacement from state i to state j during t

$b_{ij}(t)$ state benefits paid for the transfer of state i to state j during t [3]

Thus, the obtained solution of equation (II.1) are as follows.

$$V_i(t) = \sum_{j=0}^{n} \int_t^n exp\left(-\int_0^{s-t} r d\tau\right) p_{ij}(x+t, x+t+s-t)\, b_j(s)\, ds$$
$$+ \sum_{j=0}^{n} \sum_{l=0, l\neq j}^{n} \int_t^n \exp\left(-\int_0^{s-t} r d\tau\right) p_{ij}(x+t, x+t+s-t) \times \mu_{jl}(x+s)\, b_{jl}(s)$$

3. Case Study

For example, individuals born in February 1983 B and B is a permanent employee at one of the private university in Jakarta since March 2012. B directly following the pension fund belonging to the University. So that working lives will be passed if the B reached the normal retirement age (56 years) is 27 years and 1 month. Basic earnings amounted to 1.3079 million B/bulan. Assuming that no increase salaries and B are married and an annual interest rate of 10% applies then the pension contributions and benefits will be calculated. Next major actuarial liability of the individual person B in the active state in the next year until the age of 56 years.The following illustration will be given the possibility of payment of dues and benefit from individual B.

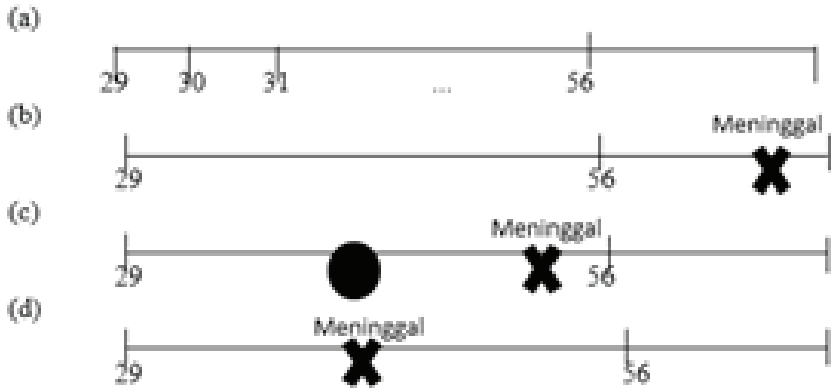

Figure 4: State (a) represents the state of the B still active and required pay dues until the age of 56 years. The state of (b) is Person B state is still active until the age of 56 years. In these circumstances, B normal retirement benefit and die before the age of 80 years. Circumstances (c) represents the state of the B disability thus granted disability benefits and then die before the age of 80 years. State (d) represents the state of the B died before age 56 years.

Before calculating the actuarial liabilities that must be owned pension fund, it will be given first regulatory contributions and benefits obtained by the participants as follows:

1. Normal participants dues of 5% of basic income.

2. Normal Retirement benefits are pension benefits for participants who began to be paid upon retirement participant reaches normal retirement age. Great benefits provided is 2.5% × Work Period × basic income
3. Disability Retirement benefits are pension benefits for participants who paid for the participants to experience disability and therefore cannot work again. Great benefits provided great benefits is the same as the normal pension
4. Large pension benefits because dies before the pension is 60% of the current value to be paid at the time of retirement of participants.
5. In the case of pensioner dies, the pension benefits are paid at 60% of the pension benefits received by pensioners
6. Assumed for the payment of benefits due to disability and death participant will be paid at once. So the benefits must be multiplied by a factor of at once (FSK) in accordance with the Pension Fund.

Under these provisions, the obtained contributions and benefits as follows:

1. Normal dues Participants: 5% × 1.3079 million/month = 65 395/month = 784 740/years
2. Normal Pension Benefit done by way of annuity payments.2.5% × 27 1/12 × 1.3079 million = 885 557/month = 10,626,687/year
3. Disability Retirement benefits done by the annuity payment, which is the same as the Normal Retirement Benefits.
4. Death Benefits before a disability pension is done by a lump sum payment, which is 60% × FS × Present Values × Normal Retirement Benefit.

   The following chart obtained great benefits dies before retirement age for every possible case.

   Model general equation are as follows:

$$b_{02}(s) = 6.856.101,714 + 736.736,872s + 10.158,111s^2 + 1835,611s^3, \quad 0 \leq s \leq 27$$

5. Death benefits after normal retirement done by way of a lump sum payment of 60% × FS × Normal Retirement benefits.

   The following will be given a chart of the benefits passed after the normal retirement age for every possible case.

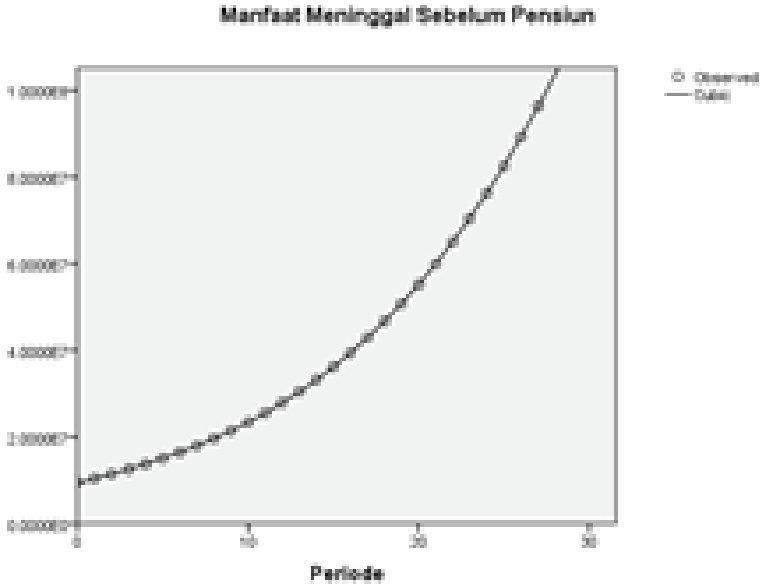

Figure 5: The longer the participants live, the greater the benefits which will be acquired by the participant when he died before pension.

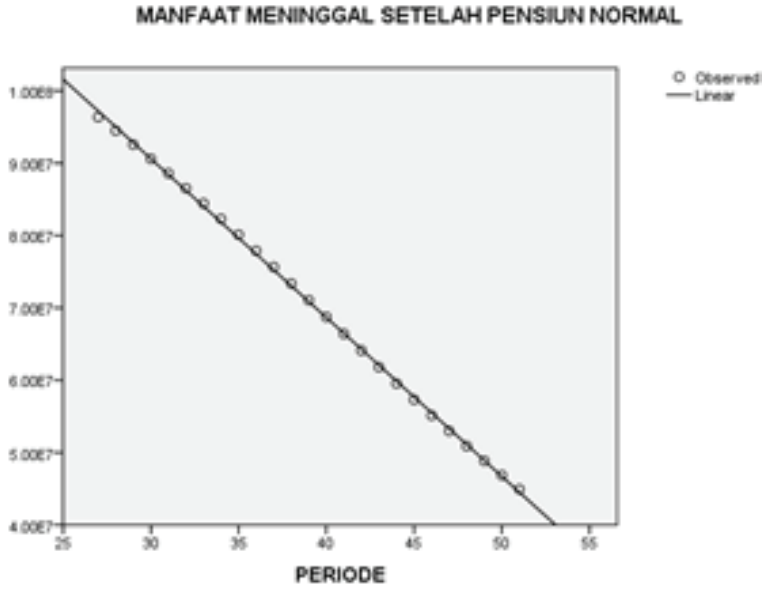

Figure 6: The longer the participants live, the greater the benefits acquired by the participant when s/he dies before the normal retirement.

Retrieved general equation is as follows:

$$b_{02}(s) = 156.500.000 - 2.195.232,66, \quad 27 < s \leq 51$$

6. Died after retirement disability benefits is done by a lump sum payment, that is, 60% × FS × Normal Retirement benefits.

The following will be given a large graph died after retirement disability benefits for every possible age occurs.

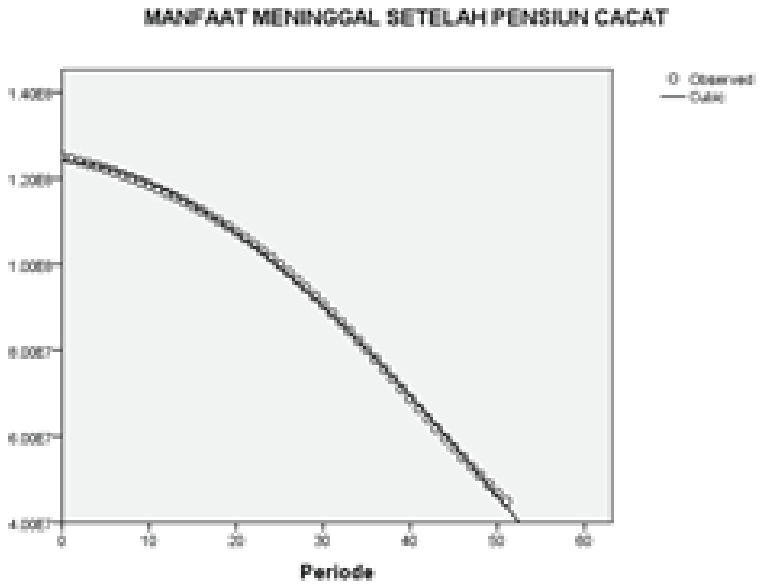

Figure 7: The longer the participants live, the less benefit obtained. This is because before dying participants already get retirement benefits, either normal retirement or disability pension.

General equation model is obtained as follows:

$$b_{12}(s) = 124.000.000 - 114.760,498s - 41.034,124s^2 + 243,189s^3, \quad 0 < s < 51$$

Thus, it can be obtained year-end actuarial liabilities for individuals with an active state at the age of 29 years is

$$\begin{aligned} V_0(1) = \Bigg[& \int_1^{27} \exp\left(-\int_1^s rd\tau\right) p_{00}(30, 29+s)\, b_0^-(s) ds \\ & + \int_{27}^{51} \exp\left(-\int_1^s rd\tau\right) p_{00}(30, 29+s)\, b_0^+(s) ds \\ & + \int_{27}^{51} \exp\left(-\int_1^s rd\tau\right) p_{00}(30, 29+s)\, \mu_{02}(29+s)\, b_{02}(s) ds \\ & + \int_1^{51} \exp\left(-\int_1^s rd\tau\right) p_{01}(30, 29+s)\, b_1(s)\, ds \\ & + \int_1^{51} \exp\left(-\int_1^s rd\tau\right) p_{01}(30, 29+s)\, \mu_{12}(29+s)\, b_{12}(s)\, ds \\ & + \int_1^{27} \exp\left(-\int_1^s rd\tau\right) p_{00}(30, 29+s)\, \mu_{02}(29+s)\, b_{02}(s)\, ds \Bigg] \end{aligned}$$

Can be obtained actuarial liability of participants at the end of the year with the active state at the age of 29 years amounted to 6,011,928, which means that the Pension Fund requires funding of 6,011,928 for the end of this year.

Furthermore, with the help of Maple software, the obtained actuarial liability of participants up to the age of 56 years are as follows:

Table 1: Actuarial liabilities with defined contributions for any period up to the age of 56 years.

Age $(x+s)$	Period (s)	Actuarial Liability $V_0(s)$
29	0	5,538,356
30	1	6,011,928
31	2	6,557,511
32	3	7,183,995
33	4	7,901,017
34	5	8,718,988
35	6	9,649,104
36	7	10,703,348
37	8	11,894,470
38	9	13,235,962
39	10	14,742,003
40	11	16,427,394
41	12	18,307,467
42	13	20,397,973
43	14	22,714,938
44	15	25,274,504
45	16	28,092,725
46	17	31,185,342
47	18	34,576,518
48	19	38,253,537
49	20	42,256,416
50	21	46,587,748
51	22	51,256,819
52	23	56,270,578
53	24	61,632,882
54	25	67,343,948
55	26	73,399,705
56	27	79,791,082

The following will show a graph of the results table above for each period as follows:

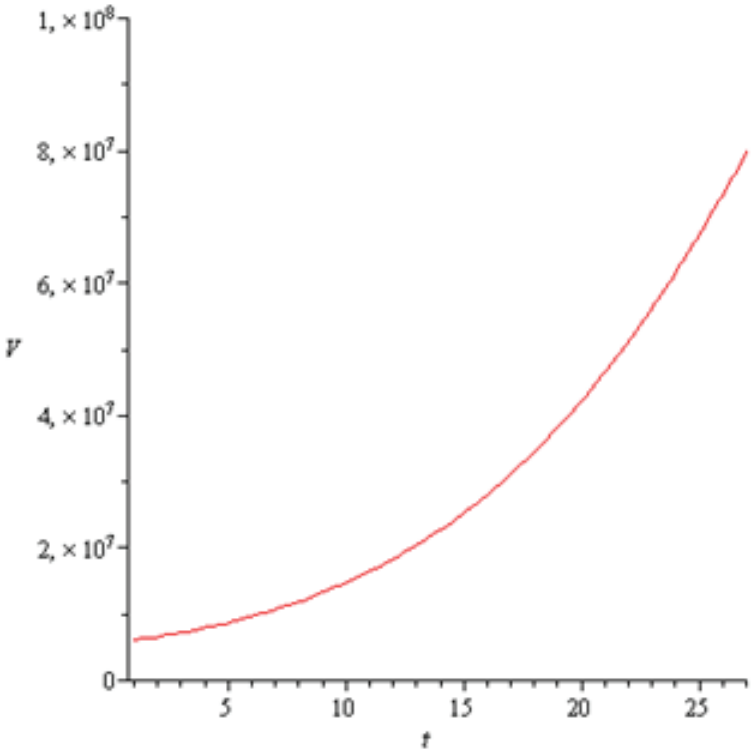

Figure 8: Graph actuarial liability for each period. Actuarial liabilities participants growing each period as time increases.

Based on the aforementioned graph, it appears that the increasing age of the participant, the participant actuarial liabilities that must be owned by the Pension Fund to increase. This is due to increasing age, the participant will be reduced time to pay dues. Furthermore, the results obtained actuarial liability of participants of each period, the pension fund can make policy in investing the funds that participants fulfilled the obligations of each period.

4. Conclusion

Calculate the actuarial liabilities on pension funds through required Markov Chain transition rate and transition opportunities. In addition, the calculation of actuarial liabilities through Markov chain can only calculate the actuarial liability to individuals of the same age. The pace of the transition can be estimated using the duration method that has been developed by Rickard Gunnvald (2014). In estimating the rate required a lot of data in the time range long enough. This will make it easier to determine the model of the rate of transition. Calculating actuarial liabilities participants through Markov chain can be calculated when an active participant or a state disability pension. In other

words, the calculation of actuarial liability through Markov chain can calculate from the participants with a variety of circumstances.

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