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# Developing a Learning Trajectory on the Conceptual Understanding Procedures of Pythagorean Theorem Material for Junior High School Students

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**Abstract.**

This research activity is motivated by students' learning obstacles that hinder their understanding of the Pythagorean theorem material. Appropriate learning design can be a solution to overcome student learning obstacles. To create high-quality learning, teachers must use careful planning, one of which is considering students' learning trajectories. Selecting the Conceptual Understanding Procedures (CUPs) learning model can help students become active learners and make learning more student-centered. Thus, it is necessary to develop a trajectory which aims to establish its feasibility based on the assessment of material experts and media experts, its practicality based on students' responses, and its effectiveness based on classical learning completeness. This study used the Gravemeijer & Cobb design research model which consists of three phases: preliminary design, experiment, and retrospective analysis. The results of this study are: (1) Validation in the material expert assessment aspect is worth 3.81 with a very valid category in terms of content feasibility, presentation feasibility, and language. (2) Validation in the media expert assessment aspect is worth 3.47 with a very valid category in terms of content, language, and format. (3) Student response was valued at3.32 with very practical criteria in terms of interest and usefulness. (4) Classical learning completeness assessment shows a value of 90.625% and is declared effective. So, the learning trajectory on the CUPs learning model is declared feasible, practical, and effective to use.

**Keywords:** learning trajectory, CUPs, learning obstacle

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## 1. INTRODUCTION

In the implementation of education, there are challenges faced by students who consider mathematics a complex subject that makes students experience learning obstacles. The learning obstacles experienced by students are having a lack of understanding of the problems in the problem, difficulty remembering the concepts or principles that must be used, and difficulty applying or mastering concepts and strategies (1).

Based on interviews with mathematics teachers at SMP Negeri 10 Surakarta, conducted on November 10, 2023, information on learning obstacles was obtained. Namely,

students consider mathematics difficult, only students with high abilities are active in learning, there are still students who cannot perform simple arithmetic operations, limited learning models, and limited learning media, and learning has not been able to fulfill student character development (independence, creativity, and innovation) in the independent curriculum, learning has not properly facilitated concept understanding, causing low student understanding abilities.

From the various learning obstacles, the teacher reveals the urgency that can be used as the focus of research, which is the level of understanding of student concepts that is still low. Concept understanding is fundamental in learning mathematics because with a good experience it can help students solve problems and be able to apply the concepts they have learned in the real world (2). The ability to understand concepts is essential for students because it makes it easier for them to overcome obstacles in subsequent learning (3). Understanding the concept is needed for the learning process to run well, so understanding the idea can be measured through the achievement of student learning outcomes (4). Data from the final summative assessment results of SMP Negeri 10 Surakarta show that students in two VIII$^{th}$ grade classes obtained an average of 50.57, and only two out of 63 (or three percent) students got a score that met the completeness requirement. The learning results of SMP Negeri 10 Surakarta students represent that the ability to understand concepts owned by students is still low. VIII$^{th}$ grade students of SMP Negeri 10 Surakarta experience learning obstacles on the material of the Pythagorean theorem. The math teacher at SMP Negeri 10 Surakarta mentioned that the material was difficult for students due to a lack of understanding of the prerequisite material, namely algebra and plane figure

Wake in (5) explains that teachers should consider presenting learning situations that can challenge students, responding to students' conjectures, and designing alternatives when designing lesson plans. To create high-quality learning, teachers must carefully plan before conducting learning activities. An appropriate learning design can be a solution to overcome students' learning obstacle. Teachers' learning design must consider students' needs when using student-centered learning methods, so teachers must consider students' learning trajectories when creating learning designs (6). The learning trajectory is a trajectory that describes the prerequisite knowledge that students have and each step from one point to the next, as well as the methods and thought processes or levels of thinking of students in learning (7). The three main components of the Hypothetical Learning Trajectory (HLT) are learning objectives, learning activities, and a conjectured learning process that presupposes the flow of student thinking (8).

In addition to designing learning with a learning trajectory, choosing a suitable model can also help students become active learners so that learning becomes student-centered, with the teacher acting as a facilitator. The Conceptual Understanding Procedures (CUPs) cooperative learning model is one of the learning models that is considered adequate to help students' concept understanding (9). The syntax of the CUPs learning model is that students work individually and in groups and present the discussion results. With the CUPs model, students can define concepts and understand and state examples or non-examples of an idea, making it easier for students to solve math problems (10). The CUPs learning model is designed to help develop students' understanding of complex concepts (11).

What distinguishes this research from previous studies is that no learning trajectory has been developed for the Conceptual Understanding Procedures (CUPs) learning model with Pythagorean Theorem material. Hence, the selection of learning trajectory development for the CUPs learning model is a novelty. Thus, the learning trajectory developed with the CUPs learning model can help teachers think earlier about preparing materials, methods, and learning strategies. Therefore, researchers want to conduct research that aims to develop a learning trajectory using the Conceptual Understanding Procedures (CUPs) learning model for the Pythagorean Theorem material.

## 2. METHOD

The type of research used is design research. Gravemeijer in (12) explains that design research is a type of research that focuses on developing learning theories and instructional stages of learning for students. Gravemeijer & Eerde's in (13) findings state that Local Instruction Theory (LIT) is a theory of learning that describes a learning trajectory on a particular topic with a set of activities that support it. The LIT is a final product of HLT designed, implemented, and analyzed the learning outcomes. This research aims to develop a learning trajectory in the CUPs learning model that is valid, practical, and effective in improving students' mathematics concept understanding ability. Thus, the validity of HLT was measured through the assessment of material and media experts, the practicality of HLT was measured through the students' practicality questionnaire, and the effectiveness of HLT could be measured through the completeness of learning outcomes classically (14). This research has contributed to helping teachers think early to prepare materials, methods, and strategies that support the achievement of learning objectives by knowing students' learning trajectories.

Gravemeijer and Cobb in (5) describe that the stages of design research consisting of preliminary design, experiment, and retrospective analysis. In the preliminary design, a literature review was conducted, interviewing teachers to develop HLT and compiling and testing the validity of HLT and research instruments. The experiment stage consists of two stages, namely, the pilot experiment and the teaching experiment. In the final stage, retrospective analysis was conducted by comparing the HLT and the learning process in the classroom.

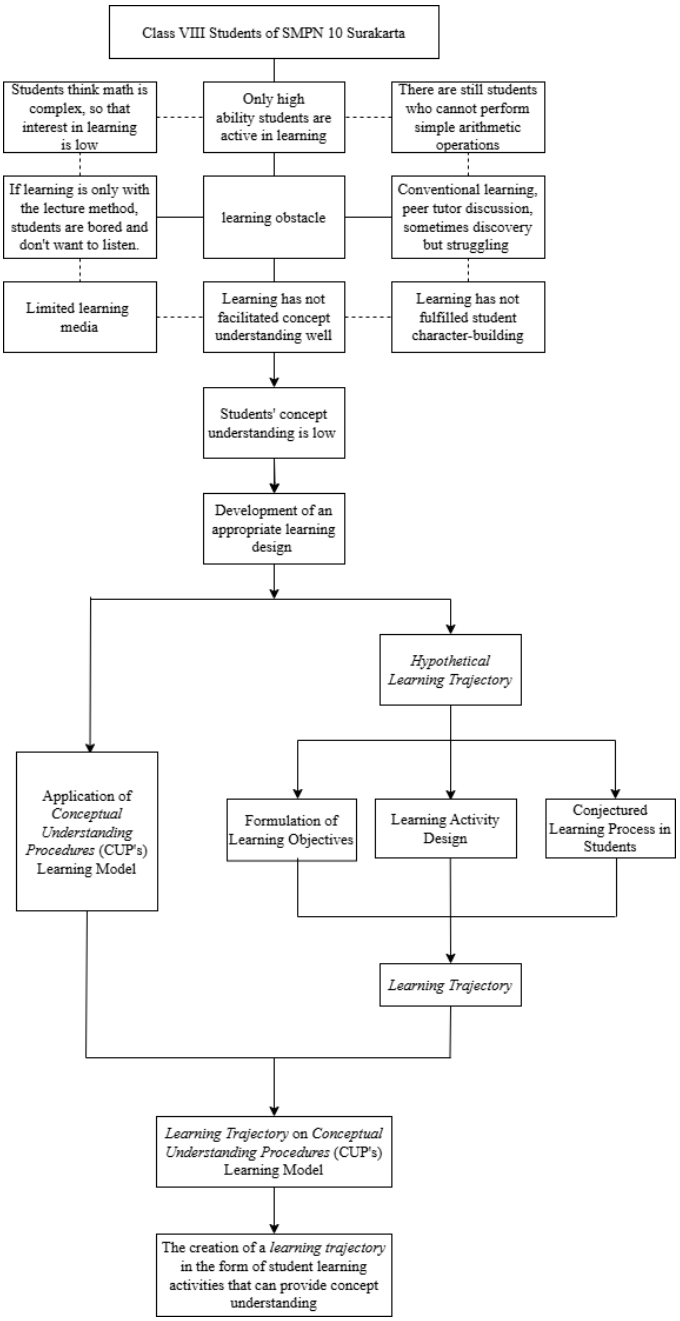


**Figure** 1: Flowchart Methodology.

The research was conducted from November 2023 to May 2024 at SMP Negeri 10 Surakarta. This research involved nine students on class VIII.G in the pilot experiment and 32 students of class VIII.H in the teaching experiment.

The data collected in this research are in word descriptions related to students' learning trajectories on Pythagorean Theorem material. The data obtained based on observations, interviews, questionnaires, and tests. The data sources in the research are mathematics teachers and students of class VIII SMP Negeri 10 Surakarta. The data analysis technique that will be used is the Miles & Huberman model, which consists of data reduction, data presentation, and conclusion drawing (15). In addition, the analysis of the HLT's feasibility, practicality, and effectiveness was categorized as in Table 1.

TABLE 1: Qualitative Categorization (16).

| Interval of Score Average | Value | Category |
|---|---|---|
| $4.00 \leq X \leq 3.00$ | A | Very Good |
| $3.00 \leq X \leq 2.00$ | B | Good |
| $2.00 \leq X \leq 1.00$ | C | Unsatisfactory |
| $1.00 \leq X \leq 0.00$ | D | Not Good |

The *Hypothetical Learning Trajectory* (HLT) was considered valid if it received good ratings from the media and material expert validators. Furthermore, HLT is considered practical if it gets student questionnaire results in the excellent category. The effectiveness of HLT was declared effective if it met the minimum classical completeness criteria of 85% (14).

## 3. RESULTS AND DISCUSSIONS

The design research stage consists of three stages, namely the preliminary design stage, the experiment stage, and the retrospective analysis stage.

### 3.1. Preliminary Design

Based on interviews with teachers, an HLT was designed for one meeting with the learning objective to find the comparison of sides in isosceles right triangles, or triangles with angles of 45˚, 45˚, and 90˚. Then, after determining the learning objectives, HLT was designed with learning activities, namely (1) students observe and listen to the teacher's explanation reviewing the Pythagorean Theorem formula material, (2) students

are asked to complete the LKPD independently, (3) students are asked to complete the LKPD in groups, (4) presentation of discussion results, (5) making conclusions about the comparison of sides in isosceles right triangles.

The HLT design that had been prepared was then tested for feasibility by validators who were considered competent with their expertise, consisting of two material expert validators and two media experts. Media expert validators and material experts consist of lecturers and teachers. The results of validation by material experts in the form of validity are contained in Table 2, and the results of validation by media experts are included in Table 3.

Table 2: Scores and Result of Validity HLT by Material Experts.

| No | Assessment Aspect | Validity | Criteria |
|---|---|---|---|
| 1. | Content Appropriateness | 3.84 | Very valid |
| 2. | Presentation Appropriateness Aspect | 4.00 | Very valid |
| 3. | Language Assessment Aspect | 3.60 | Very valid |
| | **Average** | **3.81** | **Very valid** |

Table 3: Scores and Result of Validity HLT by Media Experts.

| No | Assessment Aspect | Validity | Criteria |
|---|---|---|---|
| 1. | Contents | 3.20 | Very valid |
| 2. | Language | 3.60 | Very valid |
| 3. | Format | 3.60 | Very valid |
| | **Average** | **3.47** | **Very valid** |

Based on the validation results by the material expert and media expert validators, the HLT in the CUPs learning model is in an outstanding category, so its feasibility can be declared very valid. The following Table 4 presents the HLT design that has been tested for feasibility to be used in the experiment stage.

Table 4: HLT Design at Preliminary Design Stage.

| Learning Objectives | Learning Activities | Conjectured Learning Process |
|---|---|---|
| Find the ratio of sides in isosceles right triangles (with angles of 45˚, 45˚, and 90˚). | Students observe and listen to the teacher's explanation to review the material on the Pythagorean Theorem formula. | 1. Students observe and listen to the teacher's explanation carefully so that they can remember the Pythagorean Theorem formula and solve the problems that will be provided. |

TABLE 4: Continued.

| Learning Objectives | Learning Activities | Conjectured Learning Process |
|---|---|---|
| | 2. Students complete the LKPD to find the ratio of sides in a right triangle with angles 45˚, 45˚, and 90˚ independently. | When students complete the LKPD independently, several possible student thought processes will be found, namely:<br>• Students argue that they need to understand the elements of a right triangle, especially the element of the hypotenuse, which is the side right in front of the right angle.<br>• Students determine the length of the side of a right triangle by using the Pythagorean Theorem formula.<br>◦ Students know the concept of the hypotenuse in a right triangle to determine the length of sides a, b, and c with the Pythagorean Theorem formula.<br>◦ Students determine the length of side a, but some students have not been able to find the size of sides b and c because they experience the sloping side misconception in the LKPD.<br>• Students find that there is a pattern of similarity in the ratio of the length of the side of an isosceles right triangle to the size of the angle in front of it.<br>◦ Students write the answer to the question of the length of the hypotenuse of an isosceles right triangle when one of the side lengths is known using a previously found pattern.<br>◦ Students know there is a pattern in the side lengths of an isosceles right triangle but cannot answer the question of the length of the hypotenuse of an isosceles right triangle if one of the side lengths is known with a previously found pattern.<br>• Students summarize the comparison of the side lengths of isosceles right triangles according to the pattern found for the size of the angle in front of them.<br>◦ Students write the conclusions about comparing the side lengths of isosceles right triangles according to the pattern found and the size of the angle in front of it.<br>◦ Students cannot conclude the side length comparison of isosceles right triangles even though they know a pattern has been found previously. |
| | 3. Students in groups to complete the LKPD to find the ratio of sides in a right triangle with angles of 45˚, 45˚, and 90˚. | Students who experience difficulties convey the difficulties encountered when solving LKPD. Students discuss solving problems from the first problem to the last problem on the LKPD with several possible student thought processes, namely:<br>• Fellow group mates assist students who still have difficulty determining the hypotenuse, which is the side right in front of the right angle.<br>• Students and their group partners find a similar pattern in the length of an isosceles right triangle to the size of the angle in front of it.<br>• Students and their group partners write down the answer to the question and compare the side lengths of isosceles right triangles according to the pattern found.<br>• Students, together with their group colleagues, conclude the comparison of the length of the side of an isosceles right triangle according to the pattern that has been found against the size of the angle in front of it that the ratio of the length of the side in front of the 45˚ angle is one while the ratio of the length of the side in front of the 90˚ angle is $\sqrt{2}$. In this case, to find the ratio of sides in a right triangle with angles of 45˚, 45˚, and 90˚ is to know the elements of a right triangle, then find the length of one side using the Pythagorean formula and see the concept of comparing the length of the side to the size of the angle in front of it. |

Table 4: Continued.

| Learning Objectives | Learning Activities | Conjectured Learning Process |
|---|---|---|
| | 4. Students present the results of the discussion. | One group presents the results of the LKPD completion discussion, and other groups respond to or ask questions about the answers of the group that has presented. |
| | 5. Students make conclusions about comparing sides in right triangles with angles of 45˚, 45˚, and 90˚. | Students make conclusions about the ratio of sides in a right triangle with angles 45˚, 45˚, and 90˚ = 1: 1: $\sqrt{2}$. |

## 3.2. Experiment

The experiment stage consisted of two cycles: pilot experiment and teaching experiment. The pilot experiment stage, or HLT trial, aimed better to estimate the learning process before the teaching experiment. The pilot experiment stage was conducted on March 28, 2024, and April 1, 2024, at SMP Negeri 10 Surakarta. This stage involved nine students of class VIII G who were divided into three groups, each consisting of students with high, medium, and low mathematical concept understanding abilities. After the pilot experiment stage, the HLT practicality analysis was conducted. The practicality of the HLT at the pilot experiment stage based on the student's response questionnaire results is presented in Table 5.

Table 5: Scores Practicality of HLT Pilot Experiment.

| No | Indicator | Value |
|---|---|---|
| 1 | Interest | 3.41 |
| 2 | Usefulness of the Material | 3.20 |
| Average | | 3.30 |

Based on the results of the HLT practicality questionnaire on the CUPs learning model, it can be declared very practical. Furthermore, a retrospective analysis was conducted to obtain an improved HLT based on the pilot experiment results and then continued to conduct a teaching experiment.

The teaching experiment stage was held on April 25 and 30, 2024. There were 32 students of class VIII.H involved in this stage and divided into eight groups consisting of students with high, medium, and low concept understanding abilities based on data provided by the teacher. Furthermore, the practicality and effectiveness of HLT were analyzed after the teaching experiment stage was conducted. The results were that the HLT in the CUPs learning model was very practical and effective for students to use for

learning. The results of the HLT practicality analysis at the teaching experiment stage based on are presented in Table 6, and the HLT effectiveness results are presented in Table 7.

Table 6: Scores Practicality of HLT Teaching Experiment.

| No | Indicator | Value |
|---|---|---|
| 1 | Interest | 3.30 |
| 2 | Usefulness of the Material | 3.33 |
| **Average** | | **3.32** |

Table 7: Scores HLT Effectiveness.

| Value | Number of students | Percentage (%) | Criteria | Classical Completeness |
|---|---|---|---|---|
| 85-100 | 7 | 21,875 | Completed | **90,625 %** |
| 80-84 | 11 | 34,375 | Completed | |
| 75-79 | 11 | 34,375 | Completed | |
| 70-74 | 1 | 3,125 | Not completed | **9,375 %** |
| 65-69 | 1 | 3,125 | Not completed | |
| 60-64 | 1 | 3,125 | Not completed | |
| 55-59 | - | - | - | |
| 50-54 | - | - | - | |
| <50 | - | - | - | |
| Total | 32 | 100 | | |

## 3.3. Retrospective analysis

### 3.3.1. Retrospective Analysis Pilot Experiment

This stage aims to analyze the HLT experiment at the pilot experiment stage. Based on the HLT experiments piloted, there are still shortcomings in the alleged HLT learning process that need to be improved. The following are the results of the HLT improvement:

In the HLT, the conjectured learning process in learning activity two at the point where students determine the length of side a, but some students have not been able to find the length of sides b and c because they experience the misconception of the hypotenuse in the LKPD can be eliminated. All students could find the lengths of sides a, b, and c, even though one of the nine students experienced a slightly sloping side misconception. To minimize these misconceptions, the solution can be to add an explanation to learning activity one, namely, the researcher reviews the Pythagorean

Theorem formula material accompanied by the elements of a triangle, especially the hypotenuse of a triangle

In the HLT, the conjectured learning process in learning activity two at the point students cannot conclude the comparison of the side lengths of isosceles right triangles even though they know there is a pattern that has been found before can be eliminated. This is because all students were able to conclude that the ratio of side lengths of right triangles with angles of 45˚, 45˚, and 90˚ is 1: 1: $\sqrt{2}$.

In the HLT, the conjectured learning process in learning activity three at the point of students who still have difficulty determining the hypotenuse assisted by group mates that the hypotenuse is the side located right in front of the right angle is corrected to students with group mates discussing the results of the length of sides a, b, and c with the Pythagorean Theorem formula by knowing the hypotenuse element.

The improved *Hypothetical Learning Trajectory* (HLT) based on the retrospective analysis of the *pilot experiment* is presented in **Table 8**.

Table 8: HLT Pilot Experiment Improvement Results.

| **Learning Objectives** | **Learning Activities** | **Conjectured Learning Process** |
|---|---|---|
| Find the ratio of sides in isosceles right triangles (with angles of 45˚, 45˚, and 90˚). | 1. Students observe and listen to the teacher's explanation to review the material on the Pythagorean Theorem formula. | Students observe and listen to the teacher's explanation carefully so that they can remember the elements in a right triangle and the Pythagorean theorem formula to solve the problems that will be provided. |
| | 2. Students complete the LKPD to find the ratio of sides in a right triangle with angles 45˚, 45˚, and 90˚ independently. | When students complete the LKPD independently, several possible student thought processes will be found, namely:<br>• Students argue that they need to understand the elements of a right triangle, especially the element of the hypotenuse, which is the side right in front of the right angle.<br>• Students determine the length of the side of a right triangle by using the Pythagorean Theorem formula.<br>◦ Students know the concept of the hypotenuse in a right triangle to determine the length of sides a, b, and c with the Pythagorean Theorem formula.<br>◦ Students determine the length of side a, but some students have not been able to find the size of sides b and c because they experience the sloping side misconception in the LKPD.<br>• Students find that there is a pattern of similarity in the ratio of the length of the side of an isosceles right triangle to the size of the angle in front of it.<br>◦ Students write the answer to the question of the length of the hypotenuse of an isosceles right triangle when one of the side lengths is known using a previously found pattern.<br>◦ Students know there is a pattern in the side lengths of an isosceles right triangle but cannot answer the question of the length of the hypotenuse of an isosceles right triangle if one of the side lengths is known with a previously found pattern.<br>• Students summarize the comparison of the side lengths of isosceles right triangles according to the pattern found for the size of the angle in front of them.<br>◦ Students write the conclusions about comparing the side lengths of isosceles right triangles according to the pattern found and the size of the angle in front of it. |

Table 8: Continued.

| Learning Objectives | Learning Activities | Conjectured Learning Process |
|---|---|---|
| | 3. Students in groups to complete the LKPD to find the ratio of sides in a right triangle with angles of 45˚, 45˚, and 90˚. | Students who experience difficulties convey the difficulties encountered when solving LKPD. Students discuss solving problems from the first problem to the last problem on the LKPD with several possible student thought processes, namely:<br>• Students with group mates discussing the results of the length of sides a, b, and c with the Pythagorean Theorem formula by knowing the hypotenuse element.Students and their group partners find a similar pattern in the length of an isosceles right triangle to the size of the angle in front of it.<br>• Students and their group partners write down the answer to the question and compare the side lengths of isosceles right triangles according to the pattern found.<br>• Students, together with their group colleagues, conclude the comparison of the length of the side of an isosceles right triangle according to the pattern that has been found against the size of the angle in front of it that the ratio of the length of the side in front of the 45˚ angle is one while the ratio of the length of the side in front of the 90˚ angle is $\sqrt{2}$. In this case, to find the ratio of sides in a right triangle with angles of 45˚, 45˚, and 90˚ is to know the elements of a right triangle, then find the length of one side using the Pythagorean formula and see the concept of comparing the length of the side to the size of the angle in front of it. |
| | 4. Students present the results of the discussion. | One group presents the results of the LKPD completion discussion, and other groups respond to or ask questions about the answers of the group that has presented. |
| | 5. Students make conclusions about comparing sides in right triangles with angles of 45˚, 45˚, and 90˚. | Students make conclusions about the ratio of sides in a right triangle with angles 45˚, 45˚, and 90˚ = 1: 1: $\sqrt{2}$. |

### 3.3.2. Retrospective Analysis Teaching Experiment

The retrospective analysis stage aims to analyze the HLT results in the teaching experiment. The HLT results refined at the teaching experiment's retrospective analysis stage provide accurate learning trajectory results. The following are the results of the HLT improvement:

In the HLT, the conjectured learning process in learning activity one was improved by students observing and listening to the explanation given by the teacher well and actively giving responses so that they were able to remember the elements in a right triangle and the Pythagorean Theorem formula to solve the problems that would be given. This is because during learning, not only observing and listening to explanations from the teacher, but students also actively participate in providing responses.

In the HLT, the conjectured learning process in learning activity two, at the point of students finding a pattern of similarity in the comparison of the length of the side of an isosceles right triangle to the size of the angle in front of it, is corrected by adding the point at which students can see a pattern or similarity. Still, students have not been able to write down the pattern they find in the length of the hypotenuse of an isosceles right triangle to the size of the angle in front of it.

The improved *Hypothetical Learning Trajectory* (HLT) for the *Conceptual Understanding Procedures* (CUPs) learning model resulted in the final HLT presented in **Table 9**.

TABLE 9: Final HLT Results on CUPs Learning Model.

| **Learning Objectives** | **Learning Activities** | **Conjectured Learning Process** |
|---|---|---|
| Find the ratio of sides in isosceles right triangles (with angles of 45˚, 45˚, and 90˚). | 1. Students observe and listen to the teacher's explanation to review the material on the elements of a right triangle and the Pythagorean Theorem formula. | Students observe and listen to the explanation given by the teacher well and are active in responding so that they can remember the elements in a right triangle and the Pythagorean Theorem formula to solve the problems that will be given. |
| | 2. Students complete the LKPD to find the ratio of sides in an isosceles right triangle independently. | When students complete the LKPD independently, there will be several possible thought processes experienced by students, namely:<br>● Students argue that they need to understand the elements of a right triangle, especially the element of the hypotenuse, which is the side right in front of the right angle.<br>● Students use the Pythagorean Theorem to determine the lengths of sides a, b, and c of a right triangle<br>● Students find a pattern of similarity in the ratio of the length of the side of an isosceles right triangle to the size of the angle in front of it.<br>◦ Students can see that there is a pattern or similarity, but students are not yet able to write down the pattern they find in the length of the hypotenuse in an isosceles right triangle against the size of the angle in front of it.<br>◦ Students write the answer to the question of the length of the hypotenuse of an isosceles right triangle when one of the side lengths is known using a previously found pattern.<br>◦ Students know there is a pattern in the side lengths of an isosceles right triangle but cannot answer the question of the hypotenuse length of an isosceles right triangle if one of the side lengths is known with a previously found pattern.<br>● Students summarize the comparison of the side lengths of isosceles right triangles according to the previously found pattern concerning the magnitude of the angle in front of it. |

Table 9: Continued.

| Learning Objectives | Learning Activities | Conjectured Learning Process |
|---|---|---|
| | Students in groups to complete the LKPD to find the ratio of sides in a right triangle with angles of 45˚, 45˚, and 90˚. | Students who experience difficulties convey difficulties encountered when solving the LKPD. Students discuss solving problems from the first problem to the last problem on the LKPD with several possible student thought processes, namely:<br>• Students and their group mates discuss the results of the length of sides a, b, and c using the Pythagorean Theorem formula by knowing the hypotenuse element.<br>• Students and their group partners find a similar pattern in the length of an isosceles right triangle to the size of the angle in front of it.<br>• Students, together with group colleagues, conclude the comparison of the length of the side of an isosceles right triangle according to the pattern that has been found previously to the size of the angle in front of it that the pattern of the comparison of the length of the side in front of the 45˚ angle is one while the comparison of the length of the side in front of the 90˚ angle is $\sqrt{2}$.<br>In this case, to find the ratio of sides in a right triangle with angles of 45˚, 45˚, and 90˚ is to know the elements of a right triangle, then find the length of one side using the Pythagorean formula and see the concept of comparing the length of the side to the size of the angle in front of it. |
| | 4. Students present the results of the discussion. | One group presents the results of the discussion completion of LKPD, and other groups provide responses or questions to the answers of the group that has presented. |
| | 5. Students make conclusions about comparing sides in right triangles with angles of 45˚, 45˚, and 90˚. | Students make conclusions about the ratio of sides in a right triangle with angles 45˚, 45˚, and 90˚ = 1: 1: $\sqrt{2}$. |

A learning trajectory is a series of activities students go through to understand a concept and achieve predetermined learning objectives. The learning trajectory obtained from the HLT design has been tested. This research developed HLT on the CUPs model of Pythagoras Theorem material at SMP Negeri 10 Surakarta. The HLT components comprised learning objectives, activities, and the conjectured learning process. The learning objective formulated in the HLT was to find the comparison of sides in isosceles right triangles (with angles of 45˚, 45˚, and 90).

Three stages of design research were passed in developing a learning trajectory on the CUPs learning model of Pythagoras Theorem material at SMP Negeri 10 Surakarta: preliminary design, experiment, and retrospective analysis. The preliminary design stage aimed to design an HLT by conducting a literature review, interviewing teachers to compile the HLT, and designing the HLT. The learning activities in the HLT were designed based on the syntax of the CUPs learning model. The syntax of the CUPs learning model is that students work individually, in groups, and present discussion

results. Therefore, the activities arranged in the HLT are: (1) students observe and listen to the teacher's explanation reviewing the material of the elements in a right triangle and the Pythagorean Theorem formula, (2) students complete the LKPD independently, (3) students in groups complete the LKPD, (4) students present the results of the discussion, (5) students make conclusions. In addition to designing HLT, learning tools (LKPD and teaching modules) were also prepared to support learning. The HLT design was then tested for feasibility through two material expert validators and two media expert validators and was declared very valid. The validity of the HLT on the material expert obtained a score of 3.81, which was assessed from the aspects of the feasibility of content, presentation, and language so that it was included in the very good category. Furthermore, the validity of HLT on media experts obtained a score of 3.47, which was assessed from the aspects of content, language, and format so that it was included in the very good category.

In the experiment stage, the HLT experiment was conducted in two stages, namely, the pilot experiment stage and the teaching experiment stage. The pilot experiment stage aimed to test the HLT designed to obtain a better HLT before being used in the teaching experiment stage. The learning activities in the HLT began with students observing and listening to the teacher's explanation to review the prerequisite materials, such as the elements of a right triangle and the Pythagorean Theorem formula. At the pilot experiment stage, some students had misconceptions about the hypotenuse element in a right triangle, so HLT was improved by adding prerequisite material related to the triangle elements. The second learning activity is for students to complete the LKPD independently. Both at the pilot experiment and teaching experiment stages, students experienced difficulties in finding patterns. However, this is not a problem for researchers because researchers facilitate students' ability to ask questions during group discussions in subsequent learning activities. The third learning activity is for group students to complete the LKPD well. Students work together by participating in helping their friends who have difficulty solving problems on the LKPD. Then, the fourth learning activity is the presentation of the results of one group's discussion. Students who are active in this activity are not only students who make presentations, but students who do not make presentations also participate by asking questions and providing responses. The last learning activity is students making conclusions by comparing sides in an isosceles right triangle (with angles of 45˚, 45˚, and 90).

The last stage of design research is retrospective analysis. This stage resulted in an improved HLT by maintaining and reducing the conjectured learning process in the HLT. The study showed that almost all HLTs designed were by the experimental activities

carried out with minor improvements that had been adjusted during learning. In addition, the HLT was also analyzed for its overall practicality, with an average value of 3.32, as assessed from the indicators of interest and usefulness. The result was included in the very good category and declared very practical. The test results were also analyzed to determine the effectiveness of HLT. Classical completeness was achieved with a final score of 90.625%, so HLT in the CUPs model was effectively used in learning.

The results of the learning trajectory description on the Conceptual Understanding Procedures (CUPs) learning model on the Pythagorean Theorem material at SMP Negeri 10 Surakarta are presented in Figure 2.

## 4. CONCLUSION

Based on the results of research and discussion, it can be concluded that the learning trajectory in the Conceptual Understanding Procedures (CUPs) learning model on the Pythagorean Theorem material at SMP Negeri 10 Surakarta consists of learning objectives formulated to find the ratio of sides in an isosceles right triangle (with angles 45˚, 45˚, 90˚). Learning activities are formulated into five learning activities, namely (1) observing and listening to the teacher's explanation reviewing the material of the elements in a right triangle and the Pythagorean Theorem formula, (2) completing the LKPD to find the ratio of sides in an isosceles right triangle independently (3) completing the LKPD to find the ratio of sides in an isosceles right triangle in groups (4) presenting the results of the discussion (5) making conclusions about the ratio of sides in an isosceles right triangle. The results of this study are: (1) validation in the aspect of material expert assessment is worth 3.81 with a very valid category, (2) validation in the aspect of media expert assessment is worth 3.47 with a very valid category, (3) student response value of 3.32 with very practical criteria and (4) classical learning completeness assessment shows a value of 90.625% and is declared effective.

The research that has been prepared is inseparable from any shortcomings because this study is limited to the material topic, finding only the ratio of sides in isosceles right triangles. Therefore, other researchers are expected to develop a learning trajectory on other material topics, such as finding the Pythagorean Theorem formula and the ratio of sides in other right triangles. Not only that, but other researchers can also develop a learning trajectory on materials other than the Pythagorean Theorem.

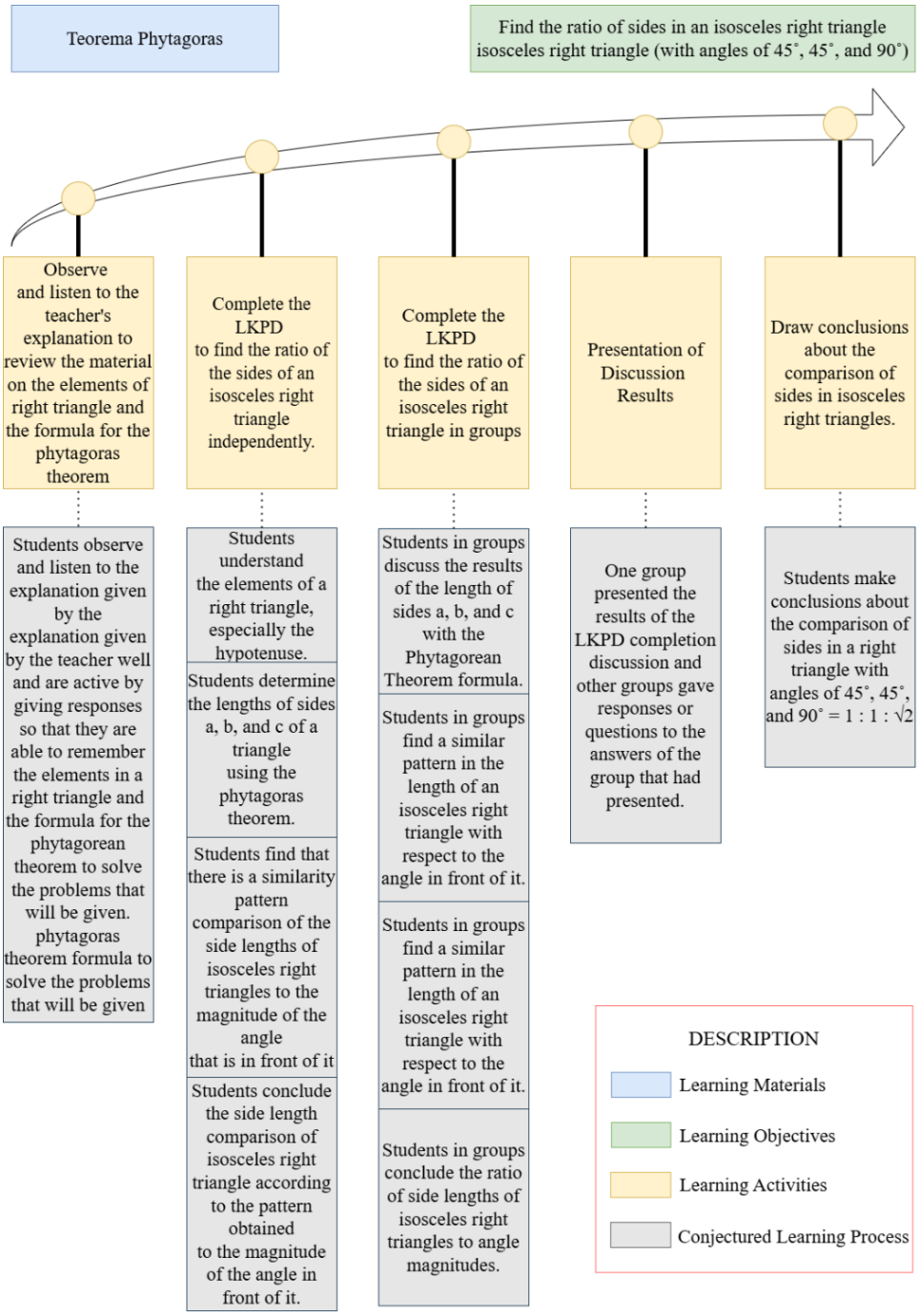


**Figure** 2: Learning Trajectory of CUPs Learning Model in General.

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