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Research Article

Developing the Learning Trajectory of Grade VII SMP Students on Prism Volume Material in Problem-based Learning

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Abstract.
This research aimed to produce a learning trajectory on prism volume material in problem-based learning, carried out in three stages: preparing for the experiment, designing the experiment, and retrospective analysis. The research subjects were students of class VII in two Wonogiri junior high schools. The trial of Hypothetical Learning Trajectory (HLT) for cycle 1, or the so-called pilot experiment, involved 6 students of class VII E and trial of HLT for cycle 2, or the so-called teaching experiment, involved 32 students of class VII G. The instruments for data collection were observation sheet, interview protocol, and learning video recording. The prism volume learning trajectory started from defining the prism, determining the elements of prism, grouping examples and non-examples of prism, and finding the formula for the volume of prism. The results showed that the learning trajectory can assist students in finding and developing an understanding of the prism volume formula.

Keywords: learning trajectory, problem-based learning, prism volume

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1. INTRODUCTION

Mathematics has an important role in education in Indonesia and the world. Mathematics is one of the fields of science that makes a positive contribution to the intellectual life of the nation. Mathematics is one of the exact sciences whose role and use cannot be separated from everyday life. According to Nurliastuti the quality of mathematics education in Indonesia is currently still relatively low, making Indonesia lag far behind other countries. Material in mathematics subjects that is abstract tends to make it difficult for students to understand the material presented. In addition, students' difficulties in understanding mathematics material are caused by the basics of mathematics that they have are still very weak. Students need to master the basic material to be able to follow the next material, because the material in mathematics will always be related. As stated by Brunner that between a concept and another concept in a mathematics lesson, there is a very close relationship.

Based on the results of interviews conducted on Wednesday, January 25, 2023 with several mathematics teachers at the school, obtained information that one of the things that makes the low quality of mathematics education is the impact of the 2-year pandemic, during the pandemic students' mathematics learning is very limited and all-round. This results in students becoming increasingly difficult to understand the material presented, because the basics of mathematics they have are still very weak. Basic math concepts such as addition, subtraction, multiplication, and division that they should have learned at the elementary school level have not been fully mastered. So before getting into learning, teachers have to repeat the basics of math from the beginning. In learning mathematics, teachers at the school have actually implemented learning models and adapted to the material to be taught, including the use of the Problem Based Learning (PBL) learning model.

Before the learning process is carried out, the learning model is an important what needs to be prepared and chosen carefully, in order to train students to think critically and be able to solve problems. Therefore, one of the efforts to improve the quality of mathematics education is to make changes to the learning model. In line with this solution, the Problem Based Learning (PBL) learning model can be used as an alternative to improve the quality of mathematics education. Actually, the learning process at SMP Negeri 2 Wonogiri has used the Problem Based Learning PBL learning model on certain materials. One of the materials whose learning applies the PBL model is the material of building space in class VII.

Problem-based learning can help students in building knowledge concepts including helping students build concepts on one of the competencies in mathematics subjects that are familiar with everyday life, namely building space. Learning the concept of building space in SMP Negeri 2 Wonogiri has been applying PBL learning model, but students still find it difficult to understand and apply the concept of building space in solving problems. Some researchers noted that most students still experience problems in understanding the concepts of geometry, especially in space. Data on the daily test results of class VII SMP Negeri 2 Wonogiri in the 2021/2022 academic year on the material of building space shows that 20 out of 32 obtained scores below 70, meaning that students who reached the KKM score were still below 50%. The PBL learning model should be able to help students in understanding the concept of building space, if learning is carried out according to the steps of the right PBL model and the teacher must understand his role in implementing the PBL learning model.

In learning activities with PBL learning model, the teacher acts as a facilitator. As a facilitator, teachers need to design learning trajectories and students' thinking flow to develop students' mathematical knowledge. The learning trajectory is prepared based on the alleged learning in the classroom so that it can be used as a guideline for implementing learning as well as an anticipatory action against possible problems faced by students during the learning process. Learning Trajectory is a learning concept that discusses various routes or learning trajectories that students have. Learning trajectory is one of the important things in supporting student learning, as stated by Wijaya et al. that tracing the appropriate learning trajectory is very important, because it can support students in developing their understanding of what is being learned. Various studies, especially in studying space building materials, researchers have succeeded in tracing students' learning trajectory. For example, in research conducted by Nursyahidah et al., through their research managed to trace the learning trajectory of junior high school students on the concept of cones and in Aisah's research which successfully designed a learning trajectory based on learning obstacles on the concept of prism volume. Tracing the appropriate learning trajectory is very important, because it can support students in developing their understanding of what is being learned. Various studies, especially in studying space building materials, researchers have succeeded in tracing students' learning trajectory. Likewise, this research will also develop a learning trajectory for seventh grade junior high school students on the concept of space, especially on the volume of prisms and tubes in problembased learning. This aims to help students understand and develop their understanding of the volume of prisms and tubes with problem-based learning.

2. METHOD

The type of research used in this study is design research which is a development research in the learning process that includes learning activities and student thinking. Design research includes a systematic study starting from designing, developing and also evaluating all interventions related to education, such as learning processes, learning programs, teaching materials, learning environments, learning products, and learning systems (1). According to Gravemeijer & Cobb (2), there are three phases in design research, namely preparing for the experiment, design experiment, and retrospective analysis. These three cycles will stop when the answers to the research questions can be obtained and the learning objectives have been achieved.

Design research contains three stages, namely preparing for the experiment, design experiment, and retrospective analysis. The first stage is preparing for the experiment. At this stage, interviews with teachers were conducted to find out the initial abilities of students, then design a hypothetical learning trajectory (HLT). Activities that contain conjectures or students' thinking trajectories are developed through a hypothetical learning trajectory (HLT) using a problem-based learning (PBL) model consisting of several learning stages, namely orienting students to the problem, organizing students, guiding individual and group investigations, developing and presenting results, analyzing and evaluating the problem-solving process. Before designing the HLT, a literature review was conducted on the volume material of prisms and tubes studied at the junior high school level. At this stage, learning tools were also prepared to support the HLT that had been made. Researchers compiled teaching modules and student worksheets (LKPD) in accordance with the HLT.

In the second stage, there were two cycles, namely pilot experiment (cycle 1) and teaching experiment (cycle 2). This stage is the stage of conducting HLT trials and improving the conjecture that has been designed at the preparing for the experiment stage so that it will get a better HLT. This trial was based on the HLT that had been made and carried out with reference to the teaching module and LKPD that had been validated. At the pilot experiment stage, the HLT design was tested to adjust the HLT that would be used in the teaching experiment. Then at the teaching experiment stage, data collection was conducted to answer the research questions.

In the third stage, retrospective analysis, all data that had been obtained at the design experiment stage was analyzed, such as student worksheets, observation sheets, interview guidelines, and video recordings. Based on the results of this analysis, it will be used to improve the HLT, in order to get a better HLT. The revised part is the part that is considered to have no influence on students' understanding and achievement of learning objectives.

The subjects in this study were seventh grade students of SMP Negeri 2 Wonogiri. The students involved in the pilot experiment process or the HLT cycle 1 trial in this study were 6 students in class VII E. The selection of subjects was based on the students' ability level, each ability level namely high, medium, and low ability was represented by two students. The students' ability level was obtained from the information from the interview with the mathematics teacher of grade VII of SMP Negeri 2 Wonogiri. At the teaching experiment stage or the HLT cycle 2 trial involved 32 students of class VII G SMP Negeri 2 Wonogiri.

The instruments used in this research are observation sheets, interview guidelines, and learning video recordings. Observations were made with the aim of directly observing the learning process in the classroom. Furthermore, another instrument used was an interview guidelines which was compiled through an interview guideline with the content of unstructured and open-ended questions so that the opinions and views of the research subjects could be raised. Video recordings in this study were used to document activities that occurred in the classroom, conversations or interactions between researchers and research subjects.

3. RESULTS AND DISCUSSIONS

At the preparing for the experiment stage, researchers designed the learning process for two meetings. The first meeting aims to recognize and determine the elements of the prism, the second meeting aims to find the volume formula of the prism. HLT on prism volume material can be seen in Table 1 below.

TABLE 1: HLT on volume of prism.

Learning Objectives	
Recognize and determine the elements of a prism	
Student Activity	**Student Thinking Flow**
Orienting to contextual problems in LKPD regarding the introduction and elements of prisms.	Students take a good look at the contextual problems in the LKPD, students will understand that to solve these problems, they must first complete the series of activities in the LKPD.
Discuss to solve problems on the LKPD about the introduction and elements of prisms	• Students in groups observe the problems contained in the LKPD • Students work together and discuss to answer every question on the LKPD given.
Developing problem solving on LKPD about the definition and elements of prisms • Determine the elements of a prism • Identify the definition of a prism after knowing its elements • Classify which images are and are not examples of prisms	• Student will determine the element of a prism ◦ The elements of a prism consist of the base side, roof side, upright side, ribs, and corner points. ◦ The formula of the elements of an n-square prism consists of, $sisi = 2 + n$, $rusuk = 3n$, $dantitiksudut = 2n$ • Students will identify the definition of a prism, that a prism is a three-dimensional shape with the same shape and size of the base side and roof side, and has a square or rectangular upright side. • In the activity of grouping pictures, there are three possibilities: ◦ Students can correctly categorize which pictures are and are not prisms. ◦ Students can only correctly categorize some of the pictures which include and exclude prisms. ◦ Students cannot categorize the pictures because they are confused about which ones include and do not include prisms.

Table 1: Continued.

Learning Objectives	
Present the results of the discussion	Between groups of students will give responses to other groups who are presenting the results of their discussions. • Each group may have different answers to the conclusion of the definition of a prism, but they all have the same core. • Each group is likely to have different answers to the grouping of pictures that include and do not include prisms.
Summarize the definition and elements of prisms	Students will summarize the definition and elements of a prism.
Find the formula for the volume of a prism	
Orienting to contextual problems in LKPD regarding the volume of prisms	Students look at the contextual problems in the LKPD, students will understand that to solve these problems, they must first complete the series of activities in the LKPD.
Discuss to solve the problems on the LKPD about the volume of prisms	• Students in groups observe the problems contained in the LKPD • Students work together and discuss to answer every question on the LKPD given.
Developing problem solving on LKPD about the volume of prisms Providing arguments about the relationship between blocks and triangular prisms Discover the concept of volume formula of triangular prism	• Students provide arguments about the relationship between blocks and triangular prisms, $prisma\ segitiga = \frac{1}{2} \times balok$ • Students will find the formula for the volume of a triangular prism from the explanation of the beam volume formula and follow the steps in the LKPD. $vol\ prisma\ segitiga = \frac{1}{2} \times vol\ balok$ $vol\ prisma\ segitiga = \frac{1}{2} \times p_{balok} \times l_{balok} \times t_{balok}$ $vol\ prisma\ segitiga = \frac{1}{2} \times t_{\Delta} \times \alpha_{\Delta} \times t_{prisma}$ $vol\ prisma\ segitiga = \left(\frac{1}{2} \times t_{\Delta} \times \alpha_{\Delta}\right) \times t_{prisma}$ $vol\ prisma\ segitiga = Luas_{alas\ prisma} \times t_{prisma}$
Present the results of the discussion	Between groups of students will give responses to other groups who are presenting the results of their discussions.
Summarize the volume formula of a prism	• Students will conclude that the formula for the volume of a prism is the area of the base of the prism multiplied by the height of the prism. • After obtaining the formula for the volume of a prism, students will know that to find the volume of the triangular prism-shaped pudding that Nasywa will make is by multiplying the area of the base of the prism by the height of the prism or by dividing two equal parts of the volume of the blockshaped mold.

After the HLT on prism volume learning was successfully designed, then the HLT design was tested at the design experiment stage, there were two cycles, namely pilot experiment as cycle 1 trial and teaching experiment as cycle 2 trial. The implementation of this pilot experiment is described as follows.

3.1. Pilot Experiment

The activities carried out at the first meeting aimed to enable students to recognize and determine the elements of the prism. In this activity, students look enthusiastic about learning in groups. However, several times during discussions working on LKPD,

there were students who felt unsure and were afraid of being wrong when answering problems in LKPD. In addition, groups with low ability levels tend to be more passive than groups with high and medium ability levels, but still share tasks to solve problems in the LKPD. In the learning activity of recognizing and determining the elements of a prism, the first stage students are asked to look at a contextual problem, then students are directed to work on the LKPD to determine the elements of a prism which are then used in identifying the definition of a prism. After that, students classify which pictures include and do not include examples of prisms among several examples of pictures in everyday life to help students get to know prisms better. In this case, students have been able to group which pictures include and do not include prisms, although there are still groups that answer incorrectly. Thus, in the end students can conclude that the definition of a prism is a three-dimensional shape with the same and parallel shape and size of the base side and roof side, and has a square or rectangular upright side and the elements of a prism consist of the base side, roof side, upright side, ribs, and corner points. Students are also able to know the formula for the elements of an n-square prism which consists of, $sisi = 2 + n$, $rusuk = 3n$, $dantitiksudut = 2n$.

The activities carried out at the second meeting aimed to enable students to find the concept of the prism volume formula. The first stage students are asked to look at contextual problems, then students are directed to work on LKPD to find the prism volume formula which is associated with the beam volume formula. In teaching the concept of prism volume formula to students, students are not just given concepts and formulas directly but students find and form the concept of prism volume formula based on their own understanding which is done by drawing conclusions about the prism volume formula from the description of the beam volume formula that they have learned before. After reflecting on contextual problems, students are asked to provide arguments regarding the relationship between beams and triangular prisms. In this activity, students are a little confused because the command sentence and information on the LKPD are not clear, so it is necessary to improve the LKPD, namely by improving the command sentence and adding clear information to the LKPD in determining the relationship between the base of the beam and the base of the triangular prism. After going through several stages in the LKPD, students will find the formula for the volume of a prism. In the end, students can draw their own conclusions that the formula for the volume of a prism is the area of the base of the prism multiplied by the height of the prism. After the implementation of the pilot experiment or the HLT cycle 1 trial, then the analysis of the HLT cycle 1 trial was conducted. When in the HLT learning trial that we have compiled there are still some shortcomings that need to be corrected, then a

revised HLT is compiled which aims to get a better quality HLT than before. Based on the results of the retrospective analysis on the HLT cycle 1, there were some revisions to improve the HLT that would be used in the HLT cycle 2. Actually, the concepts presented in the revised HLT were the same as the concepts that had been compiled in the HLT cycle 1 but there were slight changes. The revisions in HLT cycle 1 were described as follows:

1) In the LKPD regarding the introduction and elements of prisms, to help students find the formula for each element of the prism, improvements are made by separating the table on each element in the prism.

2) In the LKPD, improvements were made to the command sentence and adding clear information to the LKPD on the volume of prisms in determining the relationship between the base of the beam and the base of the triangular prism.

3) In the HLT, the first meeting students' conjecture, that students could not group the pictures because they were confused about which ones included and did not include prisms, the conjecture was eliminated, because all groups could understand the question command and were able to group pictures that included and did not include prisms, although there were groups that were less precise in grouping the pictures.

3.2. Teaching Experiment

As previously discussed, the concept of HLT cycle 2 was basically the same as the concept that had been designed in HLT cycle 1, but there were some changes or revisions to get a better HLT. At the pilot experiment stage and teaching experiment stage, the learning objectives were the same, namely: (1) students can recognize and determine the elements of prism, (2) students can find the volume formula of prism. The implementation of the learning trial in the teaching experiment or HLT cycle 2 trial is described as follows:

First Meeting. In the first meeting, the activities carried out aim to enable students to recognize and determine the elements of prism shapes. In the introductory activity, the teacher reviews the material on the types of spaces. Students are asked to mention the spaces they have learned. This aims to make students recall the various types of spaces, especially prism shapes. After that, in the core activities of this lesson using the stages of the problem-based learning (PBL) model which are described as follows:

a) Student Orientation to the Problem

At this stage, a student worksheet (LKPD) is distributed to each group. Students are asked to observe and understand a contextual problem on the LKPD regarding the introduction and elements of prisms.

b) Organizing Learners

After that, students in groups work on each command in the LKPD introduction and elements of prisms. In this activity, students are seen discussing actively and enthusiastically learning in groups.

c) Guiding Individual and Group Inquiry

After observing and understanding the contextual problems about prisms on the LKPD, students determine the shape of the tart in the problem and determine the parts of the sides, ribs, and corner points on the tart. This means that students must know the elements of a prism, because the shape of the tart resembles a prism. Based on the results of the LKPD, all groups can answer that the shape of the tart is a triangular prism. The following are students' answers regarding the shape of the tart on the LKPD problem.

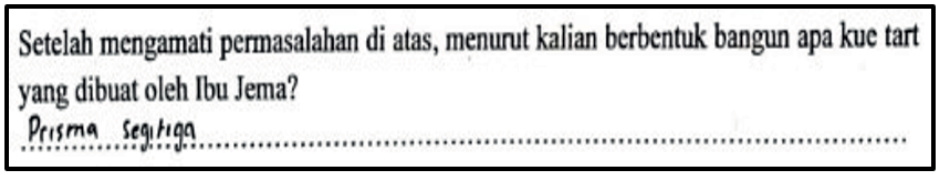


Figure 1: Students' answers regarding the shape of the tart on the LKPD.

After knowing the shape of the tart, students then answer questions about the elements of a prism by filling in the columns provided on the LKPD. Each group filled in the column correctly, according to the elements of a prism. The following is an illustration of student answers.

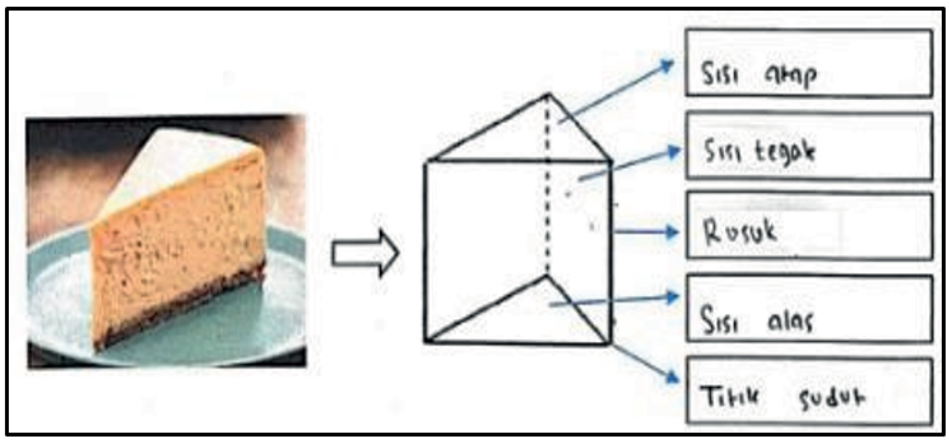


Figure 2: Students' Answers in the Teaching Experiment on the Elements of a Prism.

Judging from the students' answers, they have been able to determine the elements of a prism correctly. Furthermore, to confirm students' understanding, an interview was conducted with one group of students.

Teacher : What are the parts or elements of a prism?

Subject : Roof side, base side, upright side, ribs, and corner points bu

Teacher : Are you sure that the placement of each element of the prism on the LKPD that you have done is correct?

Subject : Sure mom, first we answer the roof side and the base side first, then answer the other elements such as the upright side, ribs and corner points.

Teacher : Yes, your answer about the elements of a prism is correct.

From the interview with one of the groups, it can be seen that students can mention and show each element of the prism correctly. Then students answer the next question to be able to conclude the meaning of the prism, the goal is to strengthen students' understanding of the prism. Based on the results of the LKPD, each group answered in various ways and sentences, but the students' answers led to the same thing. They were able to conclude the definition of a prism. Here are some answers from student groups.

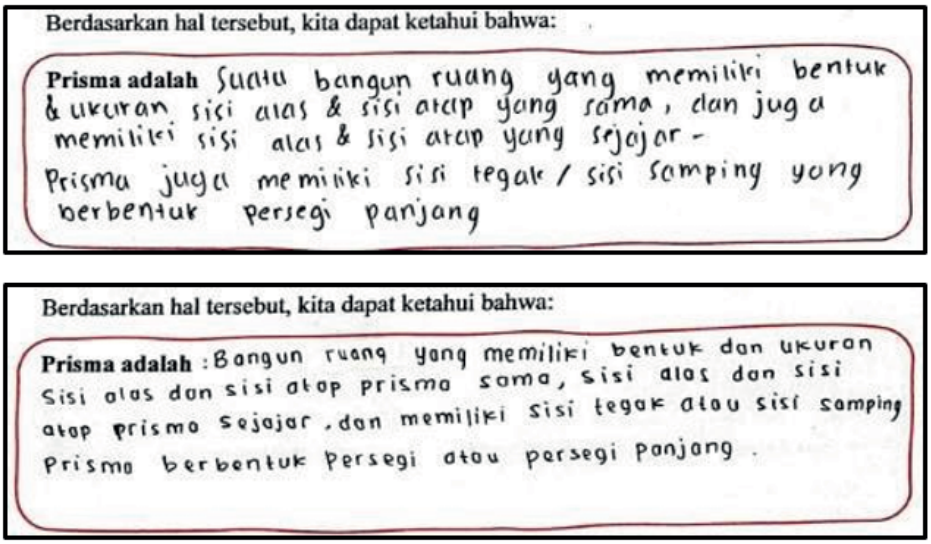


Figure 3: Students' Answers in the Teaching Experiment on the Definition of a Prism.

From the students' answers in Figure 3, it can be seen that students have been able to conclude the definition of prism with their own sentences. Based on the observation results, the process of students discussing in solving problems on the LKPD is done well. Students are able to conclude the meaning of prism after going through several processes in the LKPD.

After concluding the definition of prisms, students answered the next question, which was to classify objects in daily life that included and did not include prisms. While students were working on the LKPD, the researcher went around to observe the responses given by each group. When choosing which objects included prisms, only one group could immediately answer correctly. The most common mistake made by students is that they think that tube-shaped objects are not a type of prism, even though tube-shaped objects should be a type of prism. In addition, there are still some students who think that objects in the form of cubes and blocks are not types of prisms. To overcome this, the researcher reemphasized the understanding of the prism that they had found before, that prisms have parallel base and roof sides, the same shape and size, and other planes are square or rectangular. It is intended that students together get the correct conclusion of the grouping of images. The following is a snippet of the interview transcript conducted to guide students to get the correct grouping of images.

Teacher : From the examples presented, which one do you think is a prism?

Subject : Picture number 6,11,12 bu

Teacher : Okay, is there anything else?

Subject : Pictures number 3 and 9 are also ma'am

Teacher : Now for pictures number 3 and 9, do you all agree that they are prisms?

Subject : Agree ma'am, because blocks and cubes have the same roof side and base side shape and size.

Teacher : Yes, that's right. Then are there any other prisms in the picture other than numbers 6, 11, 12, 3, and 9?

Subject : Figures 5, 8, and 10 also have the same roof and base sides bu

Teacher : But it's a tube instead of a prism

Subject : Okee, yes it is a tube, but if we look at the definition of a prism, is a tube a prism?

Teacher : Yes ma'am, the roof and base of the tube are both circular.

Subject : Okay, do you understand?

Based on the students' final answers, there is still 1 group that is still wrong in grouping the pictures. This is because the student group is less careful and does not pay attention to the discussion delivered by the teacher that prisms have parallel base and roof sides, the same shape and size, and other fields are square or rectangular. The group of students grouped the pyramid-shaped picture into a group that included prisms.

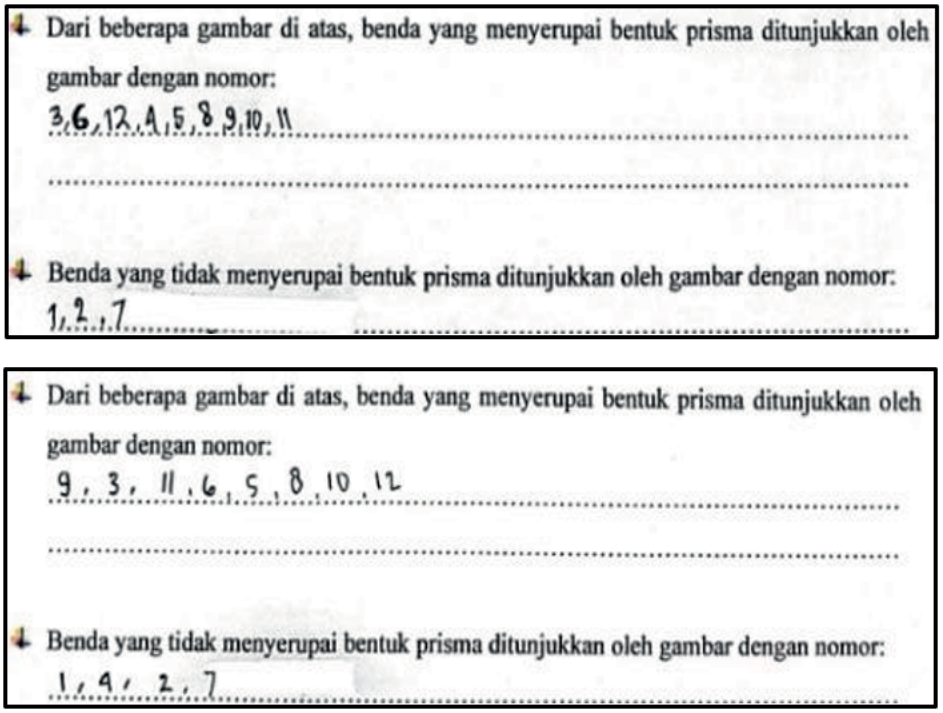


Figure 4: Students' Answers in the Teaching Experiment Regarding Pictures that Include and Do Not Include Prisms.

The next question is that students are asked to fill in the table to determine the elements of various prism shapes up to the n-square prism. The answers of all groups were correct, after making improvements to the LKPD, namely by separating the table on each element of the prism. This can reduce the difficulty of students to determine the formula of the elements of the n-square prism. The following is an illustration of students' answers.

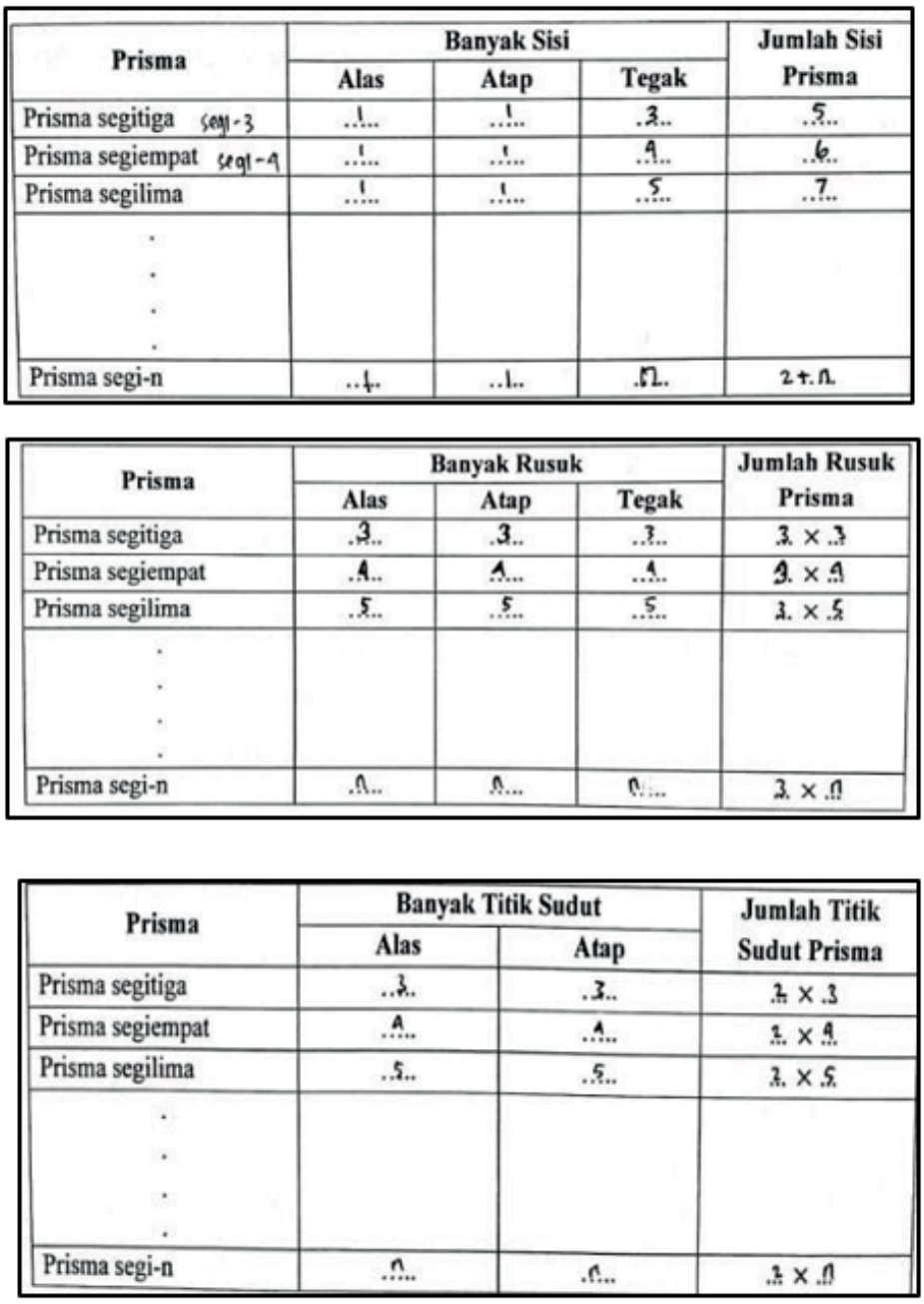
Judging from the students' answers in Figure 5, they have been able to find the formula for each element in the n-square prism consisting of, $sisi = 2 + n$, $rusuk\ 3n$, $dantitiksudut = 2n$.

a) Develop and Present Results

At the end of the lesson, after finishing the discussion, one of the groups is appointed randomly to present the results of their discussion and other groups listen carefully. Other groups get the opportunity to give responses and questions to the presenting group.

b) Analyzing and Evaluating the Problem Solving Process

With questions and answers, the teacher directs students to the conclusion of the results of the LKPD discussion of the introduction and elements of prisms. After that, as



Prisma	Banyak Sisi			Jumlah Sisi Prisma
	Alas	Atap	Tegak	
Prisma segitiga	1	1	3	5
Prisma segiempat	1	1	4	6
Prisma segilima	1	1	5	7
. . . .				
Prisma segi-n	1	1	n	2+n

Prisma	Banyak Rusuk			Jumlah Rusuk Prisma
	Alas	Atap	Tegak	
Prisma segitiga	3	3	3	3 × 3
Prisma segiempat	4	4	4	3 × 4
Prisma segilima	5	5	5	3 × 5
. . . .				
Prisma segi-n	n	n	n	3 × n

Prisma	Banyak Titik Sudut		Jumlah Titik Sudut Prisma
	Alas	Atap	
Prisma segitiga	3	3	2 × 3
Prisma segiempat	4	4	2 × 4
Prisma segilima	5	5	2 × 5
. . . .			
Prisma segi-n	n	n	2 × n

Figure 5: Students' Answers in the Teaching Experiment on the Elements of an N-Square Prism.

a closing, the teacher reinforces by explaining again the meaning and elements of the prism so that students can understand the prism as a whole.

Second Meeting. After students recognize the shape of a prism, then students will find the formula for the volume of a prism. At the beginning of the lesson, the teacher reviewed the definition of a prism, then students mentioned the elements of a prism. In addition, the teacher also reviews the area of several flat shapes that they have learned. This aims to make it easier for students to find ways to calculate the volume of a prism.

Next, enter the core activities that use the problem-based learning (PBL) learning model which are described as follows:

a) Student Orientation to the Problem

At this stage, a learner worksheet (LKPD) is distributed to each group of students. Students observe and understand a contextual problem on the LKPD.

b) Organizing Learners

After that, students were asked to work in groups on each command in the LKPD regarding the volume of prisms. During the discussion of working on the LKPD, the researcher went around to observe the responses of each group. The responses given several times there were students who felt unsure and afraid of being wrong when answering problems in the LKPD.

c) Guiding Individual and Group Inquiry

After observing and understanding the contextual problem about finding the formula for the volume of a prism on the LKPD, students are asked to determine the shape of the pudding

in the problem. In this case, all students can answer the question correctly, namely the shape of the pudding in the problem is a triangular prism. Here are the answers from students.

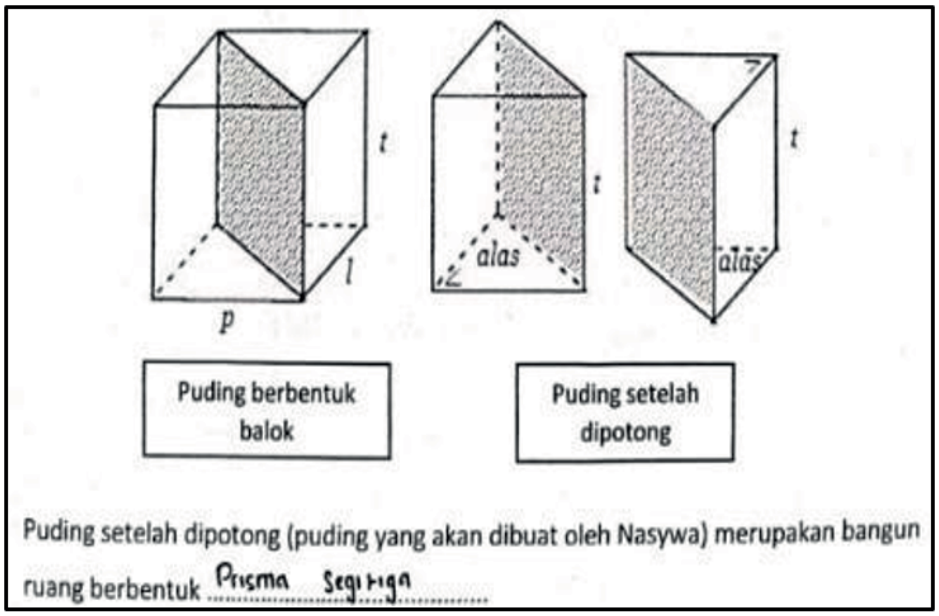


Figure 6: Students' Answers in the Teaching Experiment on the Shape of Pudding in Contextual Problems.

From these answers, students were then asked to make a temporary opinion, according to what they knew about the relationship between blocks and triangular prisms when

looking at the picture of the pudding pieces in the previous problem. While students were working on the LKPD, students seemed confused about writing the relationship between blocks and triangular prisms into mathematical sentences. Then to overcome this, the teacher explained again the meaning of the command given. The following is a snippet of the interview transcript.

Teacher : If you look at the picture on the LKPD, what do you think is the relationship between the beam and the triangular prism?

Subject : A block cut into two triangular prisms, ma'am.

Teacher : Yes, then how do you write it into a math sentence?

Subject : Block = 2x triangular prisms

Teacher : Okay that's right, anyone else have a different opinion? If it's the opposite then, $Prismasegitiga = \cdots balok$ (while writing on the blackboard)

Subject : A triangular prism is equal to half a block ma'am

Teacher : Yes, it's good, if written in a math sentence then $Prismasegitiga = \frac{1}{2} balok$

After receiving guidance from the researcher, the responses given by each group can be seen, they can understand the relationship between blocks and triangular prisms correctly. Here are some answers from the students.

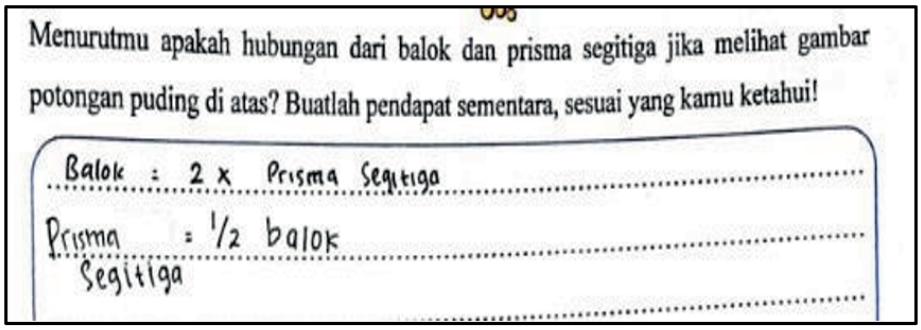


Figure 7: Students' Answers in the Teaching Experiment on the Relationship between Beams and Prisms.

From the answers to the LKPD on the relationship between blocks and triangular prisms in the pudding pieces, most students argue that $prismasegitiga = \frac{1}{2} balok$, there are also those who argue that $balok = 2 \times prismasegitiga$, this opinion is not wrong, but to make it easier for students to find the formula for the volume of a triangular prism, the researcher equalizes the perception that students can write the relationship between the beam and the triangular prism is $prismasegitiga = \frac{1}{2} balok$. Then use it in the process of finding the volume formula of the prism.

When discussing finding the prism volume formula, several times there were students who felt unsure and afraid of being wrong when answering questions on the LKPD. Students discussed with their respective groups and took a little longer to get the right answer. However, the answers written by students are correct and detailed. The following is a description of the student group's answers.

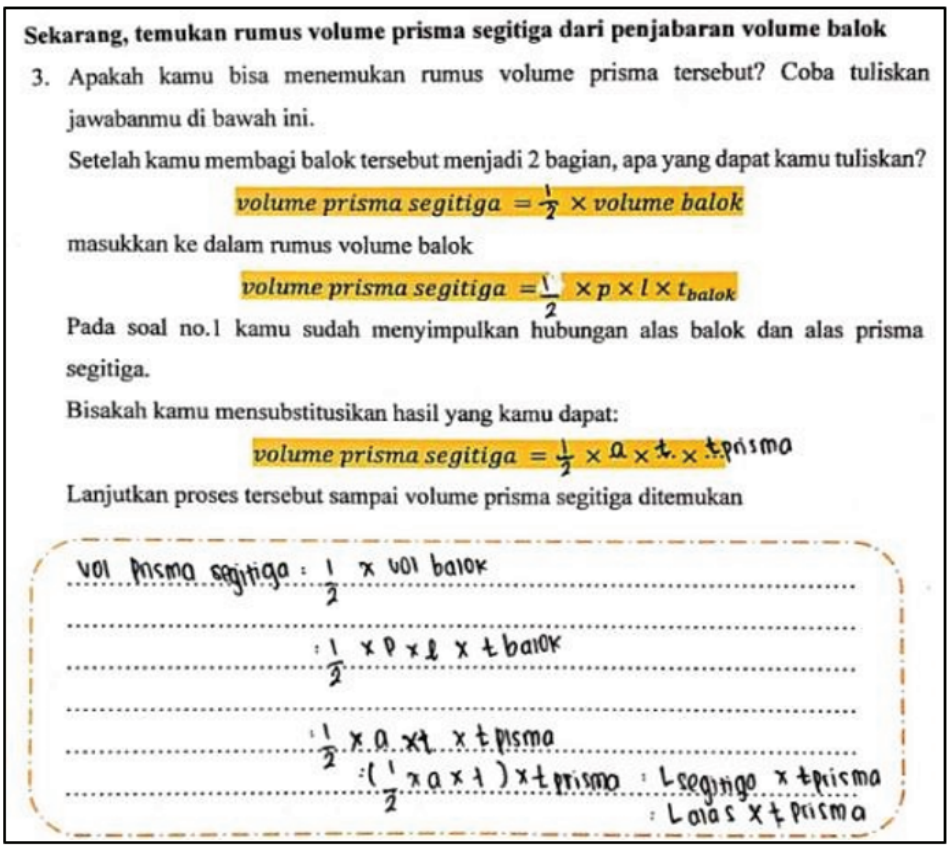


Figure 8: Students' Answers in the Teaching Experiment on Finding the Prism Volume Formula.

The next activity is for students to conclude the volume formula of the prism that they have obtained, in this activity all students can conclude the volume formula of the triangular prism correctly.

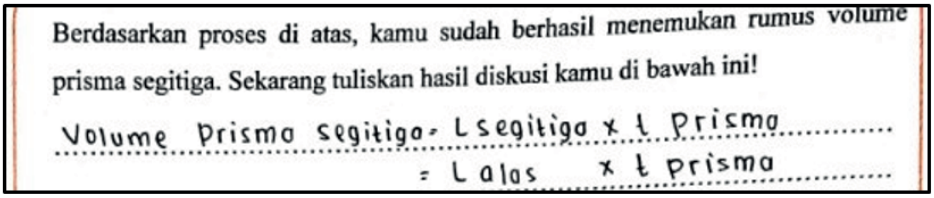


Figure 9: Students' answers in the Teaching Experiment regarding the conclusion of the formula for the volume of a triangular prism.

Based on the answers on the LKPD picture above, students have been able to find the general form of the prism volume formula which is associated with the beam volume formula, namely $volumeprisma = luasalas \times tinggiprisma.$

d) Develop and Present Results

At the end of the lesson, after finishing the discussion, one of the groups was appointed to present the results of their discussion and the other groups listened carefully. The aim is to validate every activity that has been done by students on how to calculate the volume of a prism. Other groups get the opportunity to give responses and questions to the presenting group.

e) Analyzing and Evaluating the Problem Solving Process

With questions and answers, the teacher directs students to the conclusion of the LKPD discussion on the volume of prisms. After that, as a closing, the teacher reinforces by explaining the prism volume formula again so that students can understand how to calculate the volume of a prism as a whole.

Learning trajectory is prepared based on the assumption of learning in the classroom so that it can be used as a guideline for learning implementation as well as an anticipatory action against possible problems faced by students during the learning process. The use of HLT in this study did not experience significant obstacles and produced a good HLT in supporting the learning of prism volume concepts. Students' understanding of prism volume material can be developed starting from introducing students to prism shapes, determining prism elements, grouping pictures that include and do not include prisms, until students can find the concept of prism volume formula from the description of the beam volume formula. During the HLT trial, most students responded according to the expectations in the HLT that had been designed but there were some students who responded unexpectedly, this was not a problem because the researcher could still overcome it by giving directions to students so that students could follow the learning as expected. In the introduction activity and the elements of prisms, most students can complete the LKPD well. Students can answer questions about the elements of a prism by filling in the blank column adjusted to the elements of the prism. However, when asked to determine the formula for the elements of an n-sided prism, students had little difficulty and hesitated in answering. To overcome this, the researcher assisted students in explaining the purpose of the problem of finding the formula for the elements of an n-square prism. In addition, almost all students were able to conclude the definition of a prism correctly and completely, although with various sentences. In addition, most students were also able to classify objects that included and did not include prisms.

In the activity of finding the prism volume formula, students have been able to find the prism volume formula with steps that are in accordance with the LKPD. Students discuss with their respective groups and take a little longer to get the right answer. However, the answers written by students are correct and detailed. In the process of finding the prism volume formula, there are commands in the LKPD that are not understood by students, namely regarding the command to find the relationship between the base of the beam and the base of the prism. To overcome this, the researcher helped explain the meaning of the commands in the LKPD, so that students could continue the process of finding the prism volume formula correctly.

At the retrospective analysis stage, the results of the analysis obtained were that almost all activities and hypotheses were in accordance with what was designed, but there were hypotheses of learning processes that were not in accordance with real learning, so improvements needed to be made to the HLT. The following is the learning trajectory produced in this study.

First Meeting. At the first meeting, the activities carried out aimed to enable students to recognize and determine the elements of a prism. In the first stage, namely learner orientation, a learner worksheet (LKPD) is distributed to each group of students, then students observe and understand a contextual problem on the LKPD. After that, at the stage of organizing learners, students are asked to work in groups to work on each command in LKPD 1, namely the introduction and elements of prisms. After observing and understanding contextual problems about prisms on LKPD, students are asked to determine the shape of the tart contained in the problem which is a triangular prism, determine the elements of a triangular prism, determine the definition of a prism, and classify which objects are and are not prisms, and determine the formula for the elements of an n-square prism. After that, a presentation was made by one group of students and a class discussion was held to confirm and reinforce students' answers so that students could understand the meaning and elements of prisms as a whole. The conclusions obtained at this first meeting include, the definition of a prism is a three-dimensional shape with the same and parallel shape and size of the base side and roof side, and has a square or rectangular upright side. The elements of a prism consist of the base side, roof side, upright side, ribs, and corner points. The formula of the elements of an n-square prism consists of, $sisi = 2 + n$, $rusuk = 3n$, $dantitiksudut = 2n$.

Second Meeting. After students understand about the prism, the next step is for students to find the prism volume formula. In the first phase, namely learner orientation,

a learner worksheet (LKPD) is distributed to each group of students, then students observe and understand a contextual problem on the LKPD. After that, in the phase of organizing learners, students are asked to work in groups to do each command in LKPD 2 regarding the volume of prisms. After observing and understanding contextual problems about finding the formula for the volume of prisms on the LKPD, students are asked to determine the shape of the pudding in the problem, which is a triangular prism. Then students will determine the relationship between blocks and triangular prisms, that is $prismasegitiga = \frac{1}{2}\ balok$, which will later be used to find the volume formula of triangular prisms and the volume formula of prisms in general through the explanation of the beam volume formula that they have learned before. In the end, students can draw their own conclusions that the formula for the volume of a prism is the area of the base of the prism multiplied by the height of the prism or it can be written $volumeprisma = luasalas \times tinggiprisma$. At the end of the lesson, a presentation was made by one group of students and a class discussion was held to confirm and reinforce students' answers so that students can understand how to calculate the volume of a prism as a whole.

Based on the results of the learning trajectory that has been presented, students' understanding of prism volume material can be developed starting from the activity of mentioning the definition of a prism, determining the elements of a prism, classifying pictures that include and do not include prisms, until students can find the concept of prism volume formula from the description of the beam volume formula. From each activity that students go through, the development of a learning trajectory on prism volume material can build students' understanding of the definition, elements, examples and non-examples of prisms, and the prism volume formula. This is in accordance with research conducted by (3) regarding the development of learning trajectories of pyramids and cones, the activities carried out are mentioning the definition of pyramids and cones, mentioning examples and not examples of pyramids and cones, drawing pyramids and cones, recognizing the properties of pyramids and cones, surface shapes and nets of pyramids and cones, and volume of pyramids and cones. Meanwhile, in research (4) which successfully designed a learning trajectory based on learning obstacles on the concept of prism volume, explained that to teach students the concept of prism volume formula, students are not just given concepts and formulas directly but students find and form the concept of prism volume formula based on their own understanding which is done by drawing conclusions about the prism volume formula from the description of the beam volume formula. Thus, when students forget the prism volume formula, students can reconstruct it with their learning experience. The results

of this study are expected to contribute to helping students understand prism volume material. This learning trajectory can be considered by teachers as one of the references in designing problem-based learning on prism volume material. The following is an image of students' learning trajectory on prism volume material produced in this study.

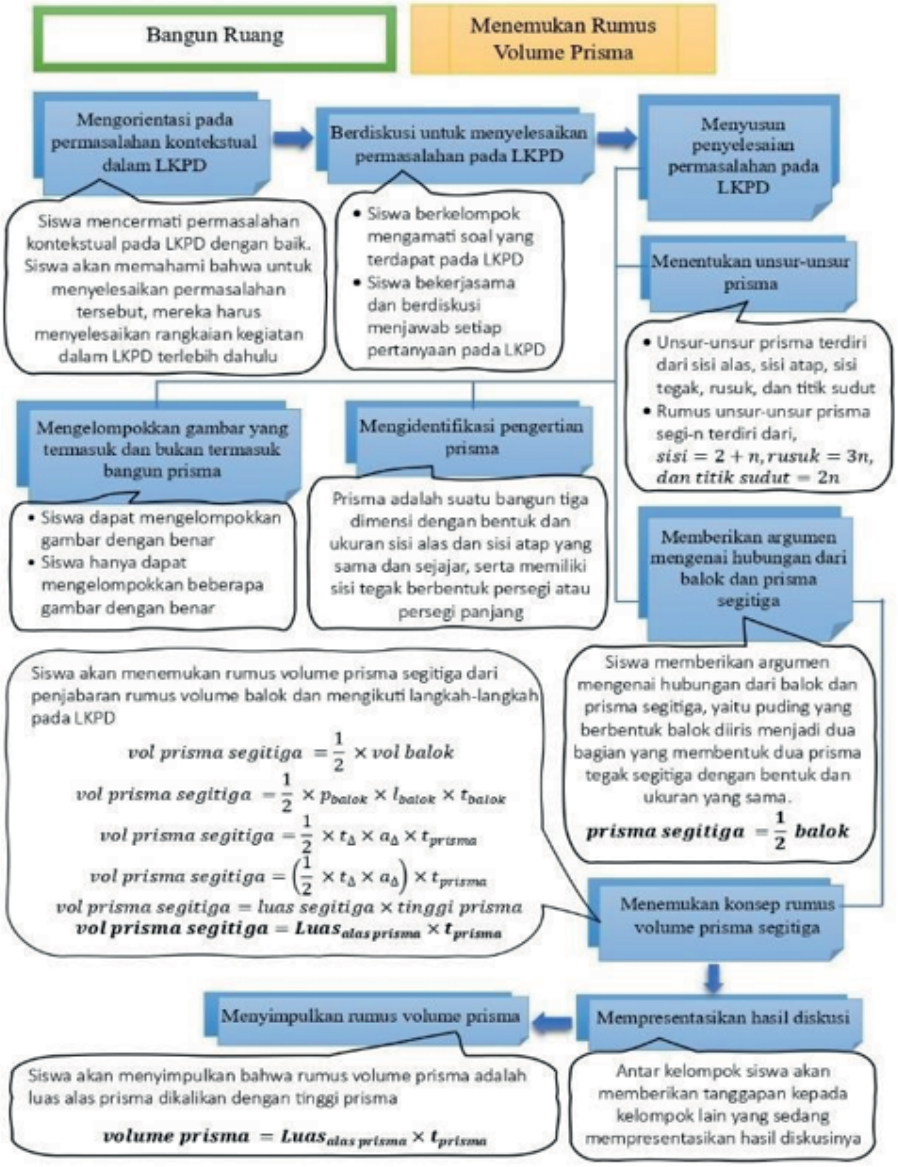


Figure 10: Students' learning trajectory of prism volume in problem-based learning.

4. CONCLUSION

Based on the results of the study, it is concluded that the learning trajectory on prism volume material in problembased learning contains activities ranging from mentioning the definition of a prism, determining the elements of a prism, classifying images that

include and do not include prisms, until students can find the concept of prism volume formula. This research is only limited to prism volume material, whereas there are other spatial shapes such as pyramids, cones, tubes, spheres that can be developed. Therefore, other researchers can develop learning trajectory on different materials.

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