

Research Article

Exploring the Student Critical Thinking in Mathematical Problem Solving

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Abstract.

This research qualitative aims to describe students' critical thinking in solving sequences and series. The research participants were 3 students from grade VIII, selected based on the results of their math tests. Data were collected by giving the task of solving problems and interviews. The student's critical thinking is described using the approach focus, reason, inference, clarity, and overview (FRISCO). The results showed that participants with high math skills can identify the main problem, analyzing questions and statements, reasoning of inductive and assessing the difference was the same, concluding the value of the difference and the solution, making pattern of sequence, and determining the sum of the first 12 terms using the arithmetic series formula, explaining the terms and symbols used in the formula, solving the problem again by adding up each term and comparing the results with the previous results. Participant with moderate mathematical ability wrote down relevant information, using inductive reasoning and mathematical principles and did not use arithmetic series formula, concluding the value of the difference and the solution, making sequence pattern, determining the sum of the first 12 terms by add up each, explaining the term and the difference symbol. Participant with low math ability, only wrote down and mentioned what is known and asked and gave reasons using question marks for question sentences. The description of critical thinking can be considered by teachers in formulating learning strategies for critical thinking.

Keywords: research qualitative, critical thinking, problem solving, arithmetic series

1. INTRODUCTION

Mathematical problem-solving fosters problem solving skills, which are important in everyday life (1,2). These problem-solving skills can be applied to science, technology, engineering, and even non-technical areas. Problem-solving activities help students develop a deeper understanding of mathematical concepts, principles, and relationships (3,4). By actively applying mathematical knowledge to solve problems, students gain a more comprehensive and meaningful understanding of the subject mathematics (5).

Mathematical problem solving provides opportunities to apply mathematical concepts and techniques to real-world situations (6). Mathematical problem solving often involves facing challenging and unfamiliar situations (7,8). By surviving through these difficulties,

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students develop perseverance and resilience (9). They learn to accept challenges, analyse setbacks, and adapt their strategies, which cultivates a growth mindset and a willingness to tackle complex problems in other areas of life.

Problem solving in mathematics encourages creative and innovative thinking (10). It involves exploring alternative approaches, making connections between different concepts, and finding multiple solutions to a problem. Engaging in creative problem solving enhances an individual's ability to think outside the box, find unique solutions, and apply innovative thinking to various real-world situations (11).

Successfully solving math problems increases students' self-confidence and self-efficacy. When students develop problem solving skills, they become more confident in their math abilities. This belief contributes to their overall self-esteem and belief in their problem-solving abilities (12).

Mathematical problem solving requires analytical thinking, logical reasoning, and the ability to evaluate information. This thinking is the essence of critical thinking. Critical thinking is the ability to analyse, evaluate and interpret information objectively and rationally (13). Students who have critical thinking ability to analyse problems and use the information they must solve problems (14).

Mathematical problem solving and critical thinking are interrelated processes that complement and reinforce one another (10). Developing strong critical thinking skills can significantly increase an individual's ability to engage in effective mathematical problem solving (15,16). Likewise, engaging in mathematical problem-solving activities fosters and strengthens critical thinking skills, giving individuals a solid foundation for solving complex problems across multiple domains (17,18).

Critical thinking is the act of analysing information to make better judgments and decisions (19). It involves using things like logic, reasoning, and creativity, to draw conclusions and generally understand things better. To make the correct solution decision requires critical thinking skills (20). Critical thinking can be known through the process of solving problems.

This research explores the cognitive processes involved in solving mathematical problems. This exploration uses six criteria in critical thinking namely FRISCO (an acronym for Focus, Reason, Inference, Situation, Clarity, and Overview). Indicators of the six critical thinking criteria (21) are presented in Table 1 below.

Different individuals tend to think critically differently. Every individual typically thinks and approaches things differently. There is a gap in critical thinking between pupils with

TABLE 1: Criteria and Indicators of Critical Thinking.

Criteria	Indicator
Focus	Figuring out the main point, issue, question, or problem
Reason	Look for reasons for and against the decision in a certain way (pro and contra reasons) Investigating something or doing an experiment Looking for evidence Identifying and assessing the grounds on which they are justified
Inference	Creates a logical inference that conforms to the reasoning.
Situation	Identifying a component which is of relevance to the cognitive process and certain principles that govern it.
Clarity	Clarification of what these terms mean and how they are used.
Overview	Looking at the things that have been chosen and deduced.

high and low abilities (22). Therefore, it is important to explore critical student thinking in solving problems based on their academic abilities. This research aims to describe students’ critical thinking in solve problems related to patterns in sequences of numbers.

In this study, students’ critical thinking in solve problems was explored so that teachers can know the characteristics of critical thinking of students in solving problems. Previous research has shown that the students showed the highest score in the explanation of issues dimension and the lowest score in the conclusions and related outcomes dimension (23). The result research (24) showed that the higher the mathematical ability level, the more diverse students’ thinking patterns in solving problems related to algebraic functions.

The results of this study reflect students’ critical thinking experience in solving problems that can be an initial reference and priority setting in training students’ critical thought skills. It is hoped that this research will provide valuable insights to help teacher design more effective teaching to improve their critical thinking skills, creating a more dynamic and relevant learning environment.

2. METHOD

This study used qualitative approach type of phenomenology to describe students’ the critical thinking in solve problems patterns in sequences of numbers. A purposive sampling technique were applied to select the 8th grade students from the State Junior High School 7 Palu. The selection of students is based on considering the student’s willingness to become a research subject and the student has studied arithmetic sequences

and series. A total of thirty-one students in the chosen class were given problem of patterns in sequences of numbers.

Based on the results of test, students are categorized into three groups, namely students with high, moderate, and low math ability. Furthermore, researchers choose one student from each group with different abilities to be used as research participants. Students selected from the high group are students with the highest mathematics test results. Students selected from the medium group are students with mathematics test results in the middle and students selected from the low group are students with the lowest scores. The participant of the research is MQ as students with high ability, DA as students with moderate ability and HR as a student with low ability.

All participants were given the first problem and were individually interviewed based on their answers. The second one was carried out one week later to ensure the validity of the data. The first problem is equivalent to the second problem. The difference between problem one and problem two is only in the numbers.

The data obtained was analysed qualitatively using the FRISCO approach through three steps including data reduction, data presentation, and conclusion. The data reduction stage is carried out by reducing unnecessary data. Data reduction is a process that includes choosing the main things, selecting, and summarizing all the data obtained. Data presentation is a process that includes data classification and identification. Namely writing a collection of data that is organized and categorized. Data from critical thinking interviews were reduced and categorized based on FRISCO criteria. By classifying and categorizing helps in drawing conclusions. Conclusions are drawn based on the results of analysis of test data and interview data regarding students' critical thinking in solving problems of arithmetic sequences and series that have been collected.

3. RESULTS AND DISCUSSIONS

The section describes the critical thinking of participant with high, moderate mathematical and low mathematical abilities in solve problems patterns in sequences of numbers. The student's critical thinking is expressed based on the criteria of critical thinking, namely Focus, Reason, Inference, Situation, Clarity, and Overview. The results of the examination showed that the critical thinking data of all participants in solving K1 and K2 was credible. Checking the credibility of the data using time triangulation, namely

checking the test results for problem 1 and the test results for problem two converge to the same meaning. Likewise, the results of the first interview and the results of the second interview converge to the same meaning. Therefore, the analysis and display only the critical thinking data of participants in solving K1. The first problem (K1) used to explore critical thinking is presented in Figure 1.

Astra Honda Motor's production results in 2019 have increased steadily every month. If Astra Honda Motor produced 150 units in January and 250 units in the 5th month, how many motorcycles did Astra Honda Motor produce in 2019?

Figure 1: Problem Solving Test (K1).

3.1. Critical thinking of students with high ability

The results of written test answers by participant MQ are shown in Figure 2

Dik : motor yang diproduksi bulan Januari = 150
 motor yang diproduksi Bulan ke-5 = 250
 $n = 12$
 produksi motor setiap bulan mengalami peningkatan yang tetap.
 Dit : Berapakah banyak motor diproduksi astra honda motor selama tahun 2019

Penyelesaian:
 Januari Februari maret april mei
 150 175 200 225 250
 +25 +25 +25 +25

$b = 25$

$$Sn = \frac{n}{2} \times (2 \times a + (n-1) \times b)$$

$$S_{12} = \frac{12}{2} \times (2 \times 150 + (12-1) \times 25)$$

$$S_{12} = \frac{12}{2} \times (300 + (11) \times 25)$$

$$S_{12} = \frac{12}{2} \times (575)$$

$$S_{12} = 6 \times (575)$$

$$S_{12} = 3.450$$

jadi banyak motor yang diproduksi astra honda motor selama tahun 2019 sebanyak 3.450.

Figure 2: MQ's answer of the first problem (K1).

The results of the researcher's interview with the participant MQ for focus, reason and clarity criteria in stage understand the problem are shown in Table 2, Table 3, and Table 4.

TABLE 2: The Results of Interview with MQ for The Focus Criteria.

Interview Code	Interview Sentence
MQ-K1 P 09	What is the main problem of the question?
MQ-K1 S 10	If what I understand from this matter, the main problem it determines how many motors are produced
MQ-K1 P 13	From this question, what information did you get?
MQ-K1 S 14	In this matter, it is known that the motorbike was produced in January 150, motors produced 5th month 250, n = 12 and production motor every month has increased steadily. Keep going who asked how many motorbikes Astra produced Honda Motor during 2019

TABLE 3: The Results of Interview with MQ for The Reason Criteria.

Interview Code	Interview Sentence
MQ-K1 P 11	How do you know if that is the main problem?
MQ-K1 S 12	that's because many motors were produced which were asked in the question.
MQ-K1 P 17	How do you know if that's what you know?
MQ-K1 S 18	From the sentence
MQ-K1 P 19	What about the sentence?
MQ-K1 S 20	Because the sentence is a statement
MQ-K1 P 21	Ok, then how did you know that was what was being asked?
MQ-K1 S 22	Because there is the word "how much"
MQ-K1 P 23	Why with what word?
MQ-K1 S 24	In my opinion, the word "how much" is a question word.

TABLE 4: The Results of Interview with MQ for The Clarity Criteria.

Interview Code	Interview Sentence
MQ-K1 P 27	In question, there is a sentence that motor production experiences a steady increase every month. What does the word fixed mean in the sentence?
MQ-K1 S 28	Means it doesn't change. So, every month the number of motorbikes produced is the same
MQ-K1 P 29	What do you mean?
MQ-K1 S 30	Every month the number of motorbikes increases and the number of motorbikes that are produced continues to be the same

The results of the researcher's interview with the participant MQ for the situation criteria in the stage of devising a plan are shown in Table 5.

The results of the researcher's interview with the participant MQ in the stage of carrying of the plan for the reason, inference, and clarity criteria are shown in Table 6, Table 7, and Table 8

TABLE 5: The Results of Interview with MQ for The Situation Criteria.

Interview Code	Interview Sentence
MQ-K1 P 31	From that question, you already know everything the information. So, is that information sufficient for Answer the question?
MQ-K1 S 32	not yet
MQ-K1 P 33	what other information is needed to answer that question?
MQ-K1 S 34	Difference
MQ-K1 P 35	Why?
MQ-K1 S 36	Because the difference will be used for the next step
MQ-K1 P 49	After getting the difference, what is the next step for solve this problem?
MQ-K1 S 50	look for the number of motors produced
MQ-K1 P 51	Next, if what you are looking for is the number of motors produced during 2019, then what formula would you used?
MQ-K1 S 52	Arithmetic series formula

TABLE 6: The Results of Interview with MQ for The Reason Criteria.

Interview Code	Interview Sentence
MQ-K1 P 39	How do you find difference?
MQ-K1 S 40	I use trial and error
MQ-K1 P 41	Why do you use that method?
MQ-K1 S 44	because in my opinion the method of experimenting is simple but it's clear, that's why I use that method
MQ-K1 P 45	Now try to explain the steps you took to find the difference?
MQ-K1 S 46	So, to find the difference, I sort the known data and make a pattern. That information known only the first month 150 and the 5 th month 250 meanwhile the 2 nd , 3 rd and 4 th months are still unknown. So, I'm looking for the 2 nd , 3 rd , and 4 th months, by trial-and-error. So, I try every month I always add with 25.
MQ-K1 P 47	Oh, I see. How many times did you try it, so you got a difference of 25?
MQ-K1 S 48	3 times, the first time I tried adding 10 but in the 5 th month the result was not 250, the second I tried again adding 20 and the 5 th month the result is not 250, the third I try to add with 25 and in the 5 th month the result is 250.
MQ-K1 P 53	Try to explain the steps you took to find the number of motorbikes produced
MQ-K1 S 54	I use the arithmetic series formula $S_n = \frac{n}{2} (2a + (n - 1) b)$
MQ-K1 P 55	What is the reason for using the arithmetic series formula?
MQ-K1S 56	Because in my opinion, if what you are looking for is the number of motorbikes produced and the difference is the same every month, then the formula used is an arithmetic series.

The results of the researcher's interview with the participant MQ in the stage of looking back for the Overview criteria are shown in Table 9.

Critical thinking of students with moderate ability

The results of written test answers by participant DA are shown in Figure 3

TABLE 7: The Results of Interview with MQ for The Inference Criteria.

Interview Code	Interview Sentence
MQ-K1 S 46	Because what is known in the first month is 150, so go ahead I added 150 + 25 so that the 2nd month the result is the same 175, the 3rd month I add 175 + 25 equals 200, months 4th 200 + 25 equals 225, 5th month 225 + 25 equals 250 the same as known. So, the 2nd month is 175, the 3rd month is 200, the 4th month is 225. Then the pattern is 150, 175, 200, 225, 250 and the difference is 25 so the difference is 25.
MQ-K1 P 57	What is the conclusion from your answer?
MQ-K1 S 58	So many motorbikes were produced by Astra Honda Motor during 2019 as many as 3,450

TABLE 8: The Results of Interview with MQ for The Clarity Criteria.

Interview Code	Interview Sentence
MQ-K1 P 59	Do you know the terms contained in your answer, for example, what is a ? b is what?
MQ-K1 S 60	a is the first term, b is the difference
MQ-K1 P 61	What is S_n ?
MQ-K1 S 62	The number of tribes I want to find
MQ-K1 P 63	Ok, where do you come from?
MQ-K1 S 64	That's from n 12
MQ-K1 P 65	why can the n be 12?
MQ-K1 S 66	Because what was asked was the production of the motorbike for 1 year
MQ-K1 P 67	Then why did a become 150?
MQ-K1 S 68	Because it is known that January = 150, then a is the first term, so a becomes 150

TABLE 9: The Results of Interview with MQ for The Overview Criteria.

Interview Code	Interview Sentence
MQ-K1 P 69	Are you sure that nothing is wrong?
MQ-K1 S 70	Yes, sure
MQ-K1 P 71	why do you know nothing is wrong?
MQ-K1 S 72	Because I have checked my answer again after finishing the work
MQ-K1 P 73	Why check the answer again?
MQ-K1 S 74	if there is something wrong in my answer, I will correct it
MQ-K1 P 75	So, every time you do a question, the answers are checked again?
MQ-K1 S 76	yes. so that my answer is not wrong, so I check my answer again
MQ-K1 P 77	How do you check your answer again?
MQ-K1 S 78	I read the questions again, then I checked what was known and what was asked that I had written down whether they were correct. I counted again everything I've done. I added up from month 1 to month 12 and the result is the same 3,450.

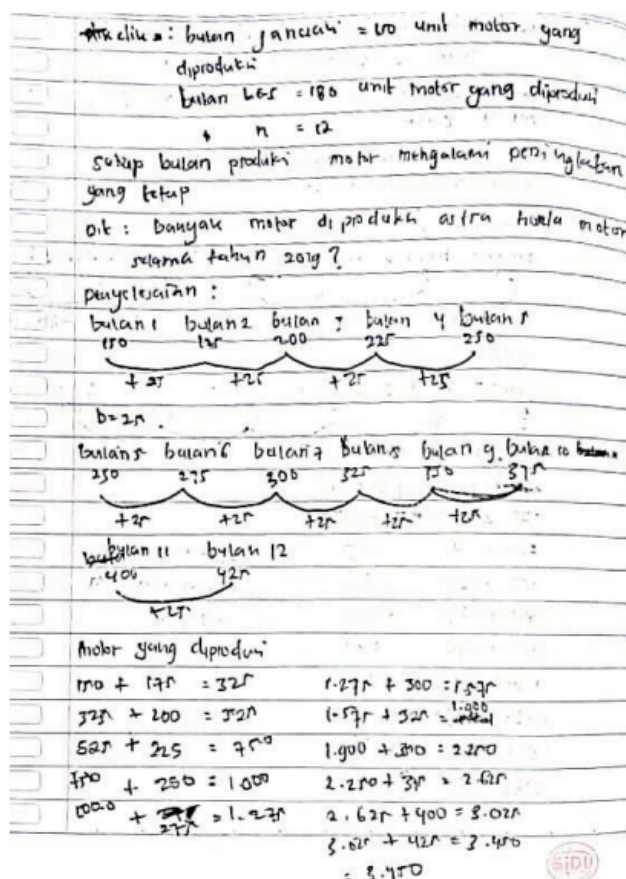


Figure 3: DA's answer of the first problem (K1)

The results of the researcher's interview with the participant DA for focus, reason and clarity criteria in stage understand the problem are shown in Table 10, Table 11, and Table 12.

The results of the researcher's interview with the participant DA for the situation criteria in the stage of devising a plan are shown in Table 13.

The results of the researcher's interview with the participant DA in the stage of carrying of the plan for the reason and inference criteria are shown in Table 14 and Table 15.

The results of the researcher's interview with the participant DA for the situation criteria in the stage of devising a plan are shown in Table 16.

Critical thinking of students with low ability

The results of written test answers by participant HR are shown in Figure 4

The results of the researcher's interview with the participant HR for focus, reason and clarity criteria in stage understand the problem are shown in Table 17, Table 18, and Table 19.

TABLE 10: The Results of Interview with DA for The Focus Criteria.

Interview Code	Interview Sentence
DA-K1 P 11	Well, in your opinion, what is the main problem with this question?
DA-K1 S 12	Determines the number of motorbikes produced during 2019
DA-K1 P15	Ok now. When viewed from this problem, what information is contained in the problem?
DA-K1 S 16	Known in January 150 units of motorcycles produced, the 5th month 250 units of motors were produced, the n was 12 and every month the production of motors has increased still.
DA-K1 P 17	Is that the only information you get?
DA-K1 S 18	Still there
DA-K1 P 19	What?
DA-K1 S 20	What is asked from the problem
DA-K1 P 21	What was asked?
DA-K1 S 22	Many motorbikes were produced by Astra Honda Motor during 2019

TABLE 11: The Results of Interview with DA for The Reason Criteria.

Interview Code	Interview Sentence
DA-K1 P 13	How did you know that this was the main problem?
DA-K1 S14	Because that's what was asked in the question, so I think that's the main problem.
DA-K1 P 25	How do you know if that's what is known in the question?
DA-K1 S 26	From the sentence in your question which says, "If the production of the motor is on in January as many as 150 units and the 5th month as many as 250 unit, how many motorbikes did Astra Honda produce Bikes for 2019."
DA-K1 P 27	Then how did you know that it was that asked
DA-K1 S 28	Because there is a question mark sis at the end of the sentence, in my opinion if there is a sentence and there is a question mark then that is what is being asked

TABLE 12: The Results of Interview with DA for The Clarity Criteria.

Interview Code	Interview Sentence
DA-K1 P 31	in the question there is a sentence for each motor production month has increased steadily. What does the meaning of the word stay in this sentence?
DA-K1 S 32	This means that it has not changed anymore. So, every month the motor increases and the same continues to increase the motor from month 1 to month 2 same month 2 to month 3 same to arrive at month 12

Based on the results of the researcher's interviews with participants for each critical thinking criterion, the results of critical thinking data analysis in solving problems of MQ, DA and HR participants are presented in Table 20.

Participant with high abilities have a deep understanding of the concepts of difference and arithmetic series formulas and students with moderate abilities master the basics of

TABLE 13: The Results of Interview with DA for The Situation Criteria.

Interview Code	Interview Sentence
DA-K1 P 33	Is the information sufficient to answer the question?
DA-K1 S 34	Not yet
DA-K1 P 35	What other information do you need to answer that question?
DA-K1 S 36	Difference (b)
DA-K1 P 37	Why?
DA-K1 S 38	to determine the results of motorcycle production in month 2, months 3, months 4 and months whose results are not yet known production.
DA-K1 P 51	After getting the difference, what is the next step to solve the problem?
DA-K1 S 52	Looking for many motors produced
DA-K1 P 53	if what you are looking for is the number of motors produced during 2019, then what formula would you used?
DA-K1 S 54	I don't use formulas

TABLE 14: The Results of Interview with DA for The Reason Criteria.

Interview Code	Interview Sentence
DA-K1 P 41	How do you find differences?
DA-K1 S 48	I use trial and error. The first, I tried to add 10 in every month but the results which in the 5th month is not the same as in the question. So, I tried again adding 20 and it still doesn't fit, I try again to add 30 but right in the 5th month the results are greater than what was asked. Then, I try again adding 25, now the result is the same in the 5th month questioned.
DA-K1 P 55	Try to explain how to look for the number of motorbikes produced in 2019
DA-K1 S 56	I immediately looked for the 6th month until the 12th month with me adding 25. I'm just left add-added from month 1 to month 12. $150 + 175 = 325$, $325 + 200 = 525$, $525 + 225 = 750$, $750 + 250 = 1,000$, $1,000 + 275 = 1,275$, $1,275 + 300 = 1,575$, $1,575 + 325 = 1,900$, $1,900 + 350 = 2,250$, $2,250 + 375 = 2,625$, $2,625 + 400 = 3,025$, $3,025 + 425 = 3,450$ so I get 3,450
DA-K1 P 57	Is that the result you got?
DA-K1 S 58	Yes
DA-K1 P 59	I saw your answer didn't use the formula, why didn't you use the formula?
DA-K1 S 60	I forgot the formula, so I solved it manually only

TABLE 15: The Results of Interview with DA for The Inference Criteria.

Interview Code	Interview Sentence
DA-K1 S 46	How I tried, I tried adding 10, 20, 30, 25 in each month, but only 25 in the 5th month results same as in question. So, the b is 25.
DA-K1 P 65	So, what is your conclusion from your sister's answer?
DA-K1 S 66	In conclusion, the number of motorbikes produced during 2019 was 3,450.

mathematics and can apply them in solving problems. There was a relationship between students' concept mastery and problem-solving skill (25,26).

TABLE 16: The Results of Interview with DA for The Overview Criteria.

Interview Code	Interview Sentence
DA-K1 P 69	From your answer, are you sure that your steps and answers are correct?
DA-K1 S 70	Yes
DA-K1 P 71	Why are you sure?
DA-K1 S 72	because I have checked again
DA-K1 P 73	Why are you checking your answers again?
DA-K1 S 74	so that my calculation is not wrong
DA-K1 P 77	how do you double check your answer?
DA-K1 S 78	By checking back one by one what I did earlier and the result is still the same 3.450

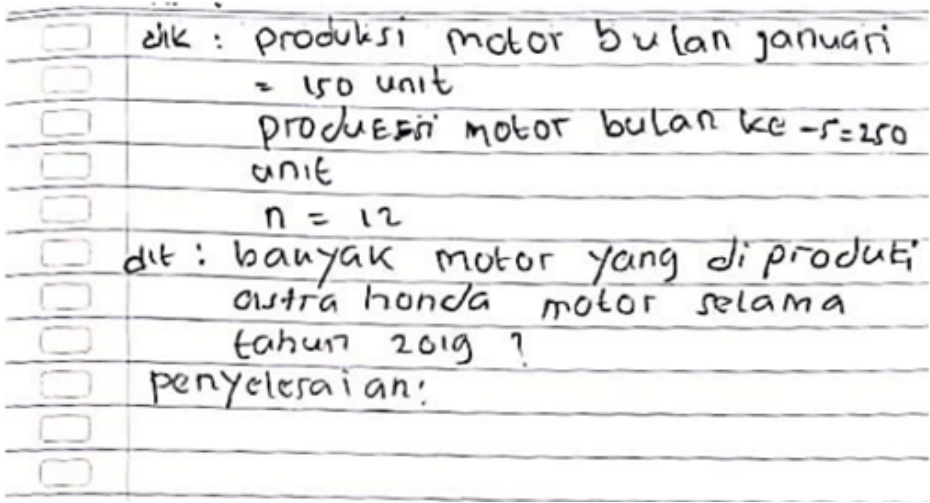


Figure 4: HR's answer of the first problem (K1).

TABLE 17: The Results of Interview with HR for The Focus Criteria.

Interview Code	Interview Sentence
HR-K1 P 13	In your opinion, what is the main problem of this question?
HR-K1 S 14	Main problem?
HR-K1 P 15	Yes, what is the main problem?
HR-K1 S 16	I don't know
HR-K1 P 17	From this question, what information did you get?
HR-K1 S 18	What is known and what is asked?
HR-K1 P 19	Try to express the information obtained from the problem
HR-K1 S 20	It is known that the production of motorbikes in January was 150 units and in the 5th month there were 250 units and then $n = 12$. Then what was asked was, how many motorbikes were produced by the factory during 2019.

Participant with high abilities tend to do detailed analysis of complex mathematical problems. They can identify relevant information, break down problems into smaller

TABLE 18: The Results of Interview with HR for The Reason Criteria.

Interview Code	Interview Sentence
HR-K1 P 25	How do you know that that's the information asked in the question?
HR-K1 S 26	From the question mark symbol

TABLE 19: The Results of Interview with HR for The Clarity Criteria.

Interview Code	Interview Sentence
HR-K1 P 29	In the question there is a sentence for each motor production month has increased steadily. What does the meaning of the word stay in this sentence?
HR-K1 S 30	the meaning of the word fixed
HR-K1 P 31	what does the meaning of the word fixed in that sentence?
HR-K1 S 32	the motor is produced the same sis every month.
HR-K1 P 33	You mean the number of motorbikes produced in the first month is the same as the number of motorbikes produced in the second month, is that so?
HR-K1 S 34	Yes

parts, and identify relationships and patterns that exist within them. Students can think critically, they can give simple explanations, conclude, which consists of deductive activities (reason), can provide further explanations on a topic (clarity) (27).

Participant with high abilities in solving mathematical problems careful problem analysis, logical thinking, creativity, clear communication skills, and high persistence in solving problems. Creativity and critical thinking help students solve problems (28). Students with moderate abilities in solving mathematical problems through critical thinking reflect basic abilities in mathematics, adequate analytical skills, logical thinking, good communication skills, and persistence in facing challenges.

Participant that struggles to think critically about mathematical problems exhibit difficulties understanding mathematical concepts, problems with problem solving analysis and techniques, illogical thinking, a lack of communication skills, limited inventiveness, and low levels of tenacity. In this case, these students need additional guidance and support to overcome their difficulties in understanding and critically applying mathematical concepts (29).

4. CONCLUSION

The results showed that participant with high math skills, identify the main problem. In Reason, analysis questions and statements, using reasoning of inductive then using an arithmetic series formula. In Inference, concluding the value of the difference in a

TABLE 20: FRISCO Criteria for Critical Thinking in Problem Solving.

Criteria	MQ	DA	HR
Focus	Identify the main problem, identify questions, and write down information relevant to the subject matter	Mention the main problem, questions, and write down information relevant to the subject matter	Write down and mention what is known and asked
Reason	Analyse questions and statements, using inductive reasoning by trial and error three times to deduce the difference value of the sequence then using the formula of an arithmetic series	Analyse questions and statements, using inductive reasoning by trial and error four times then using mathematical principles, do not formula of an arithmetic series	using question marks for question sentences
Inference	Conclude the value of the difference in the sequence of numbers then use it in concluding the answer.	Concluding the value of the difference in a sequence of numbers and the solution	
Situation	Develop a strategy by making a number sequence pattern, looking for the difference in the sequence and then determining the number of the first 12 terms using the formula	Develop a strategy by making a number sequence pattern, looking for the difference in the sequence and then add up the results from month 1 to month 12	
Clarity	Explain the terms and symbols used in the formula, using clear and precise language	Explains the term fixed increase and the difference symbol. The language used is not appropriate	Explains the term fixed increase
Overview	Solve the problem again by adding up each term and comparing the results with the previous results	Checking back one by one and the result is still the same	

sequence of numbers and the solution. In Situation, by looking for the difference in a sequence then determining the number of the first 12 terms using the formula. On clarity, explains the terms and symbols used. In Overview, solve the problem again and comparing the results with the previous results. Participant with moderate mathematical ability in the reason, using reasoning of inductive then mathematical principles and rules. In Inference, concluding the value of the difference in a sequence of numbers and the solution. In the situation, by find the difference in a sequence and then adds adding up each term. On clarity, explains the term and the difference symbol. Participant with low math ability, only write down and mention what is known and asked and give reasons using question marks for question sentences.

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