Research Article

Decision-making Process of Mathematics Teacher in Responding to Student's Mistake

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Abstract.
Students lack deep understanding and their learning needs, make it difficult for teachers to respond effectively to student errors. The case study investigated the decision-making process of teachers who respond to the student's mistakes with solutions to quadratic function problems. This type of research is qualitative research with a descriptive approach. The results are revealed based on the decision-making stages: (1) generating ideas, the subject designed strategies using the GeoGebra application, and developing problems to train students' HOTS skills. (2) Clarifying ideas, the subject considered students' ability to understand the problem more easily if it is presented in visual form. (3) Assessing the fairness of ideas, the subject believed that when students were trained from carefully correcting their work to carefully proofreading their work, they learned from their mistakes and prevented them from repeating it. In conclusion, teacher can make good decisions by constructing and assessing ideas so that students can understand their mistakes.

Keywords: decision making, mathematics teacher, quadratic function

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1. INTRODUCTION

Imagine a high school mathematics classroom where a student, grappling with quadratic functions, confidently arrives at an incorrect solution. The teacher pauses, considering the best course of action. Should they correct the mistake immediately, guide the student towards the correct method, or encourage peer discussion to explore the error collectively? Each choice carries significant implications for the student's understanding and confidence. This scenario highlights a common yet critical moment in mathematics education: the teacher's response to a student's mistake. The decision-making process behind these responses is not just a routine aspect of teaching; it is a nuanced and impactful part of fostering mathematical understanding. Research indicates that how teachers handle mistakes can profoundly affect student learning outcomes, shaping their problem-solving skills and attitudes towards mathematics [1], [2]. In this context, our study, "Decision-Making Process of Mathematics Teachers in Responding to Student's

Mistake in Solving Quadratic Functions Problems," seeks to uncover the intricacies of these pedagogical decisions. By exploring the cognitive factors that influence teachers' choices, we aim to provide insights that can enhance educational practices and support teachers in their pivotal role in guiding students through the complexities of quadratic functions.

The decision-making process of mathematics teachers in responding to student mistakes has been a subject of interest in educational research for decades. This research has provided valuable insights into the pedagogical strategies and cognitive processes that teachers employ, as well as the impacts of these strategies on student learning outcomes. Early research by Metcalfe Wong & Lim [3] emphasized the constructive role of errors in the mathematics classroom. They argue that errors should be viewed as opportunities for promote inquiry, analysis, and deeper understanding, rather than merely obstacles to correct. This perspective laid the groundwork for exploring how teachers can effectively leverage mistakes to enhance learning. A significant body of research has focused on the cognitive processes underlying teachers' decisions in the classroom. Hoth, Larrain and Kaiser [4] highlighted the importance of teachers' cognition and motivational requirements in identifying and dealing with student errors. They emphasized that decision-making skills are one of the factors that relate to knowledge and affective-motivational skills that teachers must require to identify and approach students' errors effectively. More specifically, studies have examined how teachers respond to student errors in algebra, including quadratic functions [5], [6]. Hu, Son and Hodge [5], [6] conducted research on how teachers interpret and respond to student thinking in algebraic contexts. They found that teachers' ability to diagnose the underlying misconceptions in student errors was crucial for effective intervention. Research by Köpfer [7] explored the interactions between teachers and students during error episodes. They found that teachers refer to an interaction of student attributes, their own attributes, and error attributes when reasoning their own behaviour. The specific attributes vary depending on the situation, and so do the described reasons that led to a particular behaviour as a spontaneous or more reflective decision. More recent studies have investigated the role of professional development in enhancing teachers' responses to student errors. Matitaputty, Nusantara, Hidayanto and Sukoriyanto [8], [9] highlighted the impact of collaborative teacher communities in improving error-handling strategies. Their research suggested that ongoing professional development and reflective practice were key to helping teachers make more informed and effective decisions when responding to student mistakes.

Much of the existing research on error handling and teacher decision-making focuses on general mathematical topics or broader areas of algebra without delving deeply into quadratic functions. Studies by Köpfer [7] and Wong and Lim [3] provided frameworks for understanding teacher decision-making and the constructive use of errors in mathematics, but they did not specifically address the unique challenges associated with quadratic functions. There is a lack of detailed exploration of the cognitive processes' teachers use when responding to specific types of errors in quadratic functions. Hu, Son and Hodge [5], [6] discussed professional noticing in general algebraic contexts but did not focus on the specific cognitive strategies teachers employ when dealing with quadratic functions. Investigating the cognitive strategies and thought processes teachers use when they encounter mistakes in quadratic functions can provide deeper insights into effective pedagogical practices. Additionally, Matitaputty, Nusantara, Hidayanto and Sukoriyanto [8], [9] examined error handling practices in different cultural contexts but did not delve into how varying classroom environments within the same educational system might affect decision-making. Research that considers diverse classroom contexts and how these influence teacher responses can lead to more tailored and effective instructional strategies. Unlike prior research that has primarily examined the types and frequency of student mistakes, this study specifically investigates the cognitive and affective processes that underpin teachers' decisions in responding to these mistakes. This shift in focus provides a deeper understanding of the internal mechanisms driving involves cognitive processes.

This article aims to expand research on the teacher decision-making process based on three main stages, namely: (1) generating ideas; (2) clarifying ideas, (3) assessing the fairness of ideas. The idea intended in this research is the cognitive experience and attitudes of teachers in choosing the best strategy to eliminate student errors when solving quadratic equation problems. We report the stages teachers experience in making decisions so that teachers will be better at teaching the topic of quadratic equations.

2. METHOD

This study uses qualitative research methodology with a case study approach. Qualitative research focuses on understanding the quality of relationships, activities, situations, or materials [10], and works within an interpretive paradigm [11]. Although the case study approach does not aim to generalize findings, it provides valuable insights into the complexity of the issue under investigation [12]. This study used a case study to investigate teachers' decision-making processes in responding to student errors. The case study emphasizes the important role of fixed teaching strategies in eliminating students' errors as they solve quadratic equation problems. Specifically, we highlight the stages of decision-making as suggested by [13]–[15]

The subject is a teacher who she is professional teacher in state senior high school and has experience in teaching mathematics in seven years, in East Java, Indonesia. There are two stages of data collection, namely written tests by students and diagnostic interview interviews to understand the teacher's decision-making process on students' written answers.

The first stage of the study involved collecting students' written responses in solving two quadratic function problems. These responses were analysed, and categories were generated that explained the students' errors. The literature review provided analytical guidelines in relation to common difficulties, but these were not fixed and trends in the data were allowed to emerge. Data analysis was inductive, moving from a specific analysis of individual student errors to a broad comparative analysis, meaning that comparisons occurred from error pattern to error pattern, error pattern to error category, and error category to other categories. The following presents the test question and concept and procedure of the two test questions.

The second stage of the study involved creating interview questions that were given to the research subjects to elicit responses to the results observed in the students' written tests. The questions aimed to describe the teacher's decision-making process on students' work which includes generating ideas, clarifying ideas and evaluating the reasonableness of ideas. This study adopted the framework developed by [13], [14]. This framework was developed based on the three stages of decision making according to [15]. The interviews were guided by two open-ended questions. The first question was aimed at noticing the situation of recognizing the problem through the question "how did you comment on the student's work?". Furthermore, the second question is to explore the teacher's decision-making process in three stages, namely (1) generating relevant teaching strategies to eliminate student errors, (2) clarifying teaching strategies to eliminate student errors, (3) assessing logical teaching strategies. The framework for analysing teacher response data is presented in Table 2.

The reliability analysis was done by coding. The author conducted the coding independently. Code descriptions were clarified and refined before the interviews were analysed using final coding. The author applied a member checking strategy to ensure the validity of the data and the bias of the author's interpretation. The teacher read,

TABLE 1: Test Questions.

Questions Form	**Concept and procedures involved**
What are the x -intercepts of $f(x) = 4x^2 + 5 + 1$ when y -coordinate is equal to zero on the interval $-2 \leq x \leq -1,5$!	This was the question that introduced a value of that was not 1, increasing the complexity of the task of the task if a student did attempt to utilise factorisation on a solving technique. Before that, students must understand the nature of quadratic (being that the x -intercept occur when $y = 0$, a key concept). To successfully complete this question, students needed to identify the equation as a quadratic and select an appropriate solving technique and understand the concept/s and procedure/s associated with their chosen solving technique. Next, students must pay attention to the interval given in the problem and determine whether the abscissa value falls within the given interval.
Determine the extreme value of $f(x) = 3x^2 - 5x + 8$ on the interval $1 \leq x \leq 4,3$!	In this question, students need to understand the extreme value concept. This question also assessed students' abilities to solve quadratic functions to find the extreme value of function. Students must understand the location at the vertex of the parabola which is determined by the peak (maximum or minimum point). Students can use formula to found the vertex with using $x-$coordinate of the vertext : $x = \frac{-b}{2a}$ and $y-$coordinate of the vertex, with subtitute $x = \frac{-b}{2a}$ back into the quadratic function or using the formula $y = \frac{-D}{4a}$. Next, students must analyse the set interval with the results of the vertex calculation this question, students need to understand the extreme value concept..

TABLE 2: Framework for analysing Teachers' Decision-Making Process.

Stages	Description	Evident when the teacher...
Generating ideas (GI)	Generate relevant instructional strategies to eliminate student errors	classify the possible choices of instructional strategies. detailed precisely and in detail the instructional strategies
Clarifying ideas (CI)	Clarifying of instructional strategies to eliminate student errors.	analysed existing instructional strategies and refer to the stage of building the instructional strategies. comparing or contrasting existing the instructional strategies, providing reasons or clarifications and expressing the assumptions of the instructional strategies.
Evaluating the reasonableness idea (EI)	Assess logical instructional strategies.	reasonableness of the instructional strategies, the assessment is carried out by determining accurate observations, determining reliable secondary sources or based on existing facts or logical and correct principles.

corrected, and commented on the interview transcripts as well as the author's interpretation, and then the author adjusted the interpretation of the data. Content analysis was used to analyse the interviews. The discussion below will begin with a summary of the findings that were analysed and then discussed to find explanations.

3. RESULTS

In this section, the data and analysis of the two problem formulations are presented. We will first present the description of student errors and teacher comments as the initial situation before making action decisions to correct student errors. Next, we will discuss the teacher's decision-making process on student errors.

3.1. Results of student error analysis

First, the data presented is a description of students' successes and failures for both quadratic function problems. Content analysis was conducted to determine the theme of the students' work. This analysis was based on teachers' comments on students' errors in recognizing the initial situation before making decisions to eliminate students' errors. Table 3 presents the overall percentage results for both problems.

TABLE 3: Number and percentage of students who answered the question successfully, unsuccessfully or did not attempt to answer from a sample of 33.

Questions	Correct answer	Wrong answer	Did not attempt
What are the abscissas of $f(x) = 4x^2 + 5 + 1$ when ordinate is equal to zero on the interval $-2 \le x \le -1,5!$	8 (24%)	18 (55%)	7 (21%)
Determine the extreme value of $f(x) = 3x^2 - 5x + 8$ on the interval $1 \le x \le 4!$	4 (12%)	5 (15%)	24 (73%)

Based on the data presented in table 3, it shows that most students experience challenges. The overall data illustrates that students' ability to solve quadratic function problems is very poor. Students' difficulties occur along with the complexity of the question, especially when students determine the extreme value of a quadratic function. Most students did not try to answer question number 2 because they could not remember any method to solve the problem and did not understand the meaning of extreme value. This conclusion was obtained from the teacher's perception or conjecture during the interview. The teacher mentioned that students did not understand what the extreme value of a function was and forgot the formula to find the extreme value of a function.

The teacher's responses to both questions are explored in detail in Figure 2 and Figure ?? below.

<table>
<tr>
<td>$4x + 5x + 1 = 0$
$x = \frac{(-b \pm \sqrt{(b^2 - 4ac)})}{2a}$
$a = 4,\ b = 5,\ c = 1$
$x = \frac{(-5 \pm \sqrt{(5^2 - 4(4)(1))}}{2(4)}$
$x = \frac{(-5 \pm \sqrt{9})}{8}$
$x = -1$ atau $x = -\frac{1}{2}$
Karena rentang $-2 \leq x \leq -1,5$, maka absis jika ordinatnya $= 0$ adalah $x = -1,5$.</td>
<td>Students understand that f (x) = y is the ordinate and x is the abscissa. So that students change the quadratic function into the form of a quadratic equation. Furthermore, because the value of a≠1, students choose to use the quadratic formula method to find the roots of the equation or the abscissa value that meets. After writing the formula, students determine the values of a, b and c from the given equation. Students can substitute the coefficient values correctly. Students substitute the values of a, b and c in the formula and perform calculation operations. Students are correct in calculating the determinant value, namely the root of 5 multiplied by 2 minus 4 times 4 times 1 so that the root of 25 minus 14 is 9. The root value of 9 is understood by students as 3 so that students can continue the completion process. Students notice that the value of the number under the root is a positive integer.
Students miscalculate the value of x2 by adding -5 and 3 divided by 8 to obtain $-\frac{1}{2}$.
Students pay attention to the interval range but cannot connect it with the abscissa value. Students have difficulty understanding the position of negative numbers on the real number line.</td>
</tr>
<tr>
<td colspan="2">Summary: Students understand that in the geometric interpretation of the quadratic equation ax2+bx+c=0 and find the x- point where the graph of the quadratic function ofof f(x)= 4x 2+5+1 intersects the -axis. The roots of the quadratic equation 4x2+5+1=0 re the x- points where f(x)=0. These points are the points of intersection of the paabola with the x axis.</td>
</tr>
</table>

Figure 1: Example of a student using the quadratic formula to find the x-axis intersection point and incorrectly identifying the critical points as being within the interval when they are not.

Furthermore, we analysed students' work on two test problems that students solved with the standard form $a \neq 1$ and using interval endpoints. It was found that almost a quarter of the students were successful in solving problem number 1. The student work of the unsuccessful students indicated a variety of misconceptions. The first problem not only focused on students' ability to find roots (abscissas where the ordinate is zero) using quadratic formula but also checking if the roots lie within the given interval. The second problem not only focuses on students' ability to use the vertex formula $x = \frac{-b}{2a}$, to find the x-coordinate of the vertex but also finding the critical point and evaluating the function at the interval's endpoints to determine the extreme values.

Figure 2 and Figure ?? illustrate that every problem given must have a solution. This finding is in accordance with the types of errors found in both problems where students' answers show students' answers that do not pay attention to the specified interval given in the problem. This is supported by the results of interviews with teachers when asked to respond to the work of these two students. The teacher's response included

$4x + 5x + 1 = 0$ $x = \frac{(-b \pm \sqrt{(b^2 - 4ac)})}{2a}$ $a = 4,\ b = 5,\ C = 1$ $x = \frac{(-5 \pm \sqrt{(5^2 - 4(4)(1))})}{2(4)}$ $x = \frac{(-5 \pm \sqrt{9})}{8}$ $x = -1$ atau $x = -\frac{1}{2}$ Karena rentang $-2 \leq x \leq -1{,}5$, maka absis jika ordinatnya $= 0$ adalah $x = -1{,}5$.	Students understand that f (x) = y is the ordinate and x is the abscissa. So that students change the quadratic function into the form of a quadratic equation. Furthermore, because the value of a≠1, students choose to use the quadratic formula method to find the roots of the equation or the abscissa value that meets. After writing the formula, students determine the values of a, b and c from the given equation. Students can substitute the coefficient values correctly. Students substitute the values of a, b and c in the formula and perform calculation operations. Students are correct in calculating the determinant value, namely the root of 5 multiplied by 2 minus 4 times 4 times 1 so that the root of 25 minus 14 is 9. The root value of 9 is understood by students as 3 so that students can continue the completion process. Students notice that the value of the number under the root is a positive integer. Students miscalculate the value of x2 by adding -5 and 3 divided by 8 to obtain $-\frac{1}{2}$. Students pay attention to the interval range but cannot connect it with the abscissa value. Students have difficulty understanding the position of negative numbers on the real number line.
Summary: Students understand that in the geometric interpretation of the quadratic equation ax2+bx+c=0 and find the x- point where the graph of the quadratic function ofof f(x)= 4x 2+5+1 intersects the -axis. The roots of the quadratic equation 4x2+5+1=0 re the x- points where f(x)=0. These points are the points of intersection of the paabola with the x axis.	

Figure 2: Example of students using x -coordinate of the Vertex using x=(-b)/2a and forgetting to identify the critical points as being within the interval when they are not.

“students are not used to solving problems with certain intervals so that students do not connect the answer results with the questions given”. This suggests that students do not have a deep conceptual understanding of what is meant by the solution to the two problems given. Overall, students experience success when trying to solve quadratic function problems through solving the quadratic equation to find the roots by applying the quadratic formula and using the vertex formula to find the critical point but have not checked the solution obtained with the given interval.

At this stage it was found that the responses given by the teacher showed that the teacher was able to recognize student errors including recognizing the initial situation of solving the problem, interpreting student answers, and realizing student errors. The teacher's response focuses on the ability to read and analyse the problem, use the formula, understand the interval given in the problem, and evaluate the function at the interval boundary points.

3.2. The results of the interview analysis in the teacher's decision-making process.

In this section we describe the teacher's decision-making process in correcting student errors that we have presented in the previous section. The findings are described based on the 3 stages of decision making namely generating ideas, clarifying ideas, and assessing the reasonableness of ideas for both types of student errors.

Based on the interview results, the teacher was able to generate several ideas after paying attention to the initial situation of solving problem 1. The intended idea is a learning strategy aimed at eliminating student errors. First, the teacher gave several possible choices of teaching strategies and then the teacher explained in detail the teaching strategies that had been delivered. There are three teaching strategies presented by the teacher for problem number 1. The ideas are explained in the statement:

"I will build a guided discussion with students with some questions to help students realize their mistakes (Idea 1) besides that I will try to give a similar problem and replace this interval with a limitation of $-2 \le x \le -\frac{1}{4}$ or other interval (Idea 2). I will also try to present this problem visually by first sketching the graph of the function with the specific interval (Idea 3)."

As for problem number 2 the teacher mentioned some ideas as follows:

"I will give feedback based on the conflict experienced by students. First, I will ask students to pay attention to the problem as a whole and give some explanations about extreme values (Idea 1). Next, I will guide the students to correct their answers by analysing whether the given interval satisfies or corresponds to the value of the axis of symmetry. If it does, which interval does it correspond to? Does the solution satisfy if the interval is replaced with x real numbers or x integers? (Idea2). In addition, I will also try to use the GeoGebra application to provide visual understanding (Idea 3)".

Teacher comments show that teachers can generate relevant ideas to help students correct mistakes more efficiently and reduce the likelihood of repeating the same mistakes. Teachers are also able to consider a variety of effective teaching methods, one of which is paying attention to visual learning styles. In addition, our findings show that teachers not only provide several ideas but also explain in detail each planned idea. Some of the guided guidance delivered by the teacher includes focusing students on what is known and what is asked in the problem; understanding of abscissa and ordinate and their relationship with the roots of quadratic equations; paying attention to pay attention to the operations and calculation results that students do; paying attention again to each information in detail; ensuring students' understanding of interval specifics, encouraging students to be more open to learning and error correction, using technology to help identify and analyse student errors as well as providing alternative solutions that are more effective and interactive..

Furthermore, at the clarifying idea stage the teacher's comments in the interview showed that the teacher was able to clarify the ideas mentioned in the previous stage. In problem number 1 the teacher made several assumptions about the ideas presented then compared the ideas. The clarification of ideas begins with the teacher's comment that students make mistakes in operating numbers that produce fractional values and do not understand the position of negative integers on the number line. Thus, at this stage the feedback approach emphasizes the conflict experienced by students.

Furthermore, for problem number 2. The teacher makes an argument that students do not experience problems in performing calculations, the main problem is that students have not been able to justify the interval value given does not meet the Solution obtained. Thus, the feedback given is more directed at understanding the problem by developing the problem, especially the specification of suitable and unsuitable intervals.

Teacher comments show that teachers can clarify relevant ideas to help students correct mistakes more efficiently. Teachers are also able to consider clear and constructive feedback. The feedback given by the teacher is specific and to the point of the mistakes made by the students. Some questions are given to ensure students understand the material and can overcome their mistakes. In addition, teachers also think about how to develop students' metacognitive abilities, such as self-reflection and problem-solving strategies, helping them identify and correct errors independently.

In the last stage, assessing the reasonableness of ideas is based on the previous stages. After the teacher has successfully clarified the idea then in the assessing the reasonableness of the idea stage the teacher will verify that the proposed idea is practical and effective. The teacher's activity in this stage is to make a final decision or choose the best alternative feedback idea.

Based on the results of the interview, in problem number 1 the teacher made the final decision by determining the best feedback by choosing to give similar problems and replacing this interval with the limit $-2 \leq x \leq -\frac{1}{4}$ or other intervals. the other two ideas were eliminated on the grounds that this idea was an idea that could develop learning more effectively and train students' critical thinking skills.

Furthermore, in problem number 2, the teacher determined the best option by trying to use the GeoGebra application to improve students' understanding visually. This was done with the assumption that technology can be used to provide interactive and instant feedback, as well as to identify patterns of student errors through data analysis. The teacher's explanation of using the GeoGebra application in explaining problem number 2 is presented in Figure 2.

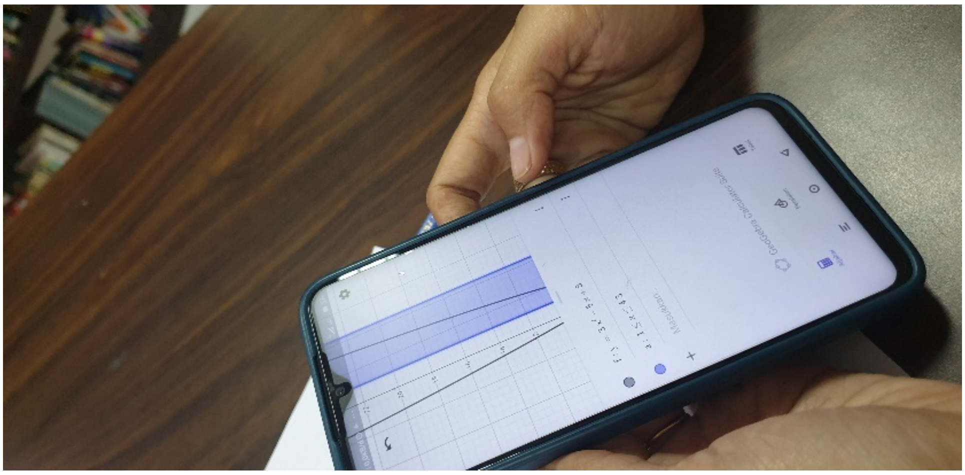

Figure 3: The use of GeoGebra application for student error correction.

Teachers in their efforts to provide several alternative corrective actions can decide on effective actions that show the teacher's decision-making process associated with the pattern of errors made by students. Overall, the teacher can provide several alternative corrective actions, make arguments about the ideas presented, consider effective decisions, and determine the best action decision based on the teacher's perception of students' errors and abilities.

4. DISCUSSION

This study aims to describe students' errors in solving quadratic functions and then guide the teacher's decision-making process in solving quadratic equations. In this way, the findings of this study provide insight into the impact of improved learning on the topic of quadratic functions.

This study adds to the collection of research on student work on algebra topics, especially quadratic function topics [16]–[19]. Overall, it was clear that students showed weaknesses in both procedural and conceptual aspects so that they could not find the right solution. After the teacher realized the students' errors, further interviews were conducted regarding the best action chosen by the teacher to correct the students' errors. The analysis of student errors aims to understand the initial situation of solving problems that students do. The interview data confirms that the teacher can pay attention to each student error both procedural and conceptual errors. In this case, this finding provides empirical evidence for [20] that "through the formative assessment, teachers can be made aware of what aspects of the lesson needed reteaching, or lacking through analysing students' solutions and answers to identify and correct their errors". Knowledge of the set of real numbers is an integrated part of the concept of quadratic functions. It becomes the basic conceptual concept that helps students to understand the problem.

In other words, this knowledge should be key in understanding and developing the topic of quadratic functions. The data presents evidence that forgetting, or lack of conceptualization occurred for most of the students in this sample.

Furthermore, the analysis of students' work has the potential to support teachers' assessment of learning practices and self-reflection. At the generating ideas stage, it shows that teachers have good knowledge by mentioning several alternative pedagogical actions that focus on developing students' thinking processes. In this case the teacher understands about students' needs and understanding [21, 22]. This shows that teachers can benefit from student errors to design remedial pedagogical strategies and develop more complex aspects of the curriculum [23]. Thus, the findings of this study indicate that it is important for teachers to make alternative improvements to teaching by integrating other concepts to enrich students' thinking in solving problems.

5. CONCLUSIONS AND RECOMMENDATIONS

It is important for teachers to pay attention to students' errors effectively so that teachers can design learning improvements by integrating several concepts on a topic can help enrich students' thinking about algebra, especially solving quadratic function problems. Choosing the best alternative learning strategy to eliminate student errors can help optimize student learning. This empirical study advances the field of teacher education and professional learning by providing evidence from teachers on how they work to assess student work specifically in terms of their assessment decisions. This study was conducted with a limited sample so the results may not be fully representative for all teachers. Further research is recommended using a larger and more diverse sample to increase the generalizability of the findings.

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