KnE Social Sciences
ICMScE
International Conference On Mathematics And Science Education
Volume 2024

Research Article

Students' Errors in Solving Powers and Roots Based on AVAEM (ARITH, VAR, AE, EQS, and MATH) Categories

Cyndana Kartika Putri*, Sufyani Prabawanto, Dadang Juandi

Mathematics Education Department, Universitas Pendidikan Indonesia Jl. Dr. Setiabudi No. 229, Bandung 40154, Indonesia

ORCID
Cyndana Kartika Putri: https://orcid.org/0009-0001-2971-7801
Sufyani Prabawanto: https://orcid.org/0000-0003-2872-6535
Dadang Juandi: https://orcid.org/0000-0001-6997-1399

Abstract.
A power can be seen as the product of multiplying a number by itself, whereas the root is the opposite of power. These topics are very important to be understood by junior high school students. This study aims to describe students' errors in solving powers and roots tasks based on AVAEM (ARITH, VAR, AE, EQS, and MATH) categories. The type of this research is qualitative research with a case-study design. The subjects of this research were 4 students of grade 9, at a junior high school in Banjarmasin, South Borneo. Data were collected using tests and interviews. During the study, it was found that errors made by students included errors in applying arithmetic operations (ARITH), errors in understanding and distinguishing the roles of literal symbols (VAR), errors in understanding algebraic expressions (AE), errors in distinguishing the meaning of the "=" sign (EQS), and errors in mathematization (MATH). Errors made by students in solving tasks are the gateway to analyzing the existence of learning obstacles. Therefore, the implication of this study is the a first step to investigate learning obstacles experienced by students in powers and root forms.

Keywords: students' errors, solving powers, roots based on AVAEM

Corresponding Author: Cyndana Kartika Putri; email: cyndanakartikaputri@upi.edu

Published: 26 April 2024

Publishing services provided by Knowledge E

Selection and Peer-review under the responsibility of the ICMScE Conference Committee.

1. INTRODUCTION

Algebra is one of the key branches of mathematics that can be used to learn more about a situation [1]. Algebra does not only stand alone, but as a unifying thread which interlaces almost all of mathematics [2]. However, the facts show that students' algebra skills in Indonesia are still weak. Results Based on the 2011 TIMSS analysis, only 18% of students in Indonesia answered correctly on the reasoning algebra content, and even only 1% answered correctly on the applied algebraic content [3]. The algebraic ability of students in Indonesia is proven by the results of previous studies related to difficulties or errors when studying algebraic topic [4–6].

One of the algebraic topics taught at the junior high school level is powers and roots. A power can be seen as the product of multiplying a number by itself, which consists of two numbers that can be written by a^b where 'a' as the base number and 'b' as the exponent. Whereas the root is the opposite of power. Powers and roots are very important topics to be understood by students because these topics are prerequisite topics in studying exponential and logarithmic equations and inequalities at the senior high school level and other more abstract topics in mathematics.

Many studies have been carried out on powers and roots [7–9]. People consider that powers and roots are easy topic, but in fact there are still many students who have difficulty solving task related to this material, especially when they use exponent rules [7]. Based on the results of daily test from four classes related to powers and roots in the 2012/2013 school year, the percentage of students' completeness in each class did not reach 35% [8]. This happens because students are weak on the basic concepts of integers, the properties of powers and roots, and the procedures and logarithms needed to solve problems [8]. The difficulties faced by students in a topic or material can be seen through the errors they make when completing the given task. Students' errors are a kind of consistent and systematic error [10].

There is a study that examines student errors in the material of powers and roots. A study concluded that grade 9 junior high school students did not understand the material about powers and roots [9]. Based on the results of the study, there were conceptual and procedural errors made by students when solving problems on the powers and roots [9]. It is possible that there are other errors that have not been identified, especially if the errors made by students are categorized into AVAEM (ARITH, VAR, AE, EQS, dan MATH) categories. Students' errors in algebra are classified into five categories, namely ARITH, VAR, AE, EQS, and MATH (AVAEM). The following is a brief explanation of these five categories [4].

1. ARITH (Arithmetic)

2. Related operations: student errors when carrying out operations on numbers or algebraic expressions.

3. Related rules: students do not follow the rules of the order of arithmetic operations on either numeric or algebraic expressions.

4. Related properties: students are unable to apply the properties of numerical operations (commutative, associative, inverse, or distributional) when dealing with numbers or algebraic expressions.

5. VAR (Variable)
6. Roles of literal symbols: students cannot distinguish the roles of literal symbols as placeholders, generalized numbers, unknowns, and varying quantities.
7. AE (Algebraic Expression)
8. Parsing obstacle: students have difficulty in understanding the order in the original language and algebraic expressions.
9. Expected answer obstacle: students expect to have a numerical answer to an algebraic expression.
10. Lack of closure obstacle: students perform addition and subtraction operations on numbers and algebraic forms to obtain algebraic forms with an algebraic expression.
11. Lack of gestalt view: students ignore visual salience of algebraic expressions, both on pattern salience (sign of inequality, variable x, negative or positive sign of an algebraic term) and local salience (algebraic expression with two terms and within a bracket).
12. EQS (Equal Sign)
13. Different meanings: students do not understand the different meanings of the equal sign in algebra (“is algebraically equivalent to”) and in arithmetic (doing calculations and writing answers).
14. MATH (Mathematization)
15. Horizontal mathematization: students have difficulty formulating or translating between problem situations and mathematics.
16. Vertical mathematization: students have difficulty in reorganizing the mathematical system or the process of moving with symbols.

Based on interview with a teacher, many students did not follow the exponent rules so that they are not able to complete the task given correctly. In operating the root forms, students tend to directly add or subtract the numbers inside the roots. If classified into the AVAEM categories, the students’ error is categorized as ARITH, which is related operations. Teachers should pay attention to how students respond, and errors made when solving problems [11]. Through the errors made by students, it can be revealed what learning obstacles are faced when studying the topic of powers and root forms

so that the teacher can use it as a reference in designing learning methods on powers and roots so that students can achieve great learning outcomes and are more ready to accept material at a higher level of education. This study aims to describe students' errors in solving power and roots task based on AVAEM (ARITH, VAR, AE, EQS, and MATH) categories.

2. RESEARCH method

This study used qualitative research method with case study design. The subjects were 4 students of grade 9, a junior high school in Banjarmasin, South Borneo. The main instrument in a qualitative research is the researcher itself. Therefore, it is important for qualitative researchers to have perception into the field under study [12]. The data in this study were collected using test and interview. Students were required to answers six tasks related to powers and roots. Unstructured interview were conducted to strengthen the information obtained from student test results. Data analysis techniques in this study include data reduction, data presentation, and conclusion. The data would be identified and classified into five types of errors according to AVAEM categories, i.e. ARITH, VAR, AE, EQS, and MATH. The procedure of data processing in this study consists of, (1) recording all student responses in completing the given task, (2) analyzing student responses and seeing patterns of errors made by students, (3) describing patterns that have been observed to in simple language and possible reasons for the mistakes made by students, and (4) interviewing students by asking them to explain how they completed the task to confirm the suspected error pattern.

3. result and discussion

This study discusses students' errors in powers and roots topic. Students' errors in this study were analyzed based on test results and interview responses. The results show that there are five categories of errors made by students in completing task related to powers and roots.

The first error identified was ARITH. The ARITH category is classified into three concerns, namely capabilities in carrying out arithmetic operations, understanding in the priority rules of arithmetic operations, and applying properties of numerical operations [4]. In this study, all students made an ARITH error. These results are in accordance with other research that ARITH error category is the most common error found in students than other error categories [13]. One of students' errors related to ARITH are shown

in Figure 1. In Figure 1, students were asked to perform arithmetic operations with exponents. One of students made errors from the start since doing the calculation where the expected result is . Based on interview response, the student said that the result was 8 because 2 to the power of 3 equals 8, and then the base number and its power (exponent) are both negative, so negative multiplied by negative results in a positive. Here it is seen that the student did not apply or even did not understand exponent rules that should be used in working the task. In addition, the student also tried to perform addition operations on rational numbers in a way such as doing multiplication on rational numbers which should be in addition on rational numbers must equalize the denominator first. The most influential factors in ARITH are students made errors when carrying out arithmetic operations and does not follow the rules of order of arithmetical operations in numerical or in algebraic expressions [4, 13].

Figure 1: Example of student's error in ARITH.

The next error made by students was VAR. Students who have difficulty in the VAR category can be seen from their inability to distinguish the roles of literal symbols as placeholders, generalized numbers, unknowns, and varying quantities [4]. In this study, one out of four students made a VAR error. Example of student's error in VAR can be seen in Figure 2. In Figure 2, students are asked to determine the value of 'a' of a given equation. As can be seen, the student has done exactly what was asked and expected until he gets the value of 'a' is equal to 7. However, the student still continues his work by substituting the value so that the student gets the final result equals 14. The response from interview shows that the student did this to get the value of 'a'. According to the student's response, the value of 'a' equals to 14, not 7. This shows that students have difficulty understanding the role of the literal symbol as an unknown [4].

The third error identified was AE. The AE category is the difficulty of students in understanding algebraic expressions [4]. In this study, two out of four students made an AE error. Example of student's error in AE can be seen in Figure 3. In that task, students are asked to simplify powers. To do this task, students need to apply some

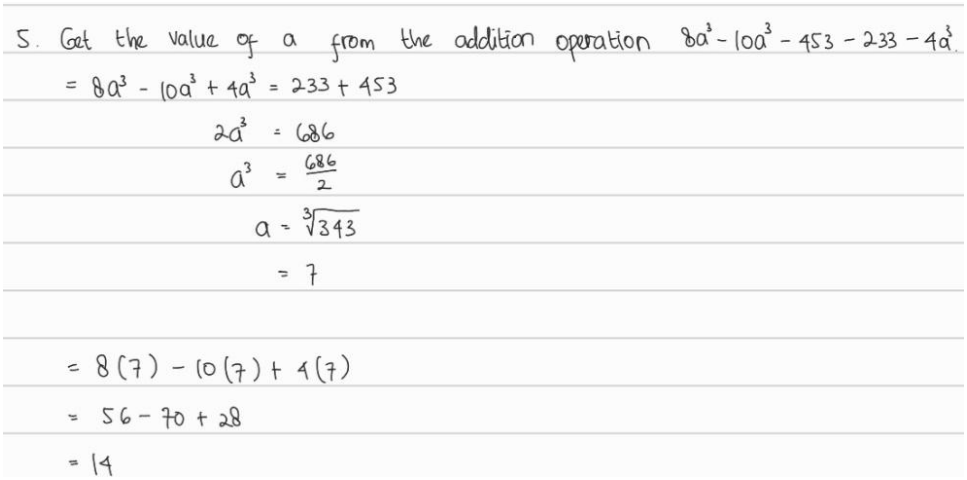

Figure 2: Example of student's error in VAR.

exponent rules so that the expected result from student is . However, on of students immediately carrying out addition on the numbers that become the numerator such as . In the denominator, the student carrying out addition on the two powers that is and to to get . This shows that the student tried to perform addition or subtraction on algebraic terms and numbers to get an algebraic term within an algebraic expression [4]. Therefore, the errors made by these students are classified into AE category.

Figure 3: Example of student's error in AE.

The fourth error was EQS. The EQS category is a student's error in understanding the meaning of the "=" sign in algebra or arithmetic [4]. In this study, one out of four students made an EQS error which shown in Figure 4. In that task, students are asked to determine whether the given statement is true (B) or false (S). The expected answer is true (B). The student's work to get is correct even though there is an incorrect way of writing. According to what is in the problem, students need to change the base of powers that is 9 to 3 so that the expected result is equal to . However, the response from the interview shows that the student directly carrying out division on the base of powers by 3 to equate it with what is in the problem without changing the power (exponent) by using the exponent rules. So, the student state that is equal to . Therefore,

it can be concluded that students do not understand the meaning of the equal sign as algebraic equivalence ("is algebraically equivalence to") [4].

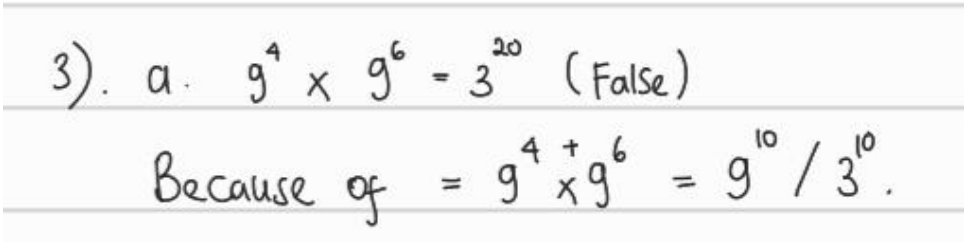

Figure 4: Example of student's error in EQS.

The last error identified was MATH. The MATH category is a student's error in doing mathematization both horizontally and vertically [4]. In this study, two out of four students made a MATH error. Example of student's error in MATH can be seen in Figure 5.

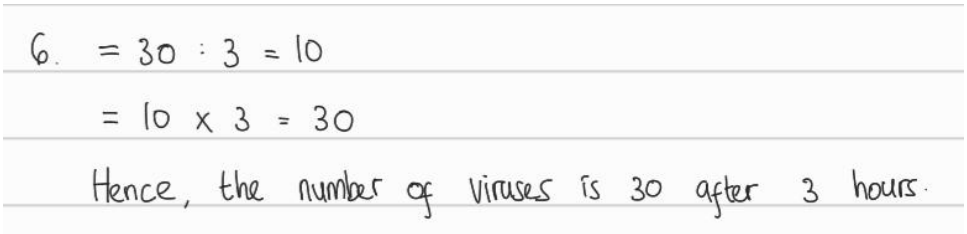

Figure 5: Example of student's error in MATH.

In that task, students were asked to determine the number of viruses in 3 hours if in the beginning there is only 1 virus and the virus divides into 3 viruses every 30 minutes. Therefore, the mathematical model expected by students is with the total viruses in 3 hours is equal to 729 viruses. Based on the students' work in Figure 5 and the interview response, the student immediately carrying out division on 30 by 3 because there is a sentence in the question which states that every 30 minutes the virus divides into 3 viruses. Such errors occurred because students could not understand the context of the given problem so they failed to formulate a mathematical model from the context of the problem [14, 15]. Students are not able to understand what is known and asked in the given task [16]. This means that the student have difficulty when dealing with word problems whose context is related to real life, namely horizontal mathematization [4].

4. CONCLUSION

Teachers need to know the errors made by students when working on task related to powers and roots so that it becomes a consideration for future learning on the same material and so that students have no difficulty when facing more abstract material in

the future. Students also need to know the errors they made so that they can correct and not repeat the same errors. The results of this study shows that there are five categories of errors made by students when solving task related to powers and roots, that is ARITH, VAR, AE, EQS, and MATH. Errors made by students in solving task can also be considered as a signs of learning obstacles experienced by students when studying powers and roots topic. Therefore, the implication of this study is as a first step to investigate learning obstacles experienced by students in powers and roots viewed by the perspective of AVAEM categories.

References

[1] Van Laren L, Moore-Russo D. The most important aspects of algebra: responses from practising South African teachers. African Journal of Research in Mathematics, Science and Technology Education. 2012;16(1):45–57.

[2] Herstein IN. Topics in algebra. John Wiley & Sons; 1991.

[3] Hadi S, Novaliyosi N. "TIMSS Indonesia (Trends in international mathematics and science study).," In:Prosiding Seminar Nasional & Call For Papers(2019).

[4] Jupri A, Drijvers P, van den Heuvel-Panhuizen M. Difficulties in initial algebra learning in Indonesia. Math Educ Res J. 2014;26(4):683–710.

[5] Muchoko C, Jupri A, Prabawanto S. "Algebraic visualization difficulties of students in junior high school.," In:Journal of Physics: Conference Series. pp. 32108.IOP Publishing(2019).https://doi.org/10.1088/1742-6596/1157/3/032108.

[6] Pratiwi V, Herman T, Suryadi D. "Algebraic thinking obstacles of elementary school students: A Hermeneutics-phenomenology study.," In:Journal of Physics: Conference Series. pp. 32115.IOP Publishing(2019).https://doi.org/10.1088/1742-6596/1157/3/032115.

[7] Karim GS, Fatmawati N, Wijayanti D. Alternative in providing proof of negative exponent for junior high school students [Jurnal Ilmiah Pendidikan Matematika]. JIPM. 2020;8(2):98–107.

[8] Herawati E. Upaya meningkatkan motivasi dan hasil belajar siswa menggunakan media pembelajaran kartu domino matematika pada materi pangkat tak sebenarnya dan bentuk akar kelas ix smp negeri unggulan sindang Kabupaten Indramayu [Jurnal Nasional Pendidikan Matematika]. JNPM. 2017;1(1):66–87.

[9] Siagian P, Surya E. Analisis kesalahan dalam menyelesaikan soal matematika materi Perpangkatan dan Bentuk Akar. Jurnal Mathematic Paedagogic; 2018. pp. 1–12.

[10] Azifa DM. “An analysis of students’errors in solving mathematics word problems related to linear equation,”MATHEdunesa. vol. 7, no. 1, p. 2018.

[11] Andini W, Jupri A. Student obstacles in ratio and proportion learning. J Phys Conf Ser. 2017;812(1):012048.

[12] Anindiya A. Analisis kesulitan siswa terhadap masalah operasi hitung bentuk aljabar dengan menggunakan teori avae (ARITH, VAR, AE DAN EQS) di Pusat Kegiatan belajar Masyarakat (PKBM). Paket B; 2019.

[13] Jupri A, Drijvers P. Student difficulties in mathematizing word problems in algebra. Eurasia J Math Sci Technol Educ. 2016;12(9):2481–502.

[14] Sumiati A, Agustini Y. Analisis kesulitan menyelesaikan soal segiempat dan segitiga siswa SMP kelas VIII di Cianjur. Jurnal Cendekia: Jurnal Pendidikan Matematika. 2020;4(1):321–31.

[15] Nur’aini JP, Munandar DR. “newman dalam menyelesaikan soal eksponen pada siswa kelas X SMA At-Taubah TirtamulyA.,”JPMI (Jurnal Pembelajaran Matematika Inovatif). vol. 4, no. 5, pp. 1065–1072, 2021.

[16] Lai CF.Error Analysis in Mathematics.Technical Report# 1012. ERIC, 2012.