Research Article

Analysis of Students' Geometrical Thinking from Geometry Task Related to HOTS from PISA

Muh. Khaedir Lutfi[1,2*], Endang Cahya Mulyaning[1], and Fitri Annisa Kusumastuti[2]

[1]*Departemen Pendidikan Matematika, FPMIPA, Universitas Pendidikan Indonesia, Bandung. Jl. Dr. Setiabudi No.229, Isola, Kec. Sukasari, Kota Bandung, Jawa Barat 40154, Indonesia*
[2]*Program Studi Pendidikan Matematika, FKIP, Universitas Tangerang Raya, Kabupaten Tangerang Kompleks Perumahan Sudirman Indah Blok E, Kec. Tigaraksa, Kabupaten Tangerang, Banten 15720, Indonesia*

ORCID
Muh. Khaedir Lutfi: https://orcid.org/0000-0002-6110-7843
Endang Cahya Mulyaning: https://orcid.org/0009-0009-8495-7873
Fitri Annisa Kusumastuti: https://orcid.org/0009-0009-0703-3968

Corresponding Author: Muh. Khaedir Lutfi; email: muh.khaedir.lutfi@upi.edu

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Abstract.
This study intends to analyze geometry tasks related to Higher Order Thinking Skills (HOTS) through geometrical thinking level by Van Hiele which consists of visualization, analysis, abstraction, deduction, and rigor. This study only focuses on visualization, analysis, and abstraction levels. The stages in this study are divided into three steps. Chronologically, the first stage begins with the process of compiling PISA tasks related to geometry and HOTS, especially for examining the students' spatial abilities. From all the tasks obtained, three tasks match the criteria that were most relevant to Van Hiele's geometrical thinking level. The second stage was continued by testing the instrument test to 23 students from one of the schools in Tangerang Regency as respondents. In the last stage, analyze the students' answers based on the steps of their work and compare them with the theory of geometrical thinking from Van Hiele. The results of this study indicate that there are differences between the steps of the students' answers and theoretical answer steps that are expected in this study. The conclusion that can be drawn is that the students' learning experiences in understanding geometrical concepts are not theoretically compatible with the principles of spatial ability, especially in completing geometry tasks related to HOTS.

Keywords: Students' Geometrical Thinking, Geometry Task, PISA

1. INTRODUCTION

In solving mathematical problems, the students must be able to understand simple concepts to higher concepts [1]. So that in developing the students' abilities in solving given problems, it is necessary to have a load of tasks that can hone the students' Higher Order Thinking Skills (HOTS) [2]. Based on the level of thinking from Bloom's Taxonomy, HOTS includes the ability to analyze, evaluate, and create [3, 4]. Therefore, the role of HOTS in the student learning is significant because it is considered to be able to help the students become good problem solvers.

Geometry is an essential topic in mathematics [5, 6], which is considered as a topic helping the students to expand their problem solving [7, 8]. Through the problem solving , the students are able to figure out a geometry problems, namely understand a text, create a diagram, find out patterns, obtain a relevant information, make a conclusion, and the last is formulate a solution [9]. In addition, geometry is also important in the students' daily lives [10].

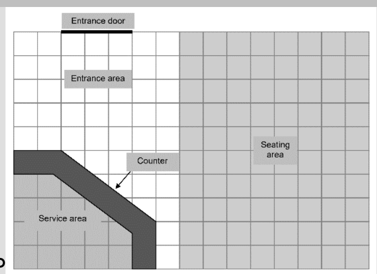
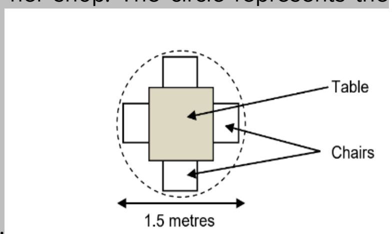
The current challenge is how the students can use geometrical concepts to complete tasks related to Higher Order Thinking Skills (HOTS). Several previous studies have also shown that there are weaknesses to the students in solving HOTS tasks [11, 12]. One source of assignments that is often used comes from the Program for International The student Assessment (PISA). Not only demanding mastery of concepts, the HOTS tasks from PISA also emphasize how these concepts are applied to various situations [13]. In addition, the HOTS tasks from PISA can train reasoning, problem solving, and analysis [14]. HOTS-oriented the tasks can train the students' spatial abilities, especially in learning geometry [15]. In addition, spatial ability is also closely related to geometry [16, 17]. The students who can complete HOTS tasks are classified as good in their spatial abilities.

One of the abilities expected in geometry learning is spatial ability, one of which is proposed by Maier which consists of Spatial Perception, Visualization, Mental Rotation, Spatial Relations, and Spatial Orientation [18]. The students' spatial ability is also influenced by their experience in learning geometry [19]. One of theories related to spatial ability is geometrical thinking [20]. However, previous research shows that the students' spatial abilities are still relatively low, especially in the process of analyzing, evaluating, developing, and creating [21]. Therefore, it is necessary to have a geometrical thinking level designed in geometry learning to see the extent to how the students' spatial abilities are [22]. Van Hiele then suggested five levels of geometrical thinking ability consisting of Visualization, Analysis, Abstraction, Deduction, and Rigor [19]. Through these components, the students can understand geometrical concepts and are skilled in using these concepts to solve everyday problems. Theory of geometrical thinking from Van Hiele can describe the different levels of understanding of each student in learning geometry [23]. Therefore, this study tries to see the process of each geometrical thinking level from Van Hiele in completing geometry tasks in HOTS and to compare the steps in solving the problem with the steps expected based on theory of geometrical thinking in this study.

2. RESEARCH METHOD

This study focuses on qualitative approaches. How this study analyze the students' geometry task, verify the process of problem solving, identify the use of geometrical concept by the students. Researchers conducted 3 stages in a qualitative study involving 23 grade IX the students in a school in Tangerang Regency. In the first stage, the researcher took three tasks from PISA as a test instrument related to HOTS in geometry material. The tasks taken involve the students' spatial abilities. So that the test instrument used can see the level of geometrical thinking of the students based on The Van Hiele theory of geometrical Thinking. The three tasks are presented in Table 1.

Table 1: Three task about geometrical thinking.

No.	Geometry Tasks
	 ICE-CREAM SHOP This is the floor plan for Mari's Ice-cream Shop. She is renovating the shop. The service area is surrounded by the serving counter . Note: Each square on the grid represents 0.5 meters × 0.5 meters.
1	**Question 1: ICE-CREAM SHOP** Let's want to put new edging along the outer edge of the counter. What is the total length of edging she needs? Show your work.
2	**Question 2: ICE-CREAM SHOP** Mari is also going to put new flooring in the shop. What is the total floor space area of the shop, excluding the service area and counter? Show your work.
3	**Question 3: ICE-CREAM SHOP** Mari wants to have sets of tables and four chairs like the one shown above in her shop. The circle represents the floor space area needed for each set.  For customers to have enough room when they are seated, each set (as represented by the circle) should be placed according to the following constraints: Each set should be placed at least at least 0.5 meters away from the walls. Each set should be placed at least at least 0.5 meters from other sets. What is the maximum number of sets that Mari can fit into the shaded seating area in her shop?

The next stage is the researchers tested the test instrument to the students. All students are required to complete the test within 60 minutes. The last step is to analyze the students' answers based on the steps of their work then the results of the analysis

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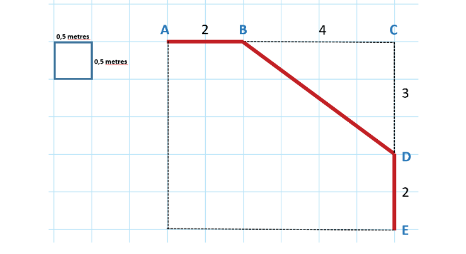
are compared with the geometrical thinking level of Van Hiele by looking at the spatial thinking process. The results of the final analysis obtained were a description of the students' abilities in applying geometrical concepts to solve the given problem, as well as the suitability of the student's working process with theory expected in this study.

3. RESULTS AND DISCUSSION

The results of analysis from the tasks that were tested on the students are presented in this discussion. The test instrument number 1 identifies the students' skills that involve the concept of the Pythagorean Theorem in identifying the overall length of the table edge. Test number 2 was tested to assess the students' ability to use basic geometrical concepts and their steps in thinking spatially based on several flat shapes identified. Then the last test instrument is used to see the students' skills in finding solutions by visualizing some of the shapes formed. Overall, the test is intended to assess the students' skills in completing the task based on HOTS by looking at their geometrical thinking level.

The expected completion steps as a solution to the test instrument number 1 can be seen in Figure 1. It can be seen from the expected process in the image that the students are required to be able to visualize the figure on the test instrument presented in their contextual circumstances. In this case, the student try to move the visual form of the problem into a simpler visualization of figure based on their shape. This process involves the students' spatial abilities, especially at the visualization level. By carrying out the visualization process, the students are easier to identify the given problem. Furthermore, the students can analyze the visualized figures based on their properties. This process is included in the analysis level. Next, at the identification stage, the students are asked to find the intended edge in the problem so that the visualization process takes place. The students are asked to relate the edge size to the given grid unit which is 0.5 meters. And finally, the students use the Pythagorean theorem to find the hypotenuse of the intended edge. Thus, the students can find the relationship between the properties of the figure that is visualized as an abstraction stage. Then, the students get an overview of the elements needed to find solutions to the problems given. From the concept of Spatial Ability, the tasks given to the students are seen as spatial spaces.

Furthermore, the expected completion stages of test instrument number 2 are detailed in Figure 2. After the students finished the task number 1, the students should be able to understand some of the problems from test instrument number 2. They just need to continue the visualization process to a further stage where they divide



Length of edge $= AB + BD + DE$

$AB = 2(0{,}5m) = 1m$

$DE = 2(0{,}5m) = 1m$

$BD = \sqrt{(BC)^2 + (CD)^2}$

$= \sqrt{\left((4)(0{,}5m)\right)^2 + \left((3)(0{,}5m)\right)^2}$

$= \sqrt{6{,}25m}$

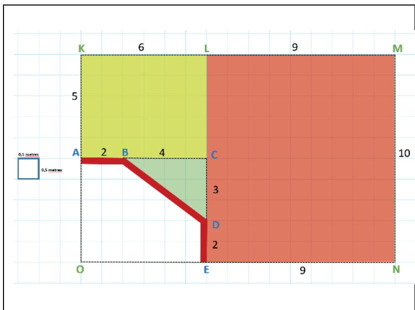
$= 2{,}5m$

So that, Length of edge $= 1m + 2{,}5m + 1m = 4{,}5m$

Length of edge $= 1m + 2{,}5m + 1m = 4{,}5m$

Figure 1: An example of expected answer process for the test instrumen 1.

the area of Ice-Cream Shop into several areas. This visualization process can make it easier for the students to identify which areas are the solutions to the problems asked. In this case, there are three areas that can be the solution, namely the yellow and red area which represent the rectangular shapes of KLCA and LMNE, and the green area which represent the right triangles BCD. At this stage, the students have visualized the given spatial problem into several shapes that are easier for them to understand. Next, the students analyze the flat shapes that become the solution area based on the properties of the flat shapes as the analysis stage. After that, at the identification stage, the students associated the areas from the previous flat shape with the requested solution based on the properties of the flat shape. This stage is part of the abstraction level in geometrical thinking. Next, the students add up all the areas obtained from the flat shapes to find a solution.



Solution Area $=$ Yellow Area Kuning $+$ Red Area $+$ Green Area

$= (KL \times KA) + (LM \times MN) + \left(\frac{BC \times CD}{2}\right)$

$= (6 \times 5) + (9 \times 10) + \left(\frac{4 \times 3}{2}\right)$

$= (30) + (90) + (6)$

$= 126$

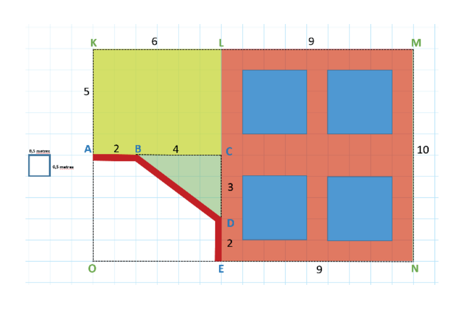
Solution Area $= 126(0{,}5m \times 0{,}5m)$

$= 126 \times 0{,}25$

$= 31{,}5\ m^2$

Figure 2: An example of expected answer process for the test instrumen 2.

Figure 3 describes the steps expected of the students in completing the test instrument number 3. Actually, this problem is easier to solve spatially than algebraically. However, in fact, there are some the students who do it algebraically, and the result were not correct. The students who solve this problem by thinking spatially first then abstractly will do much better. The visualization process carried out by the students focuses on the red area, namely the area of the seating set by considering the minimum distance of the set from the wall and the minimum distance between the sets, 0,5m or 1 grid. The students then look for the relationship between the area of the set and the grid, namely 1 set equals 3 grids so that the area is 9. The students visualize the task given to make it easier to identify them. In the end, the students find the area of the seating area that can be used to place the sets. This step was carried out by linking the previous information in the visualization stage. This stage is included in the analysis level. Furthermore, the students can find the solutions from the given task by comparing the area of the seating area (after analyzing) with the area of a set. At this stage, the students are at the level of abstraction.

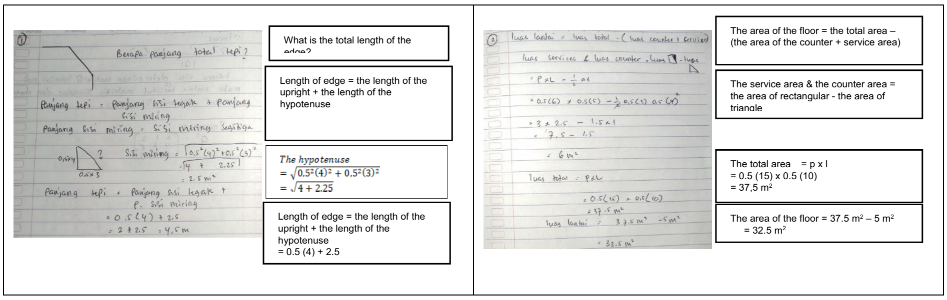


Minimum distance from the wall = 2= 2
Minimum distance between the sets = 2= 2
Total minimum distance = 4= 4
The seating area that can be used
The seating area that can be used:
$= (LM - 4) \times (MN - 4)$
$= (9 - 4) \times (10 - 4) = 30$
Area of the set $= 3 \times 3 = 9$
The maximum number of sets:
$= \dfrac{\text{The seating area that can be used}}{\text{Area of the set}}$
$= \dfrac{30}{9} = 3{,}33 \approx 4$
So, the maximum number of sets is 4 sets

Figure 3: An example of expected answer process for the test instrumen 3.

One of the students' answer on the test instrument number 1 is shown in Figure 4 (a). It can be observed that the student easily found the visual form that is asked of the problem. The student then related all the information in the task. By using the concept of the Pythagorean Theorem, the student can find the hypotenuse of the edge in the task. And finally, the student only need to add up all the edges referred to in the task. The final solution found by the student was 4,5m. From the pictures presented by the student, it can be seen that the student has been able to visualize the problem given in a simpler form. The student was also able to analyze the length of the asked edge where one of the edges is a hypotenuse. Then the student managed to state the result in meters, namely 4,5m.

The work result of the student in answering the test instrument number 2 is presented in Figure 4 (b) below. From the Figure 4 (b), we can see that the student has been able to think spatially by finding the floor area of the Ice-Cream Shop as a whole. After that, the student identified which areas of the Ice-Cream Shop do not need to add a floor. Indirectly, from this task, the student found an information that the service area and the counters do not require new floor. So, the student concluded that the area that can be the solution is the area of the Ice-Cream Shop minus the area of the service area and the counter. The student visualized the problem by inferring the existence of a square and a right triangle. In fact, the service area and the counter are a trapezoid. It is clear that the student's answer is wrong because there are parts of the counter area that do not require floors. When referring to the expected answer steps on this task, the student can find the solution easier when the student focus on area outside the service area and the counter. Therefore, the students can determine the right answer by adding up all the focused areas. So that from this task, at least three flat shapes can be visualized, namely two rectangles and one right triangle. Furthermore, the student only need to calculate the whole of the flat shape area. In this case, the student have not been able to visualize fully the given problem, so the student did mistake when analyzing the problem until determining the expected solution.



(a) (b)

Figure 4: An example of student's answer for the Task 1(a), An example of student's answer for the Task 2(b).

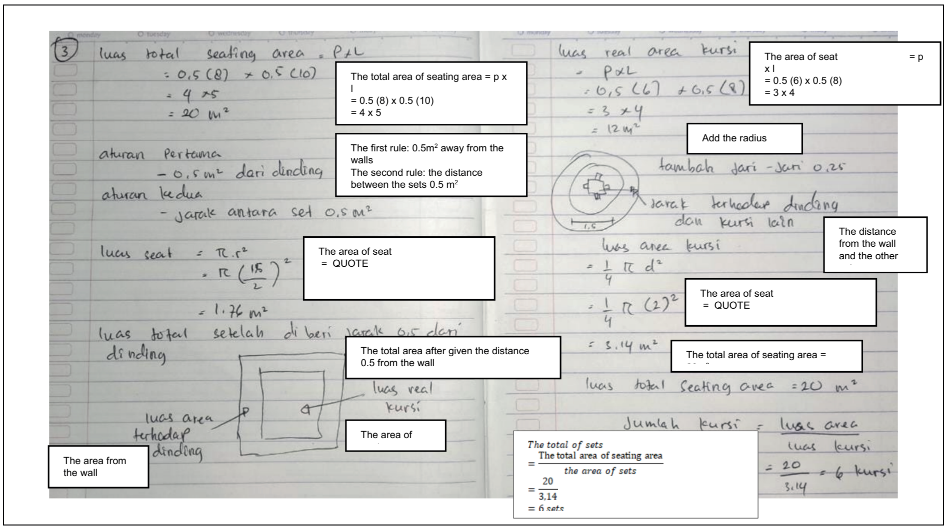


Figure 5: An example of student's answer for the Task 3.

The students' answer to the last test instrument can be seen in Figure 5. From the figure, it can be shown that the student has carried out the visualization process on the given problem. The student has tried to understand the requirements proposed in this task by describing the minimum distance of the sets from the wall and from one set to another. However, in the case of this sets, the student still mistakenly visualize the set, the student represent the area of a set as a circle. In the process, this is still understandable, but the student will experience difficulties when the student is in the analysis and abstraction stage. Therefore, the student has not been able to answer this task correctly. After observing, the student considered the set as a circle because the student saw the visual form from the given task. If based on the expected stages spatially, the student can actually solve the problem only by describing each possible set in the seating area.

4. CONCLUSION

In this section, the researchers concludes that there are still deviations from the answers given by the students with theoretically expected steps in this study. The students have not been fully able to answer accurately the HOTS tasks given. It was also revealed that the difficulties encountered by the students in answering these tasks were that the students generally had not been able to understand visualization of the exact form in the tasks given. Previous research has also shown that the students often experience obstacles in the topic of flat shapes which cause them to have difficulty in spatial thinking [24]. The majority of the difficulties experienced are at the level of geometrical thinking, especially at the level of visualization, analysis, and abstraction. This is also supported in Lutfi's research that the majority of the students in junior high school are at the maximum level of abstraction [25]. So that the students' spatial abilities are still at a low level in completing the HOTS geometry task [11]. Therefore, Van Hiele's geometrical thinking level can be used to identify obstacles that may occur when the students complete HOTS tasks, especially spatially. The role of the geometrical thinking level is seen when the students try to understand HOTS problems spatially.

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References

[1] Yenti F. Penerapan model pembelajaran Contextual Teaching and Learning (CTL) terhadap pemahaman konsep matematika siswa kelas VIII SMP Negeri 2 X Koto tahun pelajaran 2014/2015. Curricula. 2016;1(3):1–10.

[2] Subhan M, Nandari F. Analisis hots dan lots soal penilaian akhir semester muatan matematika kelas V Sekolah Dasar. Elem Sch J. 2020;10(3):137–46.

[3] Bloom BS, Airasian P, Cruikshank K, et al. A taxonomy for learning, teaching, and assessing: A revision of bloom's taxonomy of educational objectives. Longman; 2001.

[4] B.K. Suryapuspitarini, Wardono, and Kartono, "Analisis soal-soal matematika tipe higher order thinking skill (HOTS) pada Kurikulum 2013 untuk mendukung kemampuan literasi siswa.," *Prisma*. Prosiding Seminar Nasional Matematika. 2018;1:876–84.

[5] Bursill-Hall P. Why do we study geometry? answers through the ages (lecture at the opening of the faulkes institute). Part Of The Opening Festivities Of The Faulkes Institute For Geometry. Cambridge: University of Cambridge; 2002. pp. 1–33.

[6] M.D. Haryanti, T. Herman, and S. Prabawanto, "Analysis of students' error in solving mathematical word problems in geometry,." *Journal of Physics: Conference Series*. vol. 1157, no. 4, p. 2019. https://doi.org/10.1088/1742-6596/1157/4/042084.

[7] HershkowitzRina. "About reasoning in geometry.," Presented at the (1998).

[8] Mammana C. "Geometry and geometry-teaching through the ages.," Presented at the (1998).

[9] Epelboim J, Suppes P. A model of eye movements and visual working memory during problem solving in geometry. PERGAMON; 2001. pp. 1561–74.

[10] Luneta K. Understanding students' misconceptions: an analysis of final Grade 12 examination questions in geometry. Pythagoras. 2015;36(1):1–11.

[11] Dosinaeng WB, Leton SI, Lakapu M. Kemampuan mahasiswa dalam menyelesaikan masalah matematis berorientasi HOTS [Jurnal Nasional Pendidikan Matematika]. JNPM. 2019;3(2):250–64.

[12] Myelnawan M, Setyaningrum W. Kemampuan siswa SMP dalam menyelesaikan soal matematika berbasis HOTS. Jurnal Riset Pendidikan Matematika. 2021;8(1):83–95.

[13] Kurniati D, Harimukti R, Jamil NA. "Kemampuan berpikir tingkat tinggi siswa SMP Di Kabupaten Jember dalam menyelesaikan soal berstandar PISA.," *Jurnal Penelitian dan Evaluasi Pendidikan*. vol. 20, no. 2, pp. 142–155, 2016. https://doi.org/10.21831/pep.v20i2.8058.

[14] Setiawan H. Dafik, and N.D.S. Lestari, "Soal matematika dalam pisa kaitannya dengan literasi matematika dan keterampilan berfikir tingkat tinggi.," *Prosiding Seminar Nasional Matematika*. Universitas Jember. 2014;(November):244–51.

[15] Hendroanto A, Fitriyani H, Anggoro RP. Level berpikir van hiele dan kemampuan spasial: apakah pengaruhnya terhadap ketrampilan HOTS mahasiswa? JIPMat. 2019;4(1):1–6.

[16] Griffith DA. Geometry and spatial interaction. Ann Assoc Am Geogr. 1982;72(3):332–46.

[17] Karaman T, Toğrol AY. Relationship between gender, spatial visualization, spatial orientation, flexibility of closure abilities and performance related to plane geometry subject among sixth grade students. Boğaziçi University Journal of Education. 2009;26(1):1–25.

[18] Maier PH. “Spatial geometry and spatial ability–how to make solid geometry solid.,” In: *Selected papers from the Annual Conference of Didactics of Mathematics*. pp. 63–75 (1996).

[19] Usiskin Z. Van hiele levels and achivements in secondary school geometry. Chicago: The University of Chicago; 1982.

[20] Pavlovičová G, Bočková V, Laššová K. Spatial ability and geometric thinking of the students of teacher training for primary education. TEM Journal. 2022;11(1):388–95.

[21] Nurkaidah IP, Subanti S. “HOTS problem-solving ability and its relation to spatial ability.,” *Proceedings of the International Conference of Mathematics and Mathematics Education (I-CMME 2021)*. vol. 597, pp. 210–216, 2021.

[22] Choi-Koh SS. A student’s learning of geometry using the computer. J Educ Res. 1999;92(5):301–11.

[23] Van Hiele PM. A child’s thought and geometry. English Translation of Selected Writings of Dina Van Hiele-Geldof and Pierre M. Van Hiele. Brooklyn: Brooklyn College; 1984.

[24] M.K. Lutfi, D. Juandi, and A. Jupri, “Students’ ontogenic obstacle on the topic of triangle and quadrilateral,.” *Journal of Physics: Conference Series*. vol. 1806, no. 1, p. 2021. https://doi.org/10.1088/1742-6596/1806/1/012108.

[25] M.K. Lutfi and A. Jupri, “Analysis of junior high school students’ spatial ability based on van hiele’s level of geometrical thinking for the topic of triangle similarity,.” *Journal of Physics: Conference Series*. vol. 1521, no. 3, p. 2020. https://doi.org/10.1088/1742-6596/1521/3/032026.