

Research Article

Developing Phenomenological Sensitivity in Didactical Mathematics Through the Habit of Deep Observation in the Era of Industry 4.0

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A teacher needs to have the sensitivity in seeing didactic phenomena. The sensitivity can function as a vehicle or context for mathematics learning. The Konigsberg Bridge, for example, can be described without looking at the picture since it became a part of the community story. Later on, the description of the Konigsberg Bridge can be used as a context in mathematics learning, especially those related to Graph Theory. This paper discusses the phenomena such as the Konigsberg Bridge, to make mathematics teachers understand the didactic phenomena easily. By having the sensitivity of the mathematically valuable phenomena for the mathematics teachers, they can contribute more positively to mathematics education.

Keywords: phenomenological sensitivity, didactical mathematics, habit of deep observation

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1. INTRODUCTION

This short paper aims to portray how the teachers and prospective teachers develop sensitivity toward didactic phenomena by observing a phenomenon deeply. However, this process is necessary for the teachers and prospective teachers to enable them to see the phenomena and use them in teaching mathematics.

The teachers and the prospective who attended the workshop on introduction of this phenomenon became more aware of the mathematical phenomenon [1]. Because by building this sensitivity, the teachers and prospective teacher can use this didactic phenomenon as a context in learning mathematics, for example in teaching the volume of a ball using a round watermelon [2], finding the volume of a tube using an 'oval' watermelon [3], introducing the volume of pyramids with sea sand [4], teaching the

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concept of linear equations with scales [5], teaching straight line equations and their graphs using taxi fares and photocopy fees [6], and finding the concept of function using a sugar solution [7], and making camphor to show the decay process of an object [8].

These phenomena become an important part in different learning strategies. In the RME and CTL the phenomenon is used as a context [9], while in Problem Based Learning used as a problem [10], and in Project- Based Learning (PjBL) used as complex questions, problems, and challenges [11]. The introduction of the sensitivity for the teachers and prospective teachers aims to familiarize them with the didactic phenomena, so they can recognize them easily in various situation.

To increase the understanding and the sensitivity they can observe deeply by asking questions about the mathematical elements and problems what mathematics value the phenomenon has, how to define the mathematical values, when the teachers and prospective teachers regard the phenomenon as a mathematical context, and the context is presented to the students as mathematical problem. The teachers and prospective teachers should develop the ability to manipulate, give guesses, test the created conjectures, disassemble formula constructions, and generalize and re-verify the assumptions that have been tested.

2. METHOD

This research was conducted at the Faculty of Mathematics and Natural Science Education, Universitas Pendidikan Indonesia involving ten participants. The participants consist of three master degree students, 4 undergraduate students as the prospective teachers, and 3 junior high school mathematics teachers. All the participants were given some various phenomena. They were asked to read, answer the existing problems and provide comments on the findings.

After being exposed the participants indicated the interest of the subject and the increased curiosity about the related phenomena. The method used in this research paper is the phenomenology approach with the qualitative data analysis. This research is conducted by developing phenomena, introducing to the subjects, asking the participant recognize, solving the problem mathematically, and asking them to reflect their thought on the phenomenon.

Phenomenology is a scientific discipline and descriptive inquiry study that focuses on the study of appearances (phenomena), acquisition of experience, and consciousness. In short, phenomenology is the study of experiences and how it is formed. The experience in questioning is subjective and various in its intentionality. This study leads

to the analysis of possible intentionality conditions, background of social practice, and language analysis. How the level of awareness about the mathematical values in the phenomena of the teachers and prospective teachers in mathematics education is. Additionally, this research intended to investigate whether the participants are willing to grow their interest and awareness toward the existence of mathematically valuable phenomena. This research also measures how interested and how sensitive the participants are in recognizing mathematically valuable phenomena (Table 1).

TABLE 1: Participant’s sensitivity to the mathematically valuable phenomena.

Subjects	High sensitivity	Moderate Sensitive	Less Sensitive	Total
Master students	2	1	0	3
Undergraduate students	1	2	1	4
Teachers	2	1	0	3
Total	5	4	1	10

The sensitivity levels of participants vary. The interest is expressed by questioning a question such as “Some participants express their curiosity by questioning the fact that even bamboo fences can have mathematical aspects which can be used in learning and thinking at a higher level”.

One of the participants, usually did not recognized the context as deeply and interestingly as facilitator. He founded interesting because the facilitator could explain a ball into a number of pyramids. Usually, slicing watermelons is just in the shape of prisms, but the facilitator can show that a watermelon can be sliced into pyramids. Those are some responses from the participants during the workshops of the introduction mathematically valuable phenomenon. Although many participants have positive views, there are still some who have negative views, due to their little participation in this workshop. However, one participant does not like very complicated mathematical formulations. This participant rejects the involvement of mathematical thinking.

2.1. Konigsberg Bridge

When someone talks about the phenomena, he will see what the definition of the phenomena is. According to one of the online dictionaries on the website, phenomena is the plural of phenomenon which most generally refers to an observable occurrence or circumstance. In mathematics, phenomena can be situations in which a number of concepts or principles or mathematical properties are contained. Someone who already has the knowledge of mathematics may still be doubtful, but once in a while he can

become clearer after being given some explanation. An example that can be observed is the story of the Konigsberg Bridge. In the Konigsberg Bridge there are four points connected by seven arches (Figure 1 and Figure 2).



Figure 1: Konigsberg bridge.



Figure 2: Bamboo fence as a phenomenon. (Resouces: Majalah Guru, 2003).

The puzzle that arose at that time was “How do you get someone who departs from one point later to return to the original point after crossing all the bridges and each bridge is crossed once”. At the time, the town people took long walks through towns on Sundays [12]. They wondered whether it was possible to start at some location in the town, travel across all the bridges without crossing any bridge twice, and return to the starting point [12]. With repeated trials no one could answer the riddle of the Konigsberg Bridge, until it was answered by Euler several years after the riddle had been around for a long time. The answer to the riddle is that it is impossible to reach if the land is 4 and the bridge is 7 (see Figure 1), and will be passed directly once without repeating it once. This is further studied in depth in the study of graph theory. Almost all of the people who study mathematics more specifically study graph theory recognize the story of the Konigsberg Bridge.

2.2. Grid Phenomenon

The researchers would like to invite the readers to observe the phenomenon of “grids in which the grid is in the form of a rectangle of various sizes and then a diagonal is made from the rectangle.” This phenomenon is taken from the bamboo fence, as can be seen in Figure 2. Next, the question arises how many squares are scratched by the diagonal line. Of course, when observing this situation, it must be ensured that the grid has accurate measurements. The pencil used to depict should be quite pointy. The given grids must be accurate because there are situations where a diagonal line may pass through the corner where the square meets the square, which is not regarded as crossing the square. Though it is not taken seriously but if someone is careful and diligent, he can observe, contemplate, think critically and creatively, which in the end enable him to find interesting formulas. For those inside the fence, the grid is 7×2 and the number of squares scratched is 8, so we write “ $7 \times 2 = 8$ ”.

For those on the outer grid, it is said to be “ 6×3 ” if calculated manually it is clear that “ $6 \times 3 = 6$ ” because the number of squares that are scratched is 6 (see Figure 3). Is there a way or reason by using certain calculations so that the result is $7 \times 2 = 8$ and $6 \times 3 = 6$ make sense? It should be investigated on how, what is the formula, and what are the forms of the formula, if the length and width of the grid are m and n , respectively, with m and n positive numbers, of course.

The phenomenon like this becomes an interesting phenomenon that can be used by a teacher to stimulate children to think critically and creatively. First, collect as much empirical data as possible, for example there are 15 or 20 grids. Pay attention to the links that occur. To make it unique, pay special attention to 12×2 , 12×3 , 12×4 , 12×5 , 12×7 , 12×8 , 12×10 , 12×9 .

Next, the student is assigned to formulate $m \times n$ in trial and error to find the relationship between the $m \times n$ value obtained empirically (experimental) and the value of $m + n$ then students will find the difference between or the amount between $m \times n$ and $m + n$ and we get a special number which is the gcd of m and n . For this session, perhaps it takes a little bit long time, but we have to be patient in waiting for the results of their thinking. When the participant persistently works until arrived to the solution, finally they will have the formula of $m \times n = m + n - \text{gcd}(m, n)$ (see Figure 4 and Figure 5).

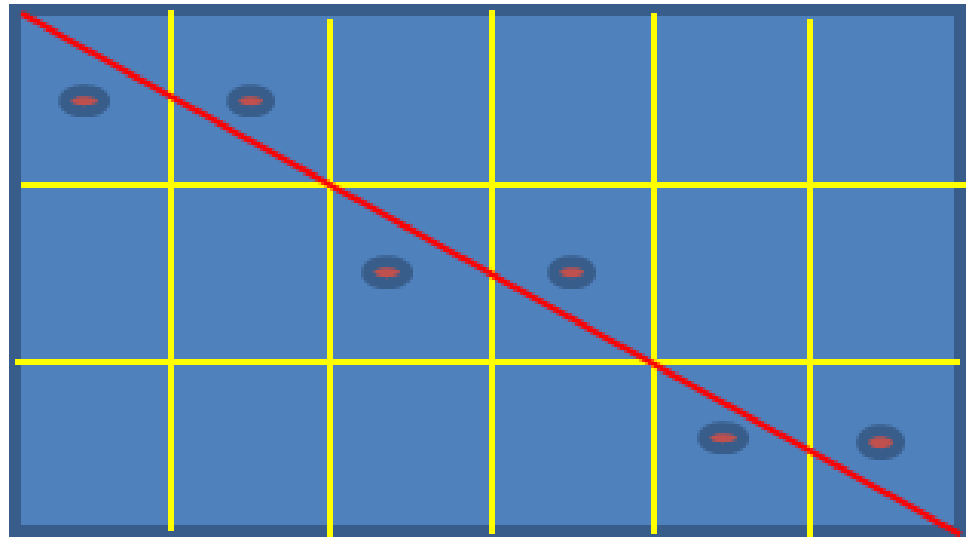


Figure 3: Formal grid taken from the fence.

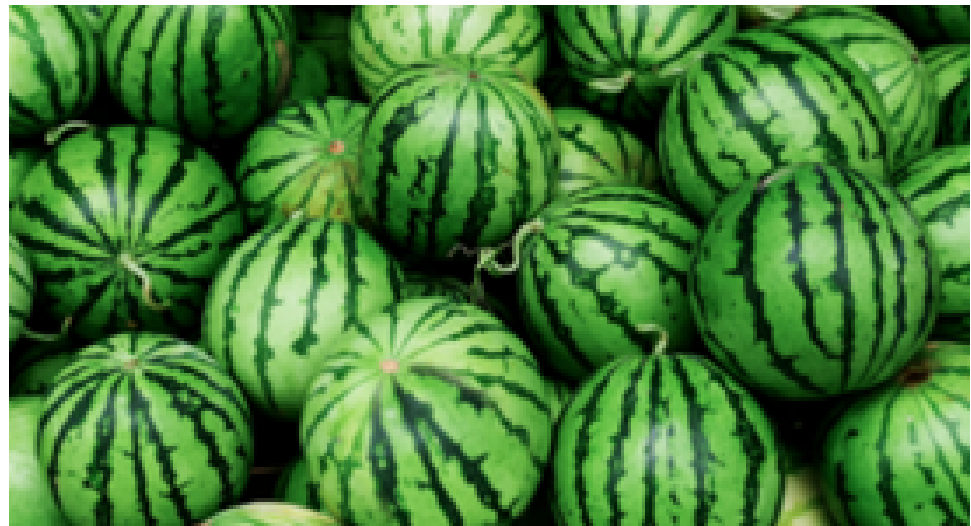


Figure 4: Watermelon as “ball-like” [13].

2.3. Watermelon Phenomenon

Very few people pay attention to watermelon for their mathematical thinking needs, generally only for the economic and culinary purposes. When you think about the volume of a geometric shape, for example a cube, it's not too difficult to determine its volume. When you think of the volume of a cuboid, we also just need to think “How many unit cubes can fit into the cuboid?” Or think about how many unit cubes can be made into a block to find the volume. In other words, talking about the volume of a building, then in essence we are looking for the capacity of the building. However, for a sphere, it is certainly not simple to ask how many cubes can occupy a block, because it is almost certainly hollow.

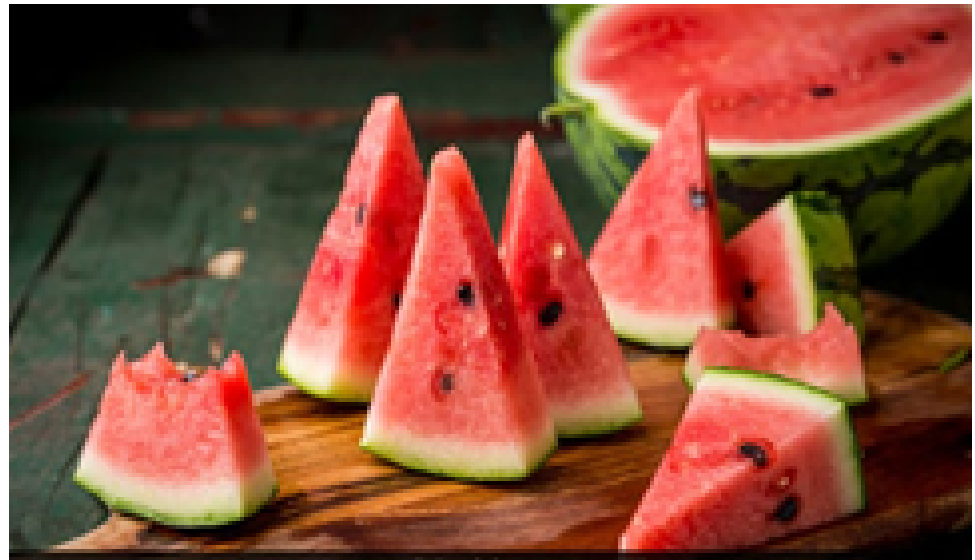


Figure 5: Slice watermelon, look like pyramids.

The most effective way to calculate the volume of the ball is by using a formula. However, the use of watermelons can confirm or strengthen the truth of the formula for the volume of the ball, or if someone doesn't know it, it means that they can find the formula for the ball. Indeed, knowing the volume of a space object is actually comparing the capacity or unit object with the cavity in the object to be measured. Here the watermelon is a solid object or can be seen as a solid ball. Precisely with this solid ball if the ball is cut or sliced, the object is still tangible and can even be enforced. Quite different from the plastic ball, or real soccer ball, if we slice it, then it would be hollow, no content except the air (Figure 6).

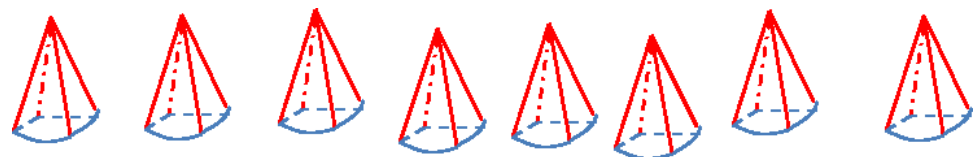


Figure 6: The pyramids from watermelon.

If a watermelon, which looks like a ball, is made of pyramids as part of the ball, then the entire watermelon can be made into a limited number of pyramids (see Figure 6). In this case the top of the pyramid is the center point of the “watermelon solid ball”. If the “pyramids-like” slice is considered to be too large, then it can be further divided into 4, so that the base of the “pyramid-like” is close to flat. Assuming that the surface area of the sphere is $4\pi R^2$ where R is the length of the radius of the sphere (watermelon), and the volume of the pyramid is defined as $\frac{1}{3}AR$, which A is the area of the pyramid base, then we can find the total volume of these pyramids as follows:

Ball Volume = Pyramid Volume 1 + Pyramid Volume 2 + Pyramid Volume 3 + ... + Pyramid Volume N

$$= \frac{1}{3} A_1R + \frac{1}{3} A_2 R + \frac{1}{3} A_3R + \dots + \frac{1}{3}A_N R$$

$$= \frac{1}{3} R (A_1+ A_2 + A_3 + \dots + A_N) \text{ whereas } (A_1+ A_2 + A_3 + \dots + A_N) = \text{area of ball}$$

$$\text{or } A_1+ A_2 + A_3 + \dots + A_N= 4R^2$$

Therefore, the volume of the ball = $\frac{1}{3} R (4R^2) = \frac{4}{3}R^3$ and this is a formula that since the researchers were in 6th grade, they never got an explanation and reason for how to get the $\frac{4}{3} R^3$ formula. With the help of watermelon (although you can actually use apples or oranges or other fruits that are round in shape), these can be used to show the volume of the ball.

3. RESULT AND DISCUSSION

The data from the research results in building sensitivity to the didactic phenomena are presented in the table 2 below:

TABLE 2: Sensitivity of the subject to the didactical phenomenology.

Subjects	High sensitivity	Moderate Sensitivity	Less sensitivity	Total
Graduate Students	2	1	0	3
Undergraduate Students	1	2	1	4
Teachers	2	1	0	3
Total	5	4	1	10

The data above show that of the ten participants being analyzed, five of them have high level of the sensitivity. This is characterized by high curiosity from the participants, enthusiasm to read, effort to find out solutions in different ways. The participants are expected to develop reflective thinking which involve the strong will to try to formulate conjectures, try to prove the conjectures that are made, and the ability to make the expression “If I am faced with a phenomenon like this, there are actually many similar things that can be used”.

The sensitivity indicators above are illustrated by the numbers which tend to be high. Five participants who consist of two master degree students, 1 undergraduate student and two mathematics teachers showed a very high level of the sensitivity, namely 85%. The second group has moderate level of the sensitivity, that reaches 70% and only one participant who has very low sensitivity level which is only 10 %.

One participant in the very high sensitivity group said in the interview as following:

TABLE 3:

Interviewer	:	<i>With a phenomenon like this can you find another phenomenon?</i>
Student-1	:	<i>With experiences like this, I can reveal other phenomena, such as the water faucet and the temperature in the water heater</i>
Interviewer	:	<i>Can you explain a little bit, about a water heater?</i>
Student-1	:	<i>Look Sir, for example, if you turn the hot water faucet, when you turn it just a little, the water coming out still feels cold. When the faucet is turned a little bigger, warm water comes out. When the faucet is almost fully turned, the water feels hot. When it is fully turned, the water coming out is very hot.</i>
Interviewer	:	<i>Good, Now try to observe empirically and formulate the relationship between the two variables</i>

This student shows two variables, namely the valve rotation angle variable which may range from only 0o to 1800, and he relates it to the temperature of the flowing water. He can empirically collect the data by rotating the faucet many times. Thus, he can measure particular angle with the particular temperature.

TABLE 4:

Interviewer	:	<i>In your opinion, what can you gain from this training?</i>
Student-2	:	<i>I found the phenomenon of the team marathon race</i>
Interviewer	:	<i>What is the real point of your findings?</i>
Student-2	:	<i>You see, sir, in an 8 km marathon to commemorate the independence of the Republic of Indonesia, there are three groups of the marathon races, each of which has 5 members. I get points in determining who is the overall group winner in the marathon race.</i>
Interviewer	:	<i>Do you have a certain method?</i>
Student-2	:	<i>Let's say there are 3 groups. B1 finished first, followed by B1, followed by C1, C2, A1, B2, B3, C3, C4, A2, A3, B4, A4, C5, A5, and B5. The question here is which group will win?</i>
Interviewer	:	<i>What criteria did you use to determine the winning group?</i>
Mhs-2	:	<i>Here's I wrote B1 , C1, C2, A1, B2, B3, C3, C4, A2, A3, B4, A4, C5, A5, and B5. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 B C C A B B C C A A B A C A B I add up the rankings and then find the mean, so that: $A = (4+9+10+12+15)/5 = 50/5 = 10$ $B = ((1+5+6+11+15)/5 = 38/5 = 7,6$ $C = (2+3+7+8+13)/5 = 33/5 = 6,6$ From the method I used, it can be concluded that the first winner is group C. The second winner is group B, and third winner is group A.</i>
Interviewer	:	<i>Do you have any other rules?</i>
Mhs-2	:	<i>I have a second suggestion First rank is given a score of 30, the 2nd rank is given a score of 29, the 3rd is given a score of 28, and so on. The winner is the group who can achieve the highest total score 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 B C C A B B C C A A B A C A B 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 So the goes this way. $B = 30 + 26 + 25 + 20 + 16 = 117$, $C = 29 + 28 + 24 + 23 + 18 = 122$ $A = 27 + 22 + 21 + 19 + 17 = 106$ So it is concluded that the group C is the First winner, Group B is the 2nd winner, and group A is the 3rd winner</i>
Interviewer	:	<i>Okay, it is possible to calculate the rankings in other ways.</i>

By using those two ways the results are consistent. As for the mathematics teacher who is quite sensitive in recognizing the phenomenon, he is interested in telling the story as follows

TABLE 5:

Interviewer	:	<i>In your opinion, what can you gain from this training?</i>
Teacher-1	:	<i>I was interested in observing the traces of a marble rolling on the table, but the table was tilted. (ramp)</i>
Interviewer	:	<i>What's the goal?</i>
Teacher-1	:	<i>The path of the marble that was rolled towards the higher level at first turned out to be curved.</i>
Interviewer	:	<i>What kind of arch did you get?</i>
Teacher-1	:	<i>If I notice the curve is meant to form a parabolic curve</i>
Interviewer	:	<i>What is the equation of the parabolic curve?</i>
Teacher-1	:	<i>If I continue and I make the coordinate axes, then I believe this will obtain the equation of the parabolic curve from the path of the marbles.</i>
Interviewer	:	<i>Well, thank you very much</i>

From some of the descriptions above, it turns out that students and teachers have the sensitivity to capture mathematical didactic phenomena and interpret them to obtain interesting learning materials in the classroom later. It is believe that if the workshop is conducted, the participants will have a high level of the sensitivity in analysing new phenomena. Therefore, they can use this skill to create more interesting mathematics learning for their students in the future.

4. CONCLUSION

The development of sensitivity to the didactic phenomena in mathematics is a necessity although the target groups and the frequency of this professional development are not very frequent. The researchers always tries to find the opportunities to include the development of the Didactic phenomena in various occasion. The participants in this study respond positively toward this development. Moreover, they generally have high sensitivity in responding to the introduction of didactic phenomena. The high desire from the participants encourages the researchers or other institutions with the mutual interest in improving the quality of education, especially in the field of mathematics education.

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