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Research Article

Symbol Sense Ability of Students to Solve Linear, Quadratic, and Rational Equations

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Abstract.
In learning algebra, one of the skills needed to be acquired by students is solving equations, such as linear equations in one variable and quadratic equations. This study aims to analyze the symbol sense ability of students by investigating strategies used to solve algebraic equations that consist of linear, quadratic, and rational equations. A qualitative descriptive method was conducted. The instrument included a test with one linear equation, one quadratic equation, and three rational equations (involving linear or quadratic factors). The test was administered to 12 students from 12th-grade high school in Bandung. Each student was asked to solve it in two different ways of answers if possible. The results showed that students were able to use symbol sense strategies for solving problems. However, there were still mistakes in solving algebraic squared value concepts, affecting their final results of the answers. We conclude that even if students can use their symbol sense abilities properly, they need to pay attention to their mathematics concepts.

Keywords: Symbol Sense Ability, Solve Linear, Quadratic, Rational Equations

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1. INTRODUCTION

Learning mathematics at high school level students is to understand mathematical concepts [1]. Based on the mathematics syllabus issue in Indonesia [2] one of the substantial topics of mathematics learning in high school includes algebra. Algebra is a branch of mathematics that studies the quantities, relationships, and structures that are formed. Algebra studies how a quantity is generalized in the form of symbols and its manipulation. When learning algebra, students begin to be introduced to symbols that are not only numbers but letters that are abstract [3,4]. Furthermore, previous studies have found that many high school students have weaknesses in algebraic manipulation because they considered algebra as abstract symbols. Students' ways of understanding the symbols without any algebraic connection because of a misconception formed at the beginning of learning [5,6]. This case made algebra a difficult topic for students to

learn and for teachers such a challenging topic to teach [7]. Along with it, Arcavi [8] found inspiration from the activities of students and teachers in algebraic learning problems, he collected it and found a wide spectrum of interesting ways of sense-making (or lack of it) with symbols emerged is named “symbol sense”.

The term of symbol sense was first used by Arcavi [8] as sensitivity to symbols. In this case, symbols are referred to symbols on algebraic problems. To describes symbol sense, Arcavi [8,9] summarized the main components of symbol sense: Friendliness with symbols; An ability to manipulate and also to ‘read through’ symbolic expressions as two complementary aspects in solving algebraic problems; The awareness that one can be successfully engineered symbolic relationship that expresses (given or desired) verbal or graphical information needed to make progress in a problem, and the ability to engineer those expressions; The ability to select one possible symbolic representation for a problem; The realization of the need to check for the symbol meanings during the implementation of a procedure, the solution of a problem, or, during the inspection of those meanings with one’s intuitions about the expected outcome; The realization that symbols can play different roles in different contexts (such as variables or parameters), and the development of an intuitive feel for those differences. For the purpose of the present study, each of these characteristics is used in the instruments of tasks.

The notion of symbol sense in Indonesia [10] has been used for examining student procedural fluency and conceptual understanding in algebra and for studying student algebraic thinking on solving substitution problems. Nevertheless, according to previous research [11], the ability of students’ symbol sense was generally lacking when solving algebraic problems. This leads us to develop further research. Therefore, the present study aims to investigate students’ strategies used to solve algebraic equations that consist of linear in one variable, quadratic, and rational equations (which involve linear and quadratic factors). The results of students’ answers will be distinguished into two strategies: procedural strategy and symbol sense strategy. The symbol sense strategy is developed based on the main components in symbol sense that are summarized by Arcavi.

The student displays the procedural strategies if they directly use a common linear equation in one variable and quadratic equation without considering the efficiency when using the procedure [12,13]. The student is said to use symbol sense strategy if (in simple terms) their solution can see the form of equality as an object that can be manipulated. As an example, if s/he can see the form of $x-p$ as an object to that can be assumed, in equation $2(x-p)^2 = 100+(x-p)^2$, then the square root of $x-p$ must be equal to 10. In the equation of $\frac{x-2a}{2x-4a} = 4$, if students can see the form of $x-2a$ is half

from $2x-4a$, then the left side of the equation is not equal to the value of 4 on the right side. From those two examples, we can conclude that students are able to involve their symbol sense ability [8,9,14].

2. RESEARCH METHOD

To investigate the symbol sense ability of twelve senior high school students in Bandung (from 12^{th} grade classes, advanced, intermediate, and low mathematics proficiency), we used a qualitative descriptive method. To do so, we developed five questions: one linear equation in one variable, one quadratic equation, and three rational equations based on symbol sense characteristics in high school (see Table 1). The rational equations involve linear and quadratic factors.

TABLE 1: Instruments of Tasks and Characteristics of Symbol Sense.

No.	Task	Characteristics of Symbol Sense
1.	$5x+10 = 4x$ What is the value of x?	Reading through symbol instead of manipulating.
2.	$6(a-1)^2-4 = 5(a-1)^2$ Determine the value of a ...	Recognize symbolic relationship, display and do symbolic generalization and proofs.
3.	Determine the solution of equation: $\frac{x-4}{2x-8} = 4$	Read through and manipulate symbolic expressions.
4.	Determine the solution of equation: $\frac{4x^2-16}{x^2-4} = \frac{5}{2}$	
5.	Determine the solution of equation: $\frac{3b-bx}{-cx+3c} = \frac{4b}{c}$	

Students were given 80 minutes to work along with the instruction of answering each question with two different solutions if possible. This instruction aims to encourage students to work on problems with strategies that involve the ability of symbol sense. Finally, we analysed their strategies based on symbol sense characteristics [8,9,14]. We classified the correct and incorrect answers. From the students' answers and its correct, it means that students are able to involve the ability of symbol sense and understanding in the material and concepts in mathematics learning.

3. RESULTS AND DISCUSSION

Table 2 shows the number of students' answers and solving strategies in linear, quadratic, and rational equations (involving linear and quadratic factors). The numbers for task 2 until 4 shows more than half students used symbol sense strategy. This

shows that the solving task by symbol sense strategy is greater than procedural strategy. However, task 1 gets none for symbol sense strategy, it means that students are not able to use their symbol sense abilities as the solving task strategy. Although in general, students have been able to involve their symbol sense abilities, in fact there were students who still found errors so that the final answers are incorrect. These errors were made by students who used symbol sense strategy and procedural strategy.

TABLE 2: Students answers and strategies solving tasks.

Task	Correct	Incorrect	Empty	Strategy	
				Procedural	Symbol Sense
1.	12	0	0	12	0
2.	9	2	1	4	7
3.	6	6	0	5	7
4.	7	5	0	5	7
5.	4	5	3	3	6

Task 1 is a representative of a student who lacks of symbol sense tried to find the value of x in equation $5x + 10 = 4x$ (see Figure 1) by the procedural strategy for solving linear equation in one variable formula. Therefore, s/he solved it by giving the same actions to both sections which was subtracting $4x$ s that the right side equal to 0 and the left side is reducing $4x$ with $5x$. then the value of 10 is omitted on the left side by similar actions on both sections so that the right side is -10. We consider this strategy does not satisfy the components of the ability to read through symbols instead of manipulating [8]. However, if they use the symbol sense's ability, they will simultaneously write $x = -10$ without cancelling on the expressions on both sides.

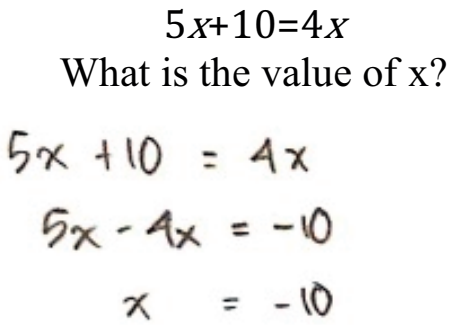

Figure 1: Representative example of student solution on Task 1.

For Task 2, seen in Figure 2 (left part) is a representative for student with symbol sense. S/he recognized the form $(a-1)^2$ as variable in its application for equation $6(a-1)^2 - 4 = 5(a-1)^2$, so that s/he was able to think of it as being variable "b". Then, s/he obtained $(a-1)^2 = b$ for equation $6(a-1)^2 - 4 = 5(a-1)^2$, which can be

manipulated into equation $6b-4=5b$. Therefore s/he substituted in equation $(a-1)^2=b$, such that $a-1=\pm 2$, it means that if $a-1>0$ then $a-1=2 \Leftrightarrow a=3$ or if $a-1<0$ then $a-1=-2 \Leftrightarrow a=-1$. As a result, the value of $a=-1$ or $a=3$. Moreover, 7 out of 12 students were able to see the form $(a-1)^2$ as a variable that applied in quadratic equation $6(a-1)^2-4=5(a-1)^2$ which give ease in determining the final solution, which the value of a. In addition, students were able to manipulate algebraic symbols, and also recognize symbolic relationships, display and do symbolic generalization and proofs [8,9]. For Task 2, in Figure 2 (right part) is seen that student lack of symbol sense used procedural strategy by calculating the result of $(a-1)^2$. Therefore, s/he did not see form $(a-1)^2$ as an object that can be manipulated. We infer s/he is unable to involve his symbol sense ability. Furthermore, there were three students who were able to see $(a-1)^2$ as the form that they can manipulate. However, they found incorrect solution at the end of the procedure. Instead of founding two solutions which are $x=2$ or $x=-2$, s/he wrote the value of x only 2, resulting in them to write the incorrect answer. This is caused by an error in the concept of algebraic square values [6,15]. We conclude the most mistake is an error in the concept of algebraic square values in their final results.

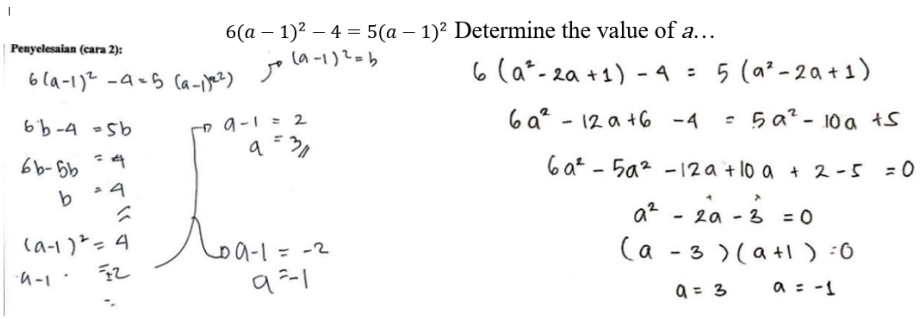

Figure 2: Representative examples of student solution strategies on Task 2.

For Task 3 is a representative of a student with symbol sense recognized in determine the solution of $\frac{x-4}{2x-8}=4$ (see Figure 3 left part). Student can find errors that score $\frac{1}{2} \neq 4$. We infer that s/he were able to read through overall symbols so that it can find errors in equation $\frac{x-4}{2x-8}=4$. This shows that students can read through and manipulate symbolic expressions, which s/he has involved their symbol sense ability [8,9].

Nevertheless, for Task 3 in Figure 3 (right part) seen that student strategy without symbol sense. The solving task strategy that used by student is in accordance to Lee and Wheeler's statement in Sharma [14], which found students choose to eliminate the numerator with the denominator to solve the equation problem. In the next stage, student were not aware of errors, then choose to look for x without knowing the meaning of the equation problem. However, students did not substitute the value of x they have

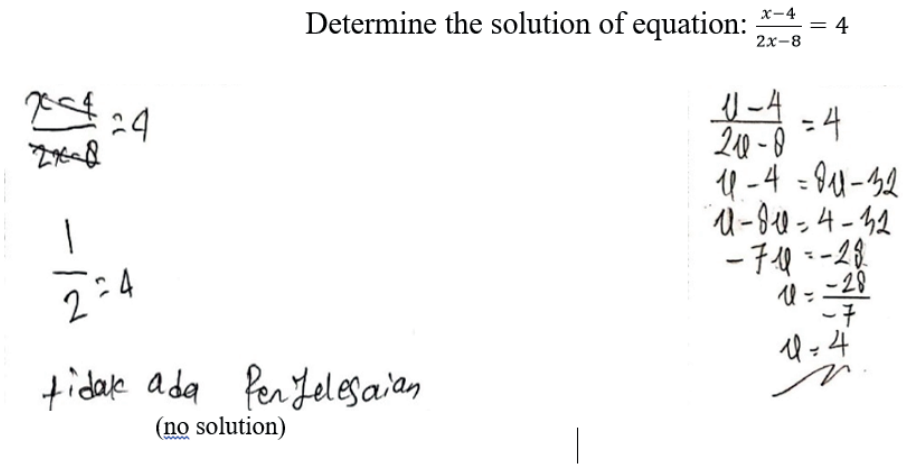

Figure 3: Representative examples of student solution strategies on Task 3.

found. To realize the errors in problem, students should re-examine the solution by substituting the value of x in equation $\frac{x-4}{2x-8} = 4$. As a result, student did not determine that the equation is incorrect and has no solution.

For Task 4, a representative of a student with symbol sense were able to find errors in equation $\frac{4x^2-16}{x^2-4} = \frac{5}{2}$ (see Figure 4 left part). Therefore, s/he found out errors that score $4 \neq \frac{5}{2}$. This shows that s/he read through and manipulated symbolic expressions which involved their symbol sense ability in this task [8, 9]. However, for Task 4 in Figure 4 (right part) gives an example of a student strategy without symbol sense. Students choose to eliminate the numerator with the denominator to solve the equation problem. In the next stage, students are not aware of errors, then choose to look for x without knowing the meaning of the equation problem [14]. However, students did not substitute the value of x they have found. Besides it, student did mistake in determining the value of a variable that has been manipulated by write the value of x is only 2 that should be have two solutions are $x = -2$ or $x = 2$ as the form from $x^2 = 2 \Leftrightarrow x = \pm\sqrt{2}$. This is caused by an error in the concept of algebraic square values [5,6]. As a result, student did not determine that the equation was incorrect and had no solution.

For task 5 a representative of a student with symbol sense were able to find errors in equation $\frac{3b-bx}{-cx+3c} = \frac{4b}{c}$ which have no solution (see Figure 5 left part). Only by 'reading' meaningfully the task can pass procedural solution, and can examine and reunite linear equation in one variable.

Therefore, in Figure 5 left part, student found the basic meaning and can eliminate parts of the task in solving the algebraic equation properly and effectively. This is according to the component of the ability of symbol sense to manipulate and see the overall results of manipulation of symbolic expressions as two complementary aspects

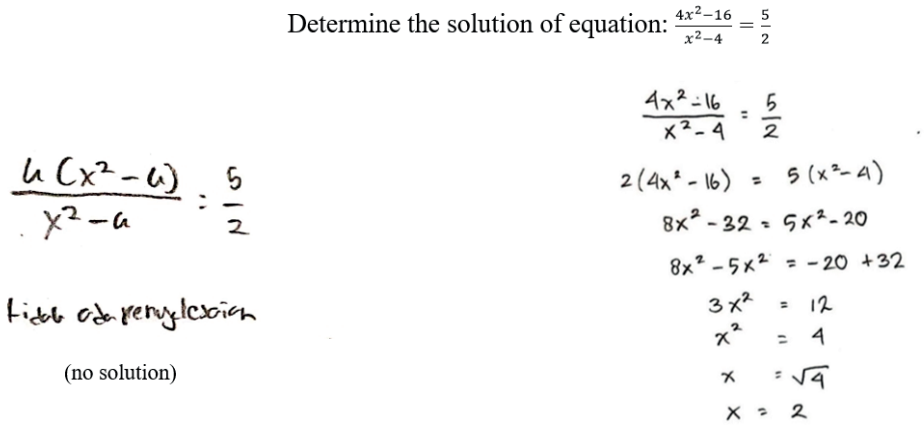

Figure 4: Representative examples of student solution strategies on task 4.

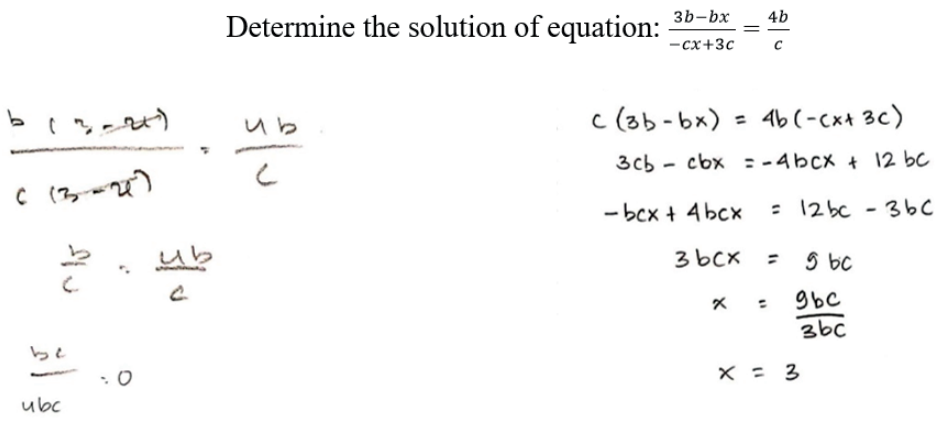

Figure 5: Representative examples of student solution strategies on task 5.

in completing algebra [8,9]. Nevertheless, for Task 5 in Figure 5 (right part) gives an example of a student strategy without symbol sense. Furthermore, s/he chosen to eliminate the numerator with the denominator to solve the equation $\frac{3b-bx}{-cx+3c}=\frac{4b}{c}$ task. Then, s/he had easily failure because of the variables b and c that are not numbers. Besides it, students were not aware of errors, s/he refused to look back for x which without knowing the meaning of the equation task. To realize the errors in task, student should re-examine the solution by substituting the value of x in equation. However, student did not substitute the value of x s/he has found.

4. CONCLUSION

Based on the results of the analysis of all students' strategies in solving tasks of questions from linear, quadratic, and rational equations, show that most of the students have symbol sense abilities. For quadratic equation task, with the task to determine

the value of a from equation $6(a-1)^2-4=5(a-1)^2$. Students were able to see the form of $(a-1)^2$ as a variable in its application in task $6(a-1)^2-4=5(a-1)^2$, so that student formulated it as variable "b". Therefore, they constructed form $(a-1)^2=b$ in equation $6(a-1)^2-4=5(a-1)^2$, which could be manipulated in equation $6b-4=5b$. For rational equation tasks (involving linear factors), with the equation $\frac{x-4}{2x-8}=4$, students were able to read the symbol as a whole of symbols form so that they can find errors in its equation. Students found errors that value of $\frac{1}{2}\neq 4$. Moreover, for rational equation (involving quadratic factors) in equation $\frac{4x^2-16}{x^2-4}=\frac{5}{2}$, students were able to read through symbols form so that they found errors in its equation. Students found that score $4\neq\frac{5}{2}$, even with a blink of their eyes.

In general, most students have been able to involve using symbol sense as their strategy to solve with the correct answers. Students see symbolically as an object in its application to the equation and find the results of the object that has been manipulated. Nevertheless, there were still difficulties experienced by students in completing it. The difficulty is students who do not understand the concept of algebraic operations, which is an error in algebraic squared values. As a result, students get the incorrect answer. Finally, we can conclude that students already have symbol sense ability. Besides, they still need to concern their mathematics concepts.

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