

## Research Article

# Cognitive Flexibility: Exploring Students' Critical Thinking Skills in Solving Mathematical Problems

Marzuki Marzuki<sup>1\*</sup>, Irfan Rusmar<sup>2</sup>, Wahyudin Wahyudin<sup>3</sup>, Dadang Juandi<sup>3</sup>

<sup>1</sup>Department of Mathematics Education, IAIN Langsa, Jl. Meurandeh, Kota Langsa, Aceh 24354, Indonesia

<sup>2</sup>Department of Palm Oil Agribusiness, Politeknik Teknologi Kimia Industri Medan 24354, Indonesia

<sup>3</sup> Department of Mathematics Education, Universitas Pendidikan Indonesia, Dr. Setiabudhi Street No. 229, Bandung 40154, Indonesia

**ORCID**

Marzuki Marzuki: <https://orcid.org/0000-0003-1706-4214>

Irfan Rusmar: <https://orcid.org/0000-0002-9268-8652>

Dadang Juandi: <https://orcid.org/000-0001-6997-1399>

**Abstract.**

Cognitive flexibility is an individual's ability to think, and critical thinking skills in the 21st century are needed by students. This study aims to obtain a substantive theory (conjecture) that is associated with students' critical thinking skills in solving mathematical problems. This type of research is qualitative with Grounded Theory (GT) systematic design procedures. Because the design of this research is GT, therefore, the data analysis was carried out in three stages, namely: open coding, axial coding, and selective coding with the help of NVivo 12 plus software. The participants involved were 30 students. The research guideline was in the form of a critical thinking ability test and semi-structured interview. In-depth interviews were conducted based on the results of the ability to think critically until the data was saturated. Data saturation occurred in the 5th interview. To test the reliability of critical thinking skills, categories and subcategories were carried out by two coders with a Cohen kappa value of 0.65, so that the coding was made reliable. The results obtained were hypothetical: if participants can think critically, then participants can solve problems, can provide arguments, can interpret algebraic derivative application functions, and can develop problem-solving strategies in critical thinking.

**Keywords:** critical thinking skills, cognitive flexibility, solving mathematical problems.

Corresponding Author: Marzuki

Marzuki; email:

marzuki@iainlangsa.ac.id

**Published:** 26 April 2024

Publishing services provided by  
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Selection and Peer-review under the responsibility of the ICMSCE Conference Committee.

## 1. INTRODUCTION

At this time, all students are important to master the concept of critical thinking skills, and their characteristics and qualities are needed to become critical thinkers. The importance of learning critical thinking skills is not only at the level of formal education but also for success in the contemporary world where the rate of creating new knowledge is increasing [1]. In the context of learning in Indonesia, critical thinking is one of the

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learning objectives, especially in mathematics [2, 3]. Teaching and learning activities should direct students to always develop critical thinking skills.

The ability to think critically is currently a challenge throughout the world [3], including in Indonesia. The importance of critical thinking skills in a social context, especially in solving daily problems and as a provision for facing the challenges of the 21st century [4–8]. Today's critical thinkers have been eroded by the issues of media opinion in our lives. This is also seen in students, educators, and parents who are less critical in receiving important information. So it is very important to have a “filter” process when someone receives information, which is part of being a critical thinker.

Previous researchers have tried to define critical thinking skills. Concerning thought processes, Dewey (1910) is seen as the father of the modern critical thinking tradition. He defines critical thinking as a continuous, active (persistent) consideration of a belief [9]. Critical thinking means sensible, reflective thinking that focuses on deciding what to believe or do [7]. However, on the other hand, critical thinking includes: estimating, evaluating, justifying, classifying, making hypotheses, analyzing, reasoning are elements of critical thinking [10–12]. However, each researcher considers these aspects separately and describes them independently. All these aspects can be integrated and considered as the fundamental characteristics of a critical thinker.

Although there are several studies in the previous literature that have been investigated on critical thinking skills [6, 13]. By using grounded theory, they only consider a few components, so far no one has explored the application material of algebraic function derivatives to obtain a substantive theory (conjecture) about students' mathematical critical thinking skills. Therefore, it is important to research students' critical thinking skills in the application material of derived algebraic functions.

## 2. RESEARCH METHOD

To answer the objectives of this research, a case study type qualitative research design with a simple systematic procedure grounded theory design is required. The case study design is used to explore causes of students' critical thinking skills in the application material of derivative algebraic functions involving 30 high school class XI students, while the systematic procedural grounded theory design is used to build substantive theories concerning the central phenomenon of students' mathematical critical thinking skills in the material. application of algebraic function derivatives with coding stages including open coding, axial coding, and selective coding [18,19,20]. The test of critical thinking skills that have been validated by experts is then continued with semi-structured

interviews, to know students' critical thinking skills until the data is saturated. The saturation of the data occurred during the 5th interview. NVivo 12 plus software is used as a tool for organizing and analyzing qualitative data. To avoid bias, 2 researchers outside the project (MI, and SP) were separately asked to code categories and subcategories obtained from data analysis for open coding, axial coding, and selective coding with a Cohen's kappa value  $\geq 0.65$ , so coding which is made reliable.

### 3. RESULTS AND DISCUSSION

The purpose of this study is to obtain a substantive theory (conjecture) about students' mathematical critical thinking skills, so the stages of the research process include; open coding, axial coding, and selective coding.

#### 3.1. Open Coding

In the open coding stage, it is carried out by providing a code from each student's answer related to ideas or notions in solving problems in the application of algebraic function derivatives, both on the critical thinking ability test data, and on the interview transcript data. The following shows three sample transcripts of the interview.

**Question-1:** In your opinion, what steps are needed to solve this problem? (Category-1: Problem Solving).

Participant A1, said by knowing the concept of limit, and the definition of the derivative. The value of  $f'(x)$  at  $x = x$  represents the slope of the line tangent to the curve at  $x = x$ , then analyzes the result of  $f'(x)$  for all values of  $x$ , with  $x \neq 0$ , and finds out the concept where a squared number is always a value positive, and draw the right conclusions.

Participant A2, said to use the concept of limit or the definition of a derivative to find its first derivative because the slope of the tangent to a function is equal to its first derivative. After using both methods, we get derived functions. If the derivative is a quadratic equation, we can use the negative definite property, to prove it. Next, we have to identify the derivative of the function  $f$ , then analyze the form of the function. If we have succeeded in proving the negative definite nature of the first derivative, we can answer point b, because for any value of  $x$  (except zero) the gradient will always be negative, then the graph will always fall for the intervals of  $x > 0$  and  $x < 0$  does not affect the monotony of the function.

Participant A3, said to look for the first derivative of the function  $f$  using the concept of limit/definition of derivative, then continued by looking for the range  $f'$ , so that the range  $f' < 0$ , for  $x \neq 0$ . From range  $f'$  identify whether the value is negative or positive so that the answer can be proven. Furthermore, it is necessary to note that the slope value of the tangent to a curve is  $f' < 0$ , so it is evident that the slope of the tangent is always negative. From this answer, it is obtained that the value of  $m$  is connected to the tangent and the rise and fall of the function. It is necessary to remember that the rise and fall of a graph are seen based on the  $m$  value of the tangent. from the answer a, we get  $f'(x) = m \text{ tangent} < 0$  for all real numbers  $x$  except 0. So, it can be concluded that for  $x > 0$  and  $x < 0$ , the graph of the function always decreases. interval test is performed if still unsure.

**Question -2:** What is your reason, in answer a, that the slope of the tangent to the  $f$  curve is always negative? (Category-2: Argument).

Participant A1, said the result obtained from the derivation of the function  $f$ ,  $f'(x)$  is  $(-2) / (x \wedge 2)$  which is the result of dividing a negative number by a positive number. Therefore, for nonzero  $x$ ,  $f'(x)$  will be negative, and  $f'(x)$  itself is the slope of the tangent to the curve  $f$ .

Participant A2, from the form of the first derived function, can identify that the function is a negative definite, because in the denominator, any value for  $x$ , squared, will result in a value that is always positive.

Participants A3, please note that the slope value of the tangent to a curve =  $f'(x)$ . from step 2, note that  $m \text{ tangent} = f'(x) < 0$ , so it is proven that the slope of the tangent is always negative ( $< 0$ ).

**Question-3:** from answer a, what is your conclusion about the increasing or decreasing of the graph of the function  $f$  when  $x > 0$  or  $x < 0$ ? (Category-3: Interpretation of descendants).

Participants A1, we have seen that  $f'(x)$  or the slope of the tangent is always negative. When drawn, the slopes of each tangent will point towards the lower right, at each  $x$  in the function  $f$ . This illustrates that the function  $f(x)$  always drops for  $x > 0$  or  $x < 0$ .

Participant A2, for any value of  $x$  (except zero) the gradient will always be negative, so the graph will always go down. The intervals  $x > 0$  and  $x < 0$  do not affect the monotony of the function.

Participant A3, for both  $x > 0$  and  $x < 0$ , the graph of the function always goes down.

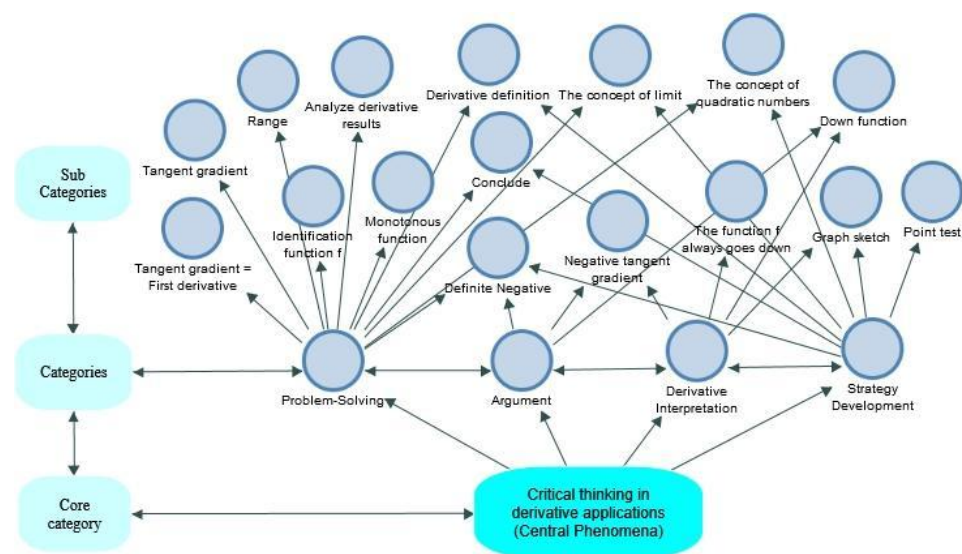
**Question -4:** in your opinion, how many ways (strategies) are there to answer the questions above? (Category-4: Strategy development).

Participant A1, said the choice of strategies to reduce the function. By using the derivative formula, the reduction of the function will be much faster when compared to using the limit concept. Then, choosing a strategy to conclude the results of  $f'(x)$  For people who do not know the concept that a square is always positive, it may be necessary to use the multipoint test and then draw conclusions from several samples that have been given.

Participant A2, to determine the first derivative, can use the concept of limit or the definition of the child. After that, proof can use form function analysis to determine the negative definite properties.

Participants A3, there are 2 ways to think about it. the first corresponds to the steps outlined above. The second way is to use the graphic method. So, the graph of the function is displayed, then tangents are drawn (tangents drawn at the point of contact with the components  $x < 0$  and  $x > 0$ ). If this is done, you will see that the tangents are all descending. This means that the slope of the tangent ( $f'(x)$ ) is negative and the function is down for both  $x < 0$  and  $x > 0$

From the extraction of the answers of the participants above, whose data comes from an emic perspective (participant), and an ethical perspective (researcher), as well as an interpretation of the relationship between the two, then analyzed using Nvivo 12 Plus, an open coding diagram is obtained as follows:



**Figure 1:** Diagram of the open coding process

After doing constant comparisons, several sub-categories are similar in reduction, so an open coding diagram is obtained as shown in Figure 1. The open coding diagram

above shows that four categories build students' critical thinking skills in mastering the application material of derivative functions which include, problem-solving, arguments, derivative interpretation, and strategy development. The next stage is the axial coding process.

### 3.2. Axial coding

In the next step, grounded theory researchers select one category and place it as a central phenomenon that is being studied, in this case, the students' critical thinking skills in the application material derived from functions. and then relating other categories to it (causal conditions), as shown in Figure 2 below.

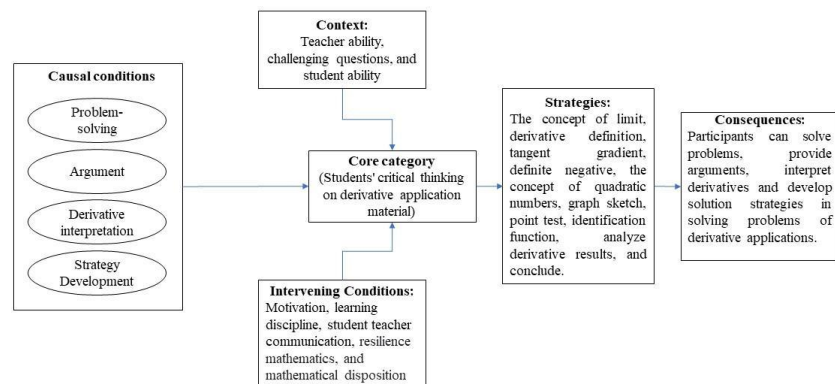
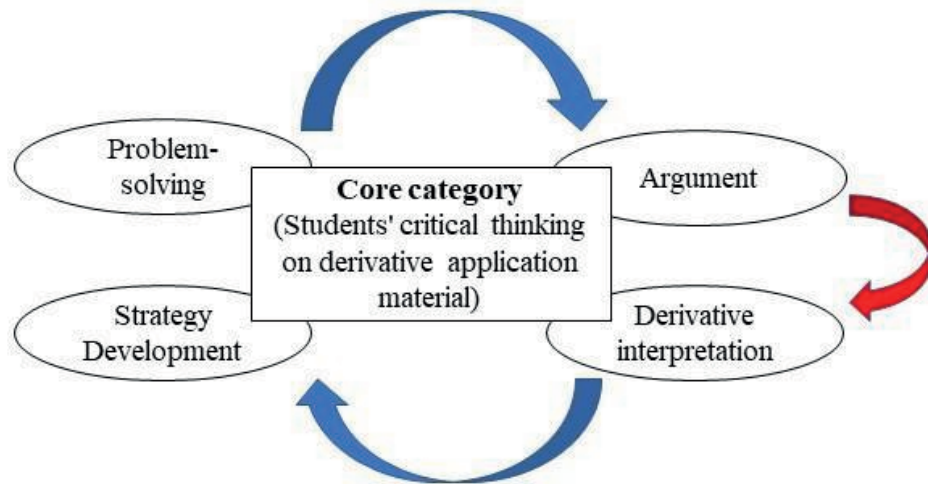


Figure 2: Diagram of the axial coding process.

From Figure 2 above, seen from left to right, six category boxes influence each other in this study. In causal conditions, four categories are found (problem-solving, arguments, derivative interpretations, and development strategies that will affect the central phenomenon (students' critical thinking abilities). Students' critical thinking abilities are influenced by two factors, namely 1). the context that includes: teacher skills, challenging questions, supported by student abilities. 2) the intervening condition is influenced by motivation, disciplined learning, the teacher's ability to communicate, mathematical resilience, and mathematical disposition, which will give birth to a diagram of students' critical strategies in solving problems of application of derivative algebraic functions (see Figure 2). In the end, the consequences obtained by participants can solve problems, be able to argue, be able to interpret derivatives, and be able to develop settlement strategies. The next step is selective coding (building hypothetical conclusions).

### 3.3. Selective coding

In this phase, the researcher writes a substance theory that is interrelated between the categories in the axial coding model and traces personal memos about the participants' ideas to build hypothetical conclusions in this study as shown in Figure 3 below.



**Figure 3:** Diagram of the selective coding.

Based on the explanation above, a hypothetical conclusion was obtained about students' critical thinking skills in the application material of algebraic function derivatives as follows:

If the participant can think critically, then the participant can solve problems, be able to provide arguments, be able to interpret problem-solving skills, in the application material derived from algebraic functions, and in the end, the participant can develop strategies in critical thinking.

Based on the findings in Fig. 3 above, the problem-solving process is part of the critical thinking process. This is in line with relevant theories and research conducted by [10, 16–18]. Students who have critical thinking skills must be able to argue [19, 20], where the argument is the most important characteristic of a critical thinker, and in the end, a critical thinker can consider the most effective problem-solving strategies

## 4. CONCLUSION

Findings of this study are expected to help education practitioners in optimizing their critical thinking skills. The foundation in critical thinking skills is problem-solving, and arguments are a special character of a critical thinker, who can produce strategies that

are considered the most appropriate in solving problems. This research is limited to the application material of algebraic function derivatives, for further researchers can test the conjectures found by conducting quantitative research, both for elementary school students and college students.

## Acknowledgments

We would like to thank all the participants who were involved in this study. Our thanks also go to Dr. M. Ikhsan, M.Pd (MI), and Dr. H. Sufyani Prabawanto, M.Ed (SP), as a coder outside this research project, and thanks to Mora for sponsoring this research.

## References

- [1] Marin LM, Halpern DF. Pedagogy for developing critical thinking in adolescents: explicit instruction produces greatest gains. *Think Skills Creativity*. 2011;6(1):1–13.
- [2] Mendikbud, “Peraturan Menteri Pendidikan dan Kebudayaan Republik Indonesia Nomor 21 Tahun 2016 Tentang Standar Isi Pendidikan Dasar dan Menengah,” (2016).
- [3] Wang Q, Woo HL. Investigating students’ critical thinking in weblogs: an exploratory study in a Singapore secondary school. *Asia Pac Educ Rev*. 2010;11(4):541–51.
- [4] Marzuki, E.C.M. Asih, and Wahyudin, “Creative thinking ability based on learning styles reviewed from mathematical communication skills.,” *Journal of Physics: Conference Series*. vol. 1315, no. 1, p. 2019.
- [5] Palinussa AL. Students’ critical mathematical thinking skills and character: experiments for junior high school students through realistic mathematics education culture-based. *Journal on Mathematics Education*. 2013;4(1):75–94.
- [6] As’ari AR, Kurniati D, Subanji S. Teachers expectation of students’ thinking processes in written works: A survey of teachers’ readiness in making thinking visible. *Journal on Mathematics Education*. 2019;10(3):409–24.
- [7] Hage G. The haunting figure of the useless academic: critical thinking in coronavirus time. *Eur J Cult Stud*. 2020;23(4):662–6.
- [8] Nardi PM. *Critical thinking: Tools for evaluating research*. Oakland (California): University of California Press; 2017. <https://doi.org/10.1525/9780520965478>.
- [9] Dewey J. “How We Think.,” D.C. Heath and Company. p. 1910. <https://doi.org/10.1037/10903-000>.
- [10] Facione P. “Critical Thinking : A Statement of Expert Consensus for Purposes of Educational.,” The California Academic Press. p. 1990.



- [11] Fisher A. "Critical Thinking An Introduction.," CAMBRIDGE UNIVERSITY PRESS. p. 2001.
- [12] Sternberg RJ. Critical thinking in psychology: It really is critical. *Critical Thinking in Psychology*; 2006. pp. 289–96.
- [13] Miller SE, Topple TA. Thinking and thinking about thinking: A qualitative study of learning in a process-centric teaching model. *J Soc Work Educ.* 2020;56(1):115–30.
- [14] Corbin A, Strauss A. J., & Strauss, "Grounded theory research: Procedures, canons, and evaluative criteria.," *Qual Sociol.* 1990;13(1):3–21.
- [15] Charmaz K, Belgrave LL. Thinking about data with grounded theory. *Qual Inq.* 2019;25(8):743–53.
- [16] Marzuki W. E. Cahya, and D. Juandi, "Students ' critical thinking skills in solving mathematical problems; A systematic procedure of grounded theory study," vol. 14, no. 4, pp. 529–548, 2021.
- [17] Alkhatib OJ. "A framework for implementing higher-order thinking skills (problem-solving, critical thinking, creative thinking, and decision-making) in engineering humanities.," 2019 *Advances in Science and Engineering Technology International Conferences, ASET 2019.* pp. 1–8, 2019. <https://doi.org/10.1109/ICASET.2019.8714232>.
- [18] Halpern, "Teaching critical thinking for transfer across domains.," *Am Psychol.* 1998;53(4):449–55.
- [19] Case R. Moving critical thinking to the main stage. *Educ Can.* 2005;45(2):45–9.
- [20] Marzuki M, Negara HR, Wahyudin W. Enhancement of students' critical thinking ability in the algebraic function derivatives application based on student learning styles during online learning. *Al-Jabar : Jurnal Pendidikan Matematika.* 2022;13(1):139–52.