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Research Article

Students' Ontogenic and Epistemological Obstacles on the Topic of Pyramid Volume

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Abstract.
The concept of pyramid volume has been studied at the starting of the elementary school (SD) level and studied more deeply in junior high school (SMP). However, many students make mistakes in solving problems related to the volume of pyramids. This study aims to identify the ontogenic and epistemological obstacles of students in understanding the concept of pyramid volume. This is a qualitative research using a Didactical Design Research (DDR). Sixteen students of grade IX in Bandung who had studied the concept of pyramid volume participated in the study. Data were obtained by analyzing respondents' ability tests and interviews to solve problems related to the volume of pyramids. Characteristics of ontogenic and epistemological obstacles were found in students' knowledge in solving problems related to the concept of pyramid volume. The results showed that students experienced several ontogenic and epistemological obstacles which resulted in errors in problem-solving.

Keywords: epistemological obstacles, pyramid volume, students' ontogenic.

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1. INTRODUCTION

Geometry is an essential branch of school mathematics in Indonesia because in the Indonesian 2006 curriculum for junior high school units, geometry has about 42% of the total content of mathematics material seen according to competency standards [1]. In Indonesia, geometry material in junior high school mathematics includes lines, angles, plane figures, congruences, solid figures, and the Pythagorean theorem [2]. In solid figures, there is a chapter on polyhedrons. According to Blanco in [3], students can still not recognize polyhedrons. Junior high school students learn in polyhedron, starting from the characteristics and nets. By firstly studying the traits and nets of the polyhedrons, it is hoped that students will be able to distinguish various polyhedrons as initial capital in understanding the concepts of surface area and volume of a polyhedrons.

Students' low ability in the polyhedrons part of the geometry material can also be caused by misconceptions about understanding the area of plane figures and difficulties in applying formulas [4]. This condition will negatively impact students in studying the next topic because students do not know enough to know the rules/procedures that must be done to plan a solution. If students do not understand the correct concepts about applying methods in solving problems, students will be wrong in solving problems, including no routine problems [5].

One of the volume concepts of the polyhedron is the volume concept of a pyramid that sixth grade elementary school students must master and study in more detail at the junior high school grade eight, as stated in the junior high school mathematics competency [6]. In Silver, Senk, and Thompson state that, in general, teachers first explain mathematical formulas, and students accept the explanation and then work on the problems given [7, 8]. This pattern may cause students to have difficulty in solving problems with different pyramid volumes like the examples given earlier because students are only trained to solve many questions without deep understanding [9, 10].

The results showed that students had difficulty solving problems related to the volume of pyramids (Figure ??) due to the gap in the difficulty of the questions given and students' mental readiness [10].

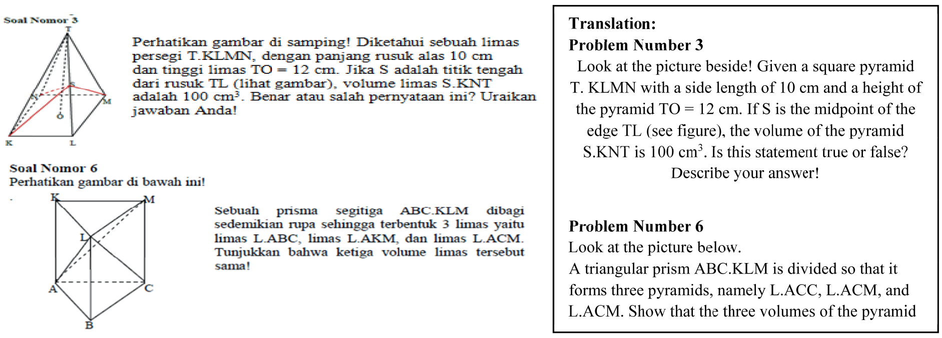


Figure 1: Problem 1: volume of the pyramid.

As for other research in the mathematics textbook used, there is the use of algebraic processes in constructing the surface area and volume of pyramids that are not following the thinking abilities of junior high school students because they are still at a concrete level, students have not been able to recognize the relationship between pyramid elements (understanding the concept of pyramids)—not owned by students as a whole [11]. Another study showed that students knew the formula for the volume of a pyramid but could not use it because they did not understand the Pythagorean theorem [3]. Meanwhile, more than 70% of students got the volume formula wrong and determined the volume of the pyramid because they did not know the properties and characteristics of the pyramid.

A study conducted on grade eight junior high school students showed obstacles in the surface area and volume of a pyramid [8]. It was caused by students having difficulty understanding the prior knowledge, namely the Pythagorean theorem, even though this prior knowledge was beneficial in understanding the concepts of surface area and volume of a pyramid. Students will have difficulty understanding the concepts of surface area and volume of a pyramid when the prerequisite concepts have not been understood and will also only memorize all the formulas without interpreting what they know.

Based on previous research and based on the statement of Turk and Arslan that students' new knowledge construction cannot be separated from epistemological obstacles, thus overcoming epistemological obstacles is one of the critical points in closing the gap between students' knowledge and understanding, the authors aim to identify ontogenic obstacles and epistemological obstacles for junior high school students in the pyramid volume material so that they can develop a didactic design that can minimize student learning obstacles to achieve learning objectives [12, 13].

According to Duroux, learning obstacles are part of knowledge or conception, not a lack of knowledge resulting in wrong responses (out of context) [14]. Brousseau [14] divides learning obstacles based on their sources into three types, namely:

1. Ontogenic obstacles relate to students' mental readiness and cognitive maturity in receiving knowledge.

2. Didactical obstacles are learning obstacles caused by sequence factors or stages of material presentation.

3. Epistemological obstacles caused by limited contexts used the first time a concept is studied.

2. RESEARCH METHOD

This research is qualitative. The research design used was Didactical Design Research (DDR). According to Suryadi [15], DDR consists of three stages, namely:

1. A didactic situation analysis was conducted before learning in the form of a study of learning obstacles.

2. Metapedadidactic analysis, namely the analysis of the teacher's ability to identify, analyze and relate thought processes before learning, during learning, and reflection after learning.

3. Retrospective analysis is an analysis that links the results of the hypothetical didactic situation analysis with the results of the metapedadidactic analysis.

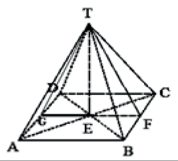
This research is the first step, namely the analysis of the didactic situation, which will later be used as a reference in designing and developing the didactic design. The subjects of this study were 16 grade nine students of one of the junior high schools in Bandung Regency who had experience learning the concept of volume pyramids. This condition was chosen because the researcher will know the students' ability to understand the concept of pyramid volume so that ontogenic and epistemological obstacles can be analyzed.

Data was collected through respondent ability test and interviews. First of all, students filled out the respondent ability test, which consisted of four questions related to the volume of the pyramid. Then, the researcher checked the students' answers and selected several students to be interviewed. Interviews were conducted to determine the extent to which students understand the volume of pyramids and to confirm the ontogenic and epistemological obstacles that students might experience based on their answers on respondent ability test. The real data that has been obtained is processed qualitatively; namely, all data is analyzed for each participant, interpreted, and identified learning obstacles experienced by students on the topic of pyramid volume based on test results and interviews. Then the writer takes some of the results of student answers based on the difficulties identified and takes the main points by selecting a sample of students based on the type of difficulty. After the data is reduced, the data is presented in narrative text and images. Through the presentation of these data, the authors relate test answers and interview results. The next step is concluding. This conclusion is expected to clarify the author's findings.

3. RESULTS AND DISCUSSION

The data in this study were obtained after 16 students completed the test, which consisted of 3 questions regarding the volume of pyramids following the applicable curriculum and interviews. The data obtained were in the form of written answer sheets from the test result and the results of interviews. Furthermore, the data that has been received is analyzed to identify ontogenic and epistemological obstacles experienced by students by looking at the answers and results of interviews. Problems 1a and 1b, listed in Figure 2, aim to examine students' knowledge about the properties and characteristics of pyramids. Point c aims to see if students can show the height of the triangle and the pyramid's height as a prior concept for the volume concept of a pyramid. Point d aims to see if students can name plane figures that can form a pyramid as a prior concept for the volume concept of a pyramid.

1. Perhatikan gambar di bawah ini!

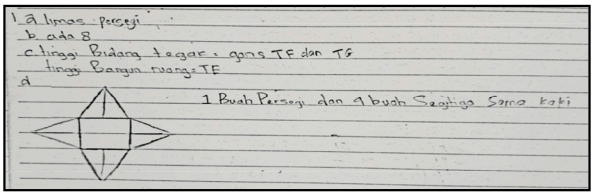


a. Apa nama bangun ruang tersebut!
b. Berapa banyak rusuknya?
c. Sebutkan garis yang merupakan tinggi bidang tegak dan tinggi bangun ruang tersebut!
d. Buatlah jaring-jaring dari bangun ruang tersebut, lalu sebutkan bangun datar apa saja yang membentuknya?

Translation:
1. Look at the image below.
 a. What is the name of the solid?
 b. How many edges?
 c. Which the height of the vertical planes and the height of the solid?
 d. Make a net from the solid, then mention what planes make it up?

Figure 2: Problem 1 about the properties and characteristics of pyramids [16].

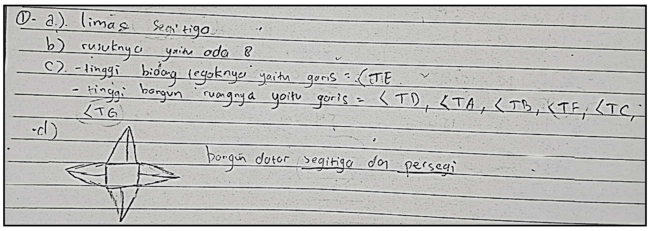
Figure 3 is the response of S5 listed on the TKR. S5 states that the name of the shape presented in the problem is a square pyramid, which has eight edges, states that lines $TFTF$ and TG are the height of the perpendicular plane of a square pyramid, the line TE is the height of a square pyramid, describes the nets of the pyramid and states that there is 1 square and The four isosceles triangles that make up the square pyramid are presented in problem 1. S5 can solve problem 1 conceptually well so that S5 can understand the nature and characteristics of the pyramid.



Translation:
a. Square Pyramid
b. There is 8
c. Slant height: TF dan TG
d. Vertical height: TE
e. 1 square and 4 isosceles triangles

Figure 3: Description of answer S5 in solving problem 1.

It understands the property and characteristics of pyramids obtained from test answers and the results of interviews with S13, as listed in Figure 4. In answering problem 1a, S13 knows that the shape of the space presented in the question is a pyramid, for a more detailed naming of pyramids, S13 understands as a triangular pyramid because, according to S13, the plane figures that form more pyramids are triangles, so the name of the shape presented is a triangular pyramid. This erroneous understanding resulted in the steps of working on the answers to questions number 3 and 4 is wrong. S13 always uses the terms base (a) and height (t) to determine the area of the pyramid's base because S13 feels that the base is also a triangle. When answering problem 1c, S13 was confused in choosing the height of the rectangular pyramid and the height of the square pyramid; S13 also included the edges of the rectangular pyramid as the height of the square pyramid. S13's answer to problem 1 shows that he does not understand the nature and characteristics of the pyramid, even though S13 has been able to determine the plane figures that make up the presented square pyramid.



Translation:

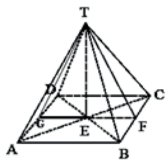
a. Triangular Pyramid
b. The edges are 8
c. The vertical height = TE
The slant height = TD, TA, TB, TF, TC, TG
d. Plane Figure: Square and Triangle

Figure 4: Description of student 13 (S13) answers in solving problem 1.

When the interview was conducted with S13, he felt that the solid material was more difficult than other mathematics materials, so he was less enthusiastic about studying it.

The following description is the student's response to problem 2, listed in Figure 5. Problem 2 is structured to understand further the height of the pyramid and the students' initial understanding of the concept of the Pythagorean theorem and the area of the plane figures as a prerequisite concept for the volume of the pyramid. The question was presented by giving a regular rectangular pyramid, knowing the length of the side that forms the base of the pyramid and the length of the hypotenuse that forms the perpendicular plane of the pyramid. Students are asked to determine the area of the perpendicular pyramid and the area of the triangle formed in the pyramid—figures 5 and 6 show student responses containing answers to solving problem 2.

2. Perhatikan gambar limas segi empat beraturan ini. Jika panjang AB = 18 cm dan panjang BT = 15 cm. Hitunglah luas ΔBCT dan luas ΔFGT!



Translation:

2. Look at this regular rectangular pyramid. If AB = 18 cm and BT = 15 cm. Calculate the area of ΔBCT and the area of ΔFGT!

Figure 5: Problem 2 on the pythagorean theorem and the area of a triangle [11].

Figure 6 is the response of S2 listed in the TKR. S2 first analyzes the presented image; he knows that to determine the area of $\triangle BCT \triangle BCT$, it is necessary to find the length of the $TFTF$ line as the height of $\triangle BCT \triangle BCT$. Then S2 uses the Pythagorean theorem to determine the length of the line $TF\, TF$ so that the length of $TF = 12\ cm TF = 12\ cm$ and the area of $\triangle BCT \triangle BCT$ is $108\ m^2. 108\ m^2.$ The length of the $TETE$ line is needed as the height of $\triangle FGT \triangle FGT$ to determine the area of $\triangle FGT \triangle FGT$; then S2 reuses the concept of the Pythagorean theorem to determine the length of $TETE$ so that the length of $TE = 3\sqrt{7}\,\mathrm{cm} TE = 3\sqrt{7}\,\mathrm{cm}$, and the area $\triangle FGT = 27\sqrt{7}\mathrm{cm}^2 \triangle FGT = 27\sqrt{7}\mathrm{cm}^2$. When determining the length of the $TETE$ line, S2 already understands how to simplify the root form so that he can solve problem 2 correctly and conceptually, and procedurally.

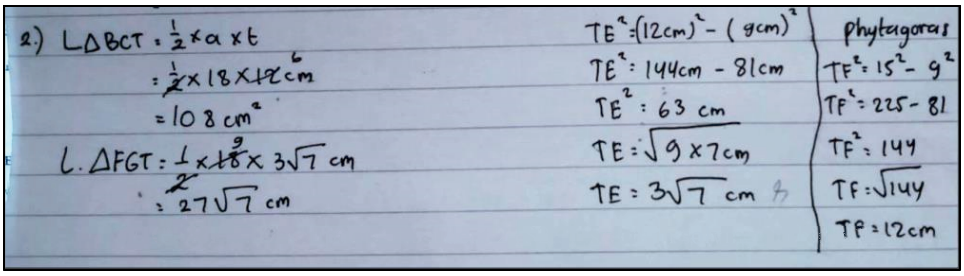
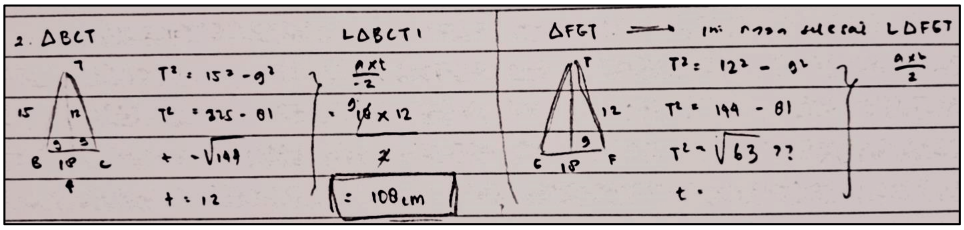


Figure 6: The response of S2 listed in the TKR.

The understanding of the Pythagorean theorem is obtained from the answers of TKR (Figure 7) and the results of interviews with S1. To solve problem 2, S1 understands that before determining the area of $\triangle BCT \triangle BCT$ and $\triangle FGT \triangle FGT$, he must first determine the height of $\triangle BCT \triangle BCT$ and the height of $\triangle FGT \triangle FGT$ using the concept of the Pythagorean theorem. S1 is correct in determining the height and area of $\triangle BCT \triangle BCT$, but he is wrong in stating the unit of area. It should be $\mathrm{cm}^2\mathrm{cm}^2$. When S1 determines the height of $\triangle FGT \triangle FGT$, it stops working on $\sqrt{63}\sqrt{63}$ and does not continue to determine the area of $\triangle FGT \triangle FGT$. When interviewed, S1 admitted that he did not understand how to work on $\sqrt{63}\sqrt{63}$ because he rarely found it in the sample questions. This is caused by the concept of the square root, which has not been fully understood by students and because students often complete the form of the square root, which is routinely studied. This finding shows that to understand the concept of the volume of a pyramid, it is not enough to understand the Pythagorean theorem and the area of a plane figures, but skills in the concept of square roots are needed.

Problem 3, listed in Figure 8, aims to identify students' understanding of the volume of pyramids. The problem presentation is that it is known that a rhombic pyramid with diagonal lengths of $10\ \mathrm{cm}10\ \mathrm{cm}$ and $15\ \mathrm{cm}15\ \mathrm{cm}$, the height of the pyramid $18\ \mathrm{cm}18\ \mathrm{cm}$, then the diagonals of the base and height of the pyramid are extended three times their original length. Students are asked to determine the change in the volume of the pyramid before and after being extended three times folding. Various answers are shown in Figure 8 and Figure 9 as student responses when solving problem 3.

Conceptual understanding of pyramid volume is obtained from TKR's answer (Figure 9) and interview results on S15. When interviewed, S15 admitted that he often got confused about determining formulas when solving problems because he memorized too many formulas, especially for plane figures and geometric shapes. This recognition



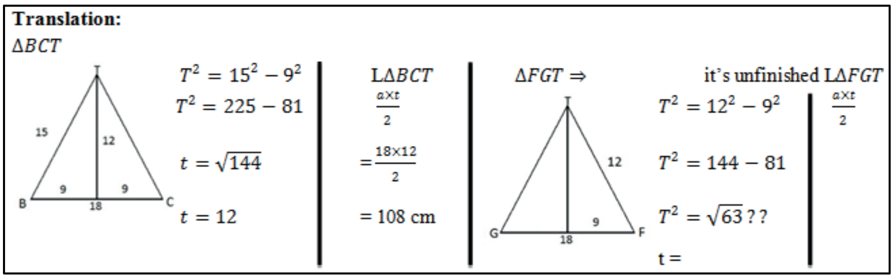


Figure 7: The Pythagorean theorem is obtained from the answers of TKR.

3. Alas sebuah limas berbentuk belah ketupat dengan panjang diagonal-diagonalnya 10 cm dan 15 cm. Tinggi limas adalah 18 cm. Jika diagonal-diagonal alas maupun tingginya diperpanjang 3 kali panjang semula, maka tentukan besar perubahan volume limas!

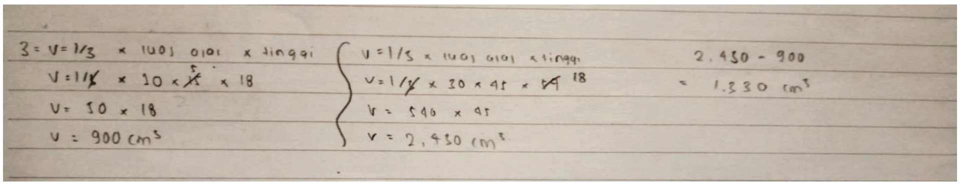
Translation:

3. The pyramid's base is a rhombus whose diagonals are 10 cm and 15 cm long. The height of the pyramid is 18 cm. If the diagonals of the base and the height are extended three times their original length, determine the change in the volume of the pyramid!

Figure 8: Problem 3 about the volume of pyramids [11].

aligns with S15's answer in solving problem 3. S15 made an error when determining the area of the base of a rhombus pyramid, after multiplying diagonal one and diagonal 2, he did not multiply by $\frac{11}{22}$. Even though S15 was correct in understanding problem 3, because he did not understand the concept of the area of a plane figures and only memorized the formula, S15 could not solve problem 3 perfectly. The expected answer to problem 3 is that students can determine the change in the volume of the two pyramids (before and after diagonal 1, diagonal two and the height is extended three times its original length) with the help of the concept of area of a plane figure. Not only S15, but 56.25% of students who did the TKR made the same mistake. This finding shows that memorizing the formula alone is not enough to solve the problem correctly but must be accompanied by understanding it conceptually.

S15's answer shows that he is able to solve problem number 3, but S15 has a poor memory of the volume of pyramids. This S15 error is indicated because he only remembers at that moment without understanding what he learned. Based on the responses given by students to answer number 3, the writer can conclude students' understanding of the volume of pyramids. Students need help understanding the concept of area of a



Translation:

$V = {}^{1}/_{3} \times base's\ area \times height$	$V = {}^{1}/_{3} \times base's \times height$	$2450 - 900$
$V = {}^{1}/_{3} \times 10 \times 15 \times height$	$V = {}^{1}/_{\cancel{3}} \times 30 \times 45 \times \cancel{54}\ ^{18}$	$= 1.330\ cm^3$
$V = 50 \times 18$	$V = 540 \times 45$	
$V = 900\ cm^3$	$V = 2.450\ cm^3$	

Figure 9: Description of student answers 15 (S15) in solving problem 3.

plane figure and concept of pyramid volume, students tend to memorize formulas, so when students forget formulas, errors occur.

4. CONCLUSION

Students experience several ontogenic and epistemological obstacles. The ontogenic obstacles that exist in the pyramid volume concept are as follows:

1. Lack of student motivation in studying the concept of pyramid volume because students view the flat-sided geometrical material as more complex than other mathematical materials.
2. Lack of understanding of students regarding prerequisite materials, such as the concept of plane figures, Pythagorean theorem, and square roots.

Epistemological obstacles that exist in the concept of volume pyramids are as follows:

1. Students do not understand the concept of simplifying the square root.
2. Students do not understand the concept of plane figures area.
3. Students do not understand the concept of unit area.
4. Students tend to memorize formulas from the concept of the volume of a pyramid, causing them to misunderstand the concept as a whole.

Each identified obstacle is expected to be considered by the teacher in designing the pyramid volume concept learning in the classroom. Teachers can provide contexts that develop students' understanding of the volume pyramid concept and some other mathematical concepts related to it. The teacher needs to provide a context that can encourage students to build new knowledge about a rule or formula. The given context must vary how the volume of the pyramid is applied. Thus, this research can be expected to be a consideration to overcome or minimize ontogenic and epistemological obstacles in learning the volume concept of pyramids in the classroom.

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References

[1] Mujahidah Annisa A, Suryadi D, Rosjanuardi R. "Design development of determinant lines materials and angles on math learning for junior high school.," In: Proceedings of the University of Muhammadiyah Malang's 1st International Conference of Mathematics Education (INCOMED 2017). pp. 1–9. Atlantis Press, Paris, France (2018).

[2] Cahyaningrum S, Sumardi. "Identifikasi kesulitan dalam menyelesaikan soal cerita pokok bahasan prisma dan lima siswa kelas VIII semester II SMP Negeri 4 Delangu Tahun Ajaran 2014/2015." In: Proceedings of the National Academy of Sciences. pp. 1–15 (2015).

[3] Fuadiah NF, Sawitri ZA. A new learning trajectory on the pyramid volume for secondary school. J Phys Conf Ser. 2020;1480(1):012028.

[4] Aziiza YF, Rosjanuardi R, Juandi D. Didactic design of the concept of surface area of flat-sided prism based on van Hiele's theory in online learning. Jurnal Pendidikan Matematika. 2022;16(1):73–88.

[5] Hidayat CR, Rosjanuardi R, Juandi D. "Epistemological obstacle on the topic of triangle and quadrilateral.," In: International Conference on Mathematics and Sience Education. IOP Publishing, Jawa Barat (2019). https://doi.org/10.1088/1742-6596/1157/4/042110.

[6] Kemdikbud, Permendikbud Nomor 37 tahun 2018 tentang perubahan atas PermenDikbud No. 24 tahun 2016 tentang kompetensi inti dan kompetensi dasar pelajaran pada kurikulum 2013 pada Pendidikan Dasar dan Pendidikan Menengah., Indonesia, 2018.

[7] Turmudi, "Pembelajaran matematika kini dan kecenderungan masa mendatang.," In: Bunga Rampai Pembelajaran MIPA. pp. 1–28. UPI, FPMIPA, UPI, Jawa Barat (2010).

[8] Nur'aeni A. Desain didaktis konsep barisan dan deret aritmetika pada pembelajaran Matematika Sekolah Menengah Atas. Journal of Mathematics Education Research.

2017;1(1):1–10.

[9] Siti Aisah L. Kusnandi, and K. Yulianti, “Desain didaktis konsep luas permukaan dan volume prisma dalam pembelajaran Matematika SMP.” M A T H L I N E : Jurnal Matematika dan Pendidikan Matematika. 2016;1(1):14–22.

[10] Sulistiawati. “Analisis kesulitan belajar kemampuan penalaran matematis siswa SMP pada materi luas permukaan dan volume limas.” In: Proceeding Seminar Nasional Pendidikan Matematika. pp. 205–225. Dan Tik Stkip Surya, Sains (2014).

[11] Rosmalia NL. “Desain didaktis luas permukaan dan volume limas pada pembelajaran matematika di SMP,” http://repository.upi.edu/18752/, (2015).

[12] Turk A, Fairclough E, Grason Smith G, Lond B, Nanton V, Dale J. Exploring the perceived usefulness and ease of use of a personalized web-based resource (care companion) to support informal caring: qualitative descriptive study. JMIR Aging. 2019 Aug;2(2):e13875.

[13] Kandaga T, Rosjanuardi R, Juandi D. Epistemological obstacle in transformation geometry based on van Hiele’s level. Eurasia J Math Sci Technol Educ. 2022;18(4):em2096. https://doi.org/10.29333/ejmste/11914.

[14] Brousseau G. Theory of didactical situations in mathematics: Didactique des mathématiques. Springer Science & Business Media. Jerman; 2006.

[15] Suryadi D. “Didactical Design Research (DDR) dalam pengembangan pembelajaran matematika.,” In: Joint Conference UPI-UiTM 2011. pp. 12., Indonesia (2011).

[16] Salamah U. Berlogika dengan matematika 2: untuk kelas VIII SMP dan MTS. PT. Tiga Serangkai Pustaka Mandiri. Solo; 2015.