KnE Social Sciences
ICESRE
6th International Conference on Education and Social Science Research
Volume 2024

Research Article

Problem-solving Strategy: Mathematical Problem-solving Model Within the Polya' Framework

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Abstract.
Mathematical problem-solving is very important for students to learn and understand, which can later be used in solving problems that exist in everyday life. This study aims to analyze the mathematical problem-solving model within the framework of Polya' theory. The subjects in this study were fifth grade students showing their strategy to solve mathematical problem. We analyzed the student' problem-solving strategy with real word problems and configured their solution. The results of this study are in the problem-solving process, the subject has completed according to the stages of Polya. At each step, the indicators work together in a unique way to solve a math problem. This research initiates that the connection between indicators in problem-solving can form a cycle of resolution.

Keywords: problem solving, mathematical problems, mathematics ability

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Published 12 March 2024

Publishing services provided by Knowledge E

Selection and Peer-review under the responsibility of the ICESRE Conference Committee.

1. Introduction

Problem solving is very important in the mathematics curriculum. Problem solving is a process of educational goals that can be viewed from the curriculum aspect [1]. Problem solving is a process of applying previously obtained information into new, previously unknown situations [2]. In solving this problem it is very important because it can be used as a basis for reference in the implementation of mathematics learning for any level. The application of problem solving requires skills such as critical thinking, so that when students have mastered a problem, students can make plans to solve problems and be able to find the right answer in solving a problem [3].

Problems in learning mathematics given are in the form of non-routine questions, such as story questions, depicting an event, illustrated pictures, and puzzles [4]. Mathematical problem solving is an effort that will be made by someone using all his efforts to solve a problem he will face [5]. This is in line with that opinion, that problem solving is an activity

that requires a high level of mentality with the aim of improving problem solving skills in learning mathematics [6]. Thus, in the application of learning mathematics students can use all their abilities to deal with problems that exist in mathematics. Mathematical problem solving ability is an ability to understand several known components, the adequacy of the required elements, which are asked, and can make plans in solving problems which in the end students will find the answers obtained.

Problem solving in a field of mathematics is very important for students to learn so that students can solve a problem that exists in everyday life. When students have the ability to solve problems, they can provide solutions to the problems they face in an organized manner (Widodo et al., 2018). The importance of mathematical problem solving is also explained by National Council Teacher Mathematics (NCTM) that problem solving has special importance in the study of mathematics [7]. The purpose of learning mathematics is to improve a skill in solving various kinds of complex mathematical problems.

Polya explains that in problem solving there are four stages including understanding the problem, planning a solution, solving the problem according to plan, and re-examining the results that have been obtained [8]. In four stages of mathematical problem solving has a unity that can be developed to solve a problem. The use of strategies in mathematics to solve mathematical problems has an influence on students' abilities and skills [9]. With a variety of strategies in solving a mathematical problem can make it easier for students to solve a problem contained in the problem.

Previous research explained that in solving a problem using the Polya stage is very effective for students, thus it can make it easier for students to solve a problem, reduce carelessness or mistakes made when solving a problem [10] can gain an experience using the knowledge and skills already possessed [11]. Furthermore, the Polya stage is very effective for developing students' mathematical problem solving abilities than with students' problem solving in ordinary learning [11]. So it can solve mathematical problems using the Polya stages can help students to solve a problem easily.

The purpose of the research is to analyze the mathematical problem solving model within the framework of Polya's theory. In this implementation, students can complete tests in the form of description questions. This will show how the ability of students to solve a problem from an existing problem using the Polya stage.

2. Method

This research uses qualitative research. Qualitative research is a research that investigates a relationship, activity, situation or material. This research use desciptive qualitative approach. The type of research that will be used is a case study.

Taking the subject in this study is a class V student. The process of taking the subject is done by giving a description of the problem in the form of mathematical problem solving. The students who were selected as subjects in this study were taken by 2 students. Students who are used as research subjects for S1 and S2. Subject 1 is female while subject 2 is male.

The procedure in this study is that researchers prepare a Mathematical Problem Test (MPT) which will be used to see the mathematical problem solving process in students (see Figure 1). In working on the MPT, subjects were given 15 minutes. After doing the Mathematical Problem Test (MPT), the researcher conducted interviews with the two subjects with the aim of seeing the process of solving mathematical problems within the framework of Polya's theory.

> Sponge cake sellers provide two types of sponge with the same thickness, but different sizes. The surface diameter of the large sponge cake is 14 cm, while the small one is 10 cm. If a large sponge cake is sold for Rp. 15,000, the price of a small cake is Rp. 10,000. Which is more profitable, buying three small sponge cakes or two large sponge cakes?

Figure 1: Mathematics Problem.

The instruments in this study are the Mathematical Problem Test (MPT) and interview guidelines. The Mathematical Problem Test used in this study is in the form of a description question. Mathematical Problem Test (MPT) is used to determine students' ability to solve a mathematical problem based on Polya's theory. Before the questions are given to students, the questions must be validated first. The interview guide contains questions to be asked. The form of this guide is in the form of a table of questions that will make it easier for researchers to carry out interviews. Problem solving indicators are presented in table 1.

Researchers used data analysis according to Miles, Huberman, and Saldana (2014) which was carried out interactively starting from data condensation with the aim of creating a problem-solving model using the Polya framework. The presentation of the data will be presented in the form of a problem-solving model diagram. Verification and drawing conclusions in this study were carried out by comparing data from the Mathematical Problem Test (MPT) and data from interviews.

TABLE 1: Indicator of problem solving step.

Problem solving step	Indicator
Understanding the problem	1.a Name the symbols used in solving the problem 1.b Meaning of the question symbol 1.c Determine the symbols with questions 1.d Presenting the symbols related to the problem
Device a plan	2.a Choose a symbol related to the problem according to the purpose 2.b Assign symbols in mathematical models 2.c Presents the meaning of mathematical models related to problems and strategies 2.d Choose the appropriate method
Carry Out the plan	3.a Using the chosen method to solve the problem 3.b Use mathematical models that have been created to solve problems 3.c Using symbols correctly in every step according to mathematics concept 3.d the relationship between symbols used in solving problems
Look back	4.a Proving the suitability of symbols and troubleshooting steps used 4.b Symbols have different meanings in different problems

3. Result and Discussion

3.1. Results

In the implementation of solving mathematical problems, subject 1 begins by describing what is known in the problem and the problems that exist in the problem. This is shown in Figure 2.

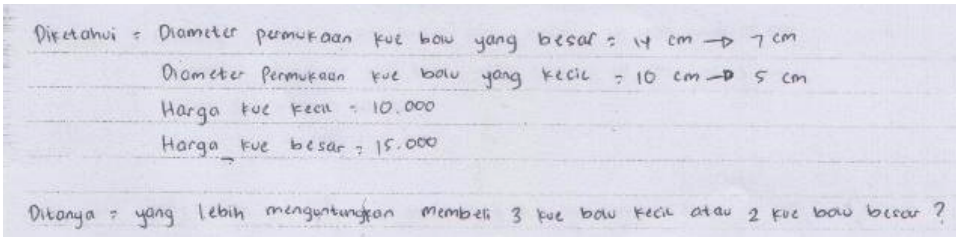

Figure 2: Understand the problem.

In the first stage, namely understanding the problem, it can be seen in Figure 2 that it is able to understand what information is contained in the question and what is asked in the question. Subjects are able to inform a meaning of mathematical symbols in problem solving. (1A, 1B). Subjects are also able to present the symbols related to the problem (1D). It can be seen in Figure 2 that the subject can determine the right symbol to solve a problem and is able to inform the symbol in the mathematical model that has been made in a problem (2A, 2B). After understanding what was asked, the subject made a plan in solving the existing problem (2D).

P : What is known about it?

S1 : The surface diameter of the large sponge cake is 14 cm, the radius is 7 cm. The surface diameter of the small sponge cake is 10 cm, the radius is 5 cm. The price of a small cake is Rp. 10,000, the price of a large cake is Rp. 15,000

P : So what is being asked in this question?

S1 : More profitable 3 small sponge cakes or 2 large sponge cakes.

The results obtained from the Mathematical Problem Test (MPT) and excerpts from interviews with subject 1, that subject 1 can inform what is in the question and what is asked on the question which is a problem that needs to be solved. In the next stage, the subject can solve a problem according to the plan that has been made previously. Subject 1 provides an explanation of the strategy that will be carried out to solve the problem in detail from the beginning until the subject finds the final result.

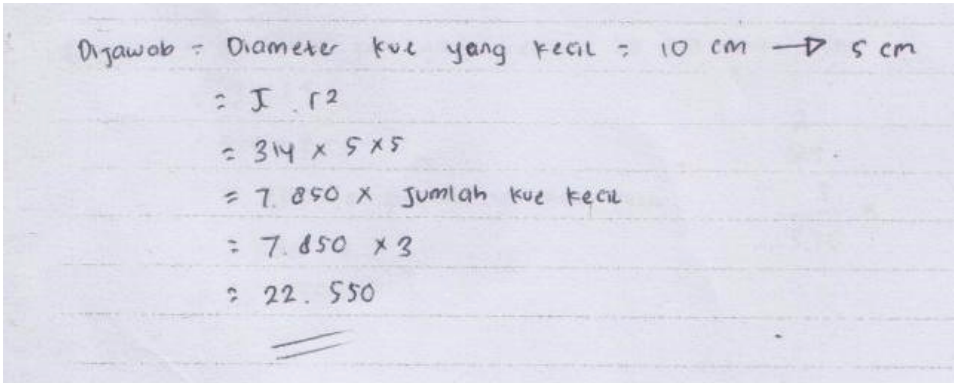

Figure 3: Solve the problem by using cirle concept.

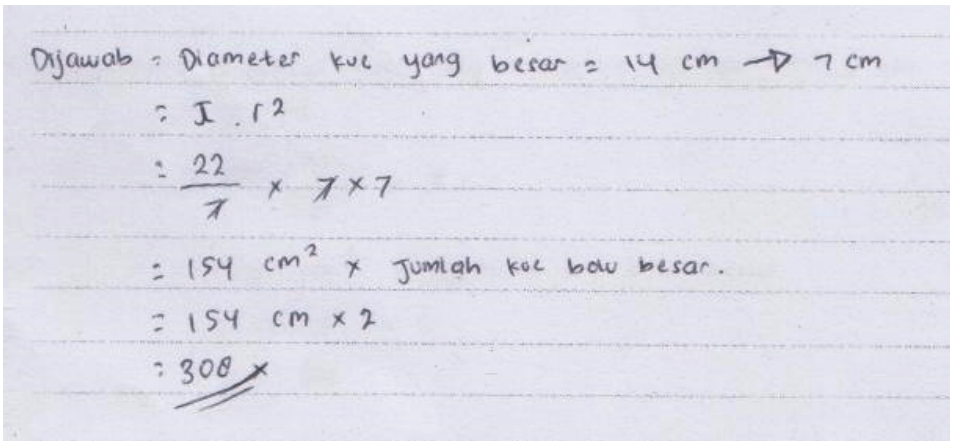

Figure 4: Comparation of two cake.

From the gains obtained by subject 1 shown in Figure 3, the subject looks for the area of the circle on a small sponge cake first. It can be seen that the subject can solve the problem but there are errors in the search for results. In the excerpt of the interview with

S1, the subject explained broadly on a small sponge cake. Next, the subject explains the area of the large sponge cake in Figure 4.

S1 finds the area of a circle on a large sponge cake with the same formula. With the explanation above, the subject has used the chosen method in solving the problem (3A). Subjects also mathematical models that have been created to solve problems and every presented steps (3B, 3C). From the excerpt from the interview with S1, the subject can make a plan in solving the problem well. Judging from the wide results on the small sponge cake, there is an error in multiplying the results of the small sponge cake.

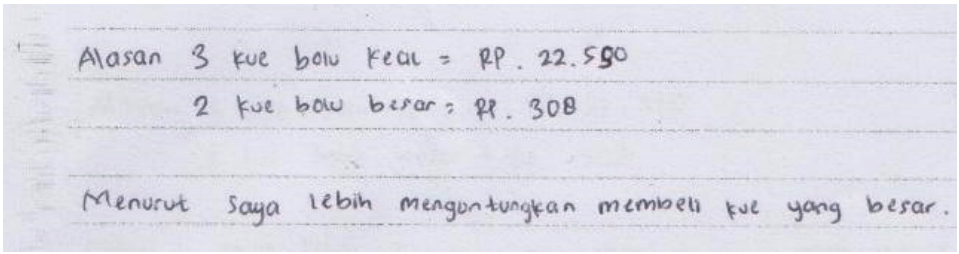

Figure 5: Making conclusion.

In Figure 6, the subject has provided a review of the results that have been found. The conclusion presented by subject 1 explains that it is more profitable to buy a large sponge cake. With this statement, the subject has confirmed the final result in solving the problem. Furthermore, subject 1 re-examined the results he had obtained to ensure that in solving the problem it was correct (4A). The subject also explained that if the symbols used had different meanings if there were different problems (4B).

P : Are you sure about this answer?

S1 : Yes, Im sure

P : After you solve this problem do you check again?

S1 : Of course

P : Do you think that if there are other problems using that formula, will the results be roughly the same?

S1 : Its depend on the problem I think

Seen in the test results and interview quotes at S1, the subject has gone through the stages of re-examining the results obtained. Thus, it can minimize errors or mistakes in working on questions.

The results obtained from the Mathematical Problem Test (MPT) and interview quotes on subject 1, the subject was able to complete the first problem-solving stage, namely understanding the problem. Furthermore, the subject can plan what will be done when solving a problem. Here the subject does not explain the plan that will be used. But seen

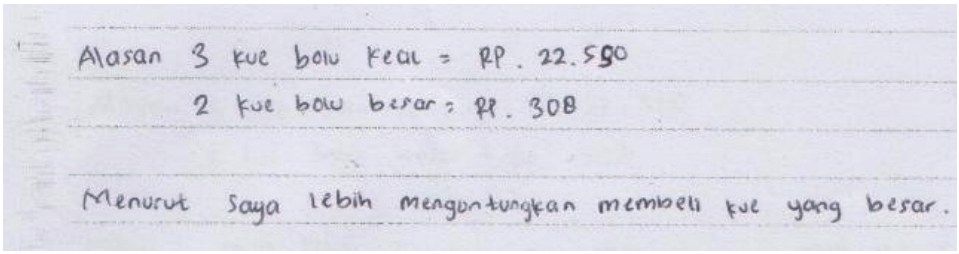

Figure 6: Problem solving model of S1.

on the test sheet, the subject immediately worked according to what was planned. After the subject finds the results obtained, the subject re-examines the results obtained to ensure whether the answers that are done are correct or not. Subject 1 also provides information about the final result in the form of a conclusion. This is done to ensure the final result he gets and further clarify the information he gets from solving the problem.

In the implementation of solving mathematical problems, subject 2 begins by describing what is known in the problem and the problems that exist in the problem. This is shown in Figure 7.

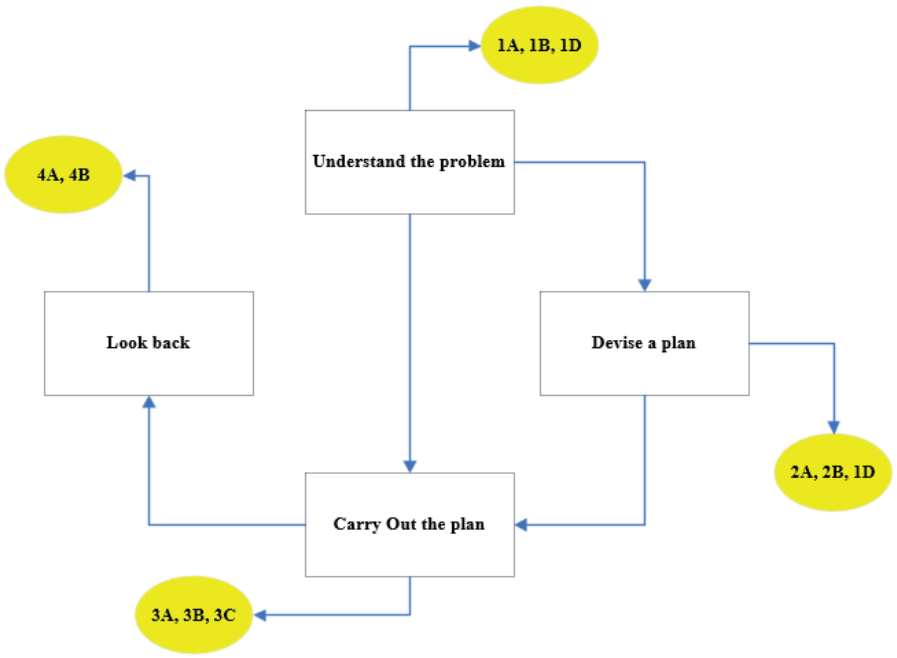

Figure 7: Understanding the problem.

In the first stage, namely understanding the problem, it can be seen in Figure 7 that subject 2 is able to understand what information is contained in the problem. However, as shown in Figure 2, subject 2 does not provide an explanation of what was asked in the question. Subject 2 can inform a meaning of mathematical symbols in problem

solving (1A, 1B). Subject 2 can also write symbols that match their meanings listed in the problem (1D). This can be strengthened by excerpts from interviews with S2.

P : Okay, what do you know from the questions you read?

S2 : What is known is a small diameter of 10 cm if the radius is made 5 cm. the diameter is 14 cm and the radius is 7 cm

P : So what is being asked in this question?

S2 : Which is more profitable

The results obtained from the Mathematical Problem Test (MPT) and interview quotes with subject 2, subject 2 can provide information about what is known in the question and is able to provide information about what is asked in the question. The subject is also able to choose the symbol to solve a problem and is able to explain the symbol model in mathematics (2A, 2B). Furthermore, the subject is able to choose the method that will be used in solving a problem (2D). The next stage is that the subject is able to solve the problem in question by using a previously planned strategy. Furthermore, the subject can explain step by step in solving the problem to find the final result that has been obtained.

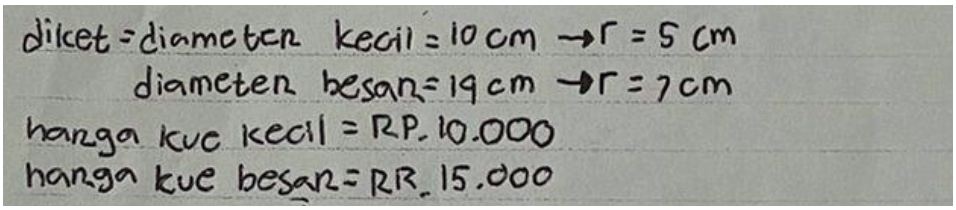

Figure 8: Comparing the area of cake.

Based on the answers in Figure 8, the subject is able to solve the questions given. Subject 2 was able to solve the problem using the previously planned method to solve the problem (3A). The subject used the formula for the area of a circle on each sponge cake to show which one was more profitable. It can be seen in Figure 8 that the subject also uses mathematical models to solve problems and uses the correct symbols in solving problems (3B, 3C). It can be seen in the interview quotes and test results, S2 which explains how to solve the problems in detail. However, for the results sought the broad part of the sponge cake is still not right. S2 also knows the meaning of the symbols used in solving the math problems used. The subject can explain that it is more profitable to give a small sponge cake. However, the subject seemed to be still unsure about the answer he got. On the answer sheet shown in Figure 9, the subject did not provide information or conclusions from the results obtained. The subject re-examines the results obtained to minimize errors when working on questions (4D).

Seen the results of the completion of subject 2, subject 2 can understand the information contained in the problem. The subject provides complete information about what is known in the question and the subject is able to explain what is being asked in the question. Subject 2 is able to formulate a strategy or plan before solving the problem. Then the subject can solve the problem using a plan that has been made previously. In the last stage, the subject has re-examined the results obtained.

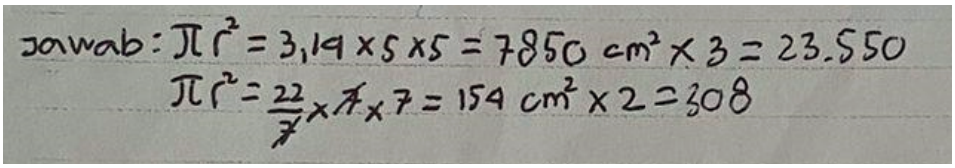

Figure 9: Problem solving model of S2.

3.2. Discussion

Based on the results of the study, several fifth grade students were given a Mathematical Problem Test (MPT) to see the mathematical problem solving process. In the implementation of problem solving there are stages, namely understanding the problem, planning a solution, solving the problem according to the plan, and re-examining the results that have been obtained.

Problem solving has a variety of methods that must be done to solve a problem. This is in line that the process of mathematical problem solving activities that focus on the stages in solving a problem that will be taken by students which later students can find the results obtained [5]. The stages of problem solving based on Polya, the outline includes understanding the problem, making plans, implementing plans, and re-examining [12].

In the first stage, namely understanding the problem, the subject is able to understand and inform what is known about the problem. The subject is able to explain what is being asked in the question, the number, relationship and values related to what is being asked in the question. In subject 1, it can be seen that the subject can understand and know what is being asked in the question. Subjects can name symbols used in solving problems. Subject 1 can also determine the meaning of a symbol and be able to write the symbol according to its meaning. This is the same as subject 2 that can also understand and know what is known and what is being asked in the question. Thus the two subjects have carried out the stages of understanding the problem properly and correctly. From the results of this study, in the stage of understanding the problem students can be said to have reached this stage if students are able to know what is

known and asked the question correctly [13,14]. At the stage of preparing a plan, the two subjects at this stage used the formula for the area of a circle with the aim of seeing which of the two types of cakes of different sizes was more profitable [15]. Subjects are able to develop plans or strategies that will be used when solving problems and can explain symbols in mathematical models. The subject is also able to choose the method that will be used to solve the problem. By planning the strategies that will be used, it will make it easier for students to work on the questions. In solving a problem, at the stage of making a plan requires experience or prior knowledge so that students can solve problems easily [16]. The third stage is to solve the problem according to the plan made. At this stage the plans that have been previously planned will be implemented at this stage. Implementation of this stage, S1 and S2 are able to solve problem solving according to the plan that has been made. However, S1 and S2 have errors in operating the result of the area of a circle on a small sponge cake. Both subjects are less careful in solving this problem. That errors in solving mathematical problems made by students are students who are less thorough, errors in providing information, errors in process skills, and errors when understanding problems [17].

The last stage is to re-examine the results that have been obtained. At this stage it is very important to do to minimize errors when solving a problem. With this it can be taken into consideration to see whether the results obtained are correct or not [12,18]. S1 and S2 have re-examined the results obtained. Both subjects are confident with the results obtained. Subject 1 provides a review of the answers it gets. With this can provide an explanation of what can be. However, subject 2 does not explain the review of the results obtained.

In working on the subject, it will be seen whether it is in accordance with the indicators that have been set. It can be seen from the results completed by the three subjects that several indicators have been set. Solving mathematical problems were carried out based on indicators in problem solving [19]. Students will be given mathematical problem solving questions according to Polya's stages. The problem faced by the subject in solving this problem is in the form of a description. Thus, when students solve mathematical problems, the students' mathematical problem solving process will be seen. When giving questions to students, students will find a problem contained in the questions. Students will try to solve problems when they are faced with existing problems. With various kinds of problems, students can also use various strategies in solving problems. In solving a problem, it is possible for students to solve the same problem and maybe not because they have their own plan in solving the problem. Students will be faced with challenges such as difficulties and understanding problems

when solving a problem [20]. Solving a problem definitely requires a high creativity, critical, and readiness in new situations [14].

4. Conclusion

Based on the discussion above, it can be concluded that the mathematical problem solving process, the subject has carried out the appropriate stages of Polya. However, not all indicators in problem solving are not implemented. In the stage of understanding the problem, students are able to understand and identify what is known in the problem. At the stage of making a plan, the subject has a strategy in solving the problem. The next stage the subject solves the problem according to the plan that has been made previously. In the last stage, the subject re-examines the results obtained to ensure the results obtained. This study has several limitations, including not linking the problem solving process with the thinking process or students' reasoning. Therefore, in order to strengthen the problem-solving model, further research can pay attention to the thinking process or mathematical reasoning process.

Acknowledgements

We would like to thank the school for facilitating the research and for cooperating in the data collection process.

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