

## Research Article

# A Management Procedure to take Advantage of Minorities

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## Abstract.

A group of companies is often formed and controlled by a single entity, namely the holding or parent company. The holding company has administrative control over the entire group, directing all investment and management decisions. The control can be either direct, meaning the parent company holds the majority of the voting rights of another economic entity of the group, or indirect, through intermediaries holding shares to a third company, and the parent can direct them to vote in its favor. The group has the same strategic mindset, and all decisions are made with the group's overall growth in mind. Subject to the fact that the companies belonging to a group are being controlled and directed by another entity, a portion of their common stock is often owned by third parties outside of the group. In accounting and consolidation processes these parties are referred to as the non-controlling interest and their rights as minority voting rights. Usually, such rights exist when a company is open to public offering and independent investors or external companies acquire part of the equity common stock, or when a partial takeover has taken place and the majority of the target's capital is acquired by the group, in any combination. Minority rights in an economic entity cease at the percentage of the company's share capital attributable to them. For the group as a whole, minority interests refer to the proportion of the group that is held by third parties. From the holding company's point of view, the objective is to maximize the value of the group and, by extension, to maximize its shareholders value. In this article, we present a method where the holding company redistributes group's shares. In addition, by allowing its own shares to trade, resulting in a greater decrease in holdings' integrated control and a higher increase in its cash inflows.

**Keywords:** group, participations, minority interest, CF, optimization

## jel CLASSIFICATION codes

C60, C61; G32

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**Published:** 1 February 2023

**Publishing services provided by**  
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Selection and Peer-review under the responsibility of the EBEEC Conference Committee.

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## 1. INTRODUCTION

In recent times, the science of managerial accounting and corporate finance has become interested in companies that group together and under a common strategic plan with links either between them or through a central company. Corporate groups, as found both in business and academic literature, although not a new phenomenon, are increasingly being developed and selected by companies and their management as a means of expansion and strategic development. Usually, groups are controlled by a parent, or holding, company that can exercise administrative control over the whole group, i.e., drive all investment and management decisions. Controlled companies are often referred to as subsidiaries.

A group of companies offers several advantages, but the most important is mitigating risk. Nowadays, the major rule is to separate different business activities in order to protect each one's value from future liabilities that might occur. Through subsidiaries, the parent company is able to expand its activities, while its liabilities reach up to the proportion of its holding in their share capital. In such a manner, the parent company avoids the creation of intra-company divisions, which would then absorb all the capital and all the resulting liabilities. In addition, subsidiaries provide the possibility for further expansion of the parent by entering into merger & acquisition procedures. At the end of the day, the way the group is structured grants benefits and protection to the holding company by directing risks and liabilities to the various companies within the group, whilst in the event of subsidiaries' profitability, cash flows are passed on to the top layer of the parent company through dividends.

Controlling a company can be either direct, meaning the parent company holds the majority of the voting rights of another economic entity of the group, or indirect, through intermediaries which hold shares to a third company, and the parent can direct them to vote in its favor. It should be stressed that the concepts of control and ownership are different. Any investor, whether an individual or a legal entity, is the owner of a company if he holds part of its share capital. However, ownership does not imply the possibility of control. The concept of control is linked to the voting rights attached to all the shares held by the shareholders. Depending on the composition of the share capital voting shares may be different among the shareholders. In companies that we meet the form of one-share-one-vote, each share gives equal voting rights to all shareholders. The one who holds the majority (whether it is a single shareholder or a coalition) also gains control of the company. In many cases, however, we find forms of shares where either they give no voting rights, or they give more than one. Although logic remains the same,

an adjustment should be made on the weight of each share to arrive at the normalized percentage of control and thus find who actually holds the majority and thus control of the company.

Corporate group structure can be simple, and the networks between the companies can be easily found and calculated, but most of the time are characterized by complexity and particularities. The latter creates problems, especially in cases where someone seeks the final controller. Cross holdings between the companies, cycle group structures, or even complex pyramid structures can easily misdrive control power within the group. Such situations, create multiple indirect voting rights allowing the holding company, to control others with minor or even no shares. An even more interesting case is when a company from the group holds shares of its parent company. According to law, in such cases trade is considered to have been made in the name of the parent company and the procedure is considered a stock buyback from the parent company. As can be seen, such a move within a group creates additional complexity in calculating the actual percentage of control.

Subject to the fact that the companies belonging to a group are being controlled and directed by another entity, a portion of their common stock is often owned by third parties outside of the group. In accounting and consolidation processes these parties are referred to as the non-controlling interest and their rights as minority voting rights. Usually, such rights exist when a company is open to public offering and independent investors or external companies acquire part of the equity common stock, or when a partial takeover has taken place and the majority of the target's capital is acquired by the group, in any combination. Minority rights in an economic entity cease at the percentage of the company's share capital attributable to them. For the group as a whole, minority interests refer to the proportion of the group that is held by third parties and are a very important part of its composition constitutes a major source of cash flow.

The paper is organized as follows: We examine previous research under the concept of control, ownership, and group's optimal structure. Then, we present a new model with own shares included which results to a new more optimal structure of the group.

## 2. Literature review

A quick search of the existing literature on business groups will mainly yield results that have to do with its structure and the concepts of control and ownership. Indeed, is true that these two concepts are so similar that sometimes seems identical, but most of the times one does not fully confirm the other. The separation between control and

ownership of a group, especially when there are multiple network connections between the companies is of high importance between the researchers.

Ownership of a company is held by anyone who owns at least one share in the company. The degree and size of ownership is defined by the number of shares held by the shareholder, i.e., the percentage of shares held in relation to the company's total share capital. The concept of ownership is accompanied with the cash flow received from shareholders in the form of dividends. On the other hand, control, is depending on the voting rights that each shareholder receives from its shares. Voting rights regulate the behaviour and power of shareholders to wrest control on a company. The more dispersed a company's shares are among the shareholders, the more difficult it is for one of them to accumulate a large control percentage.

Control of a company is exercised by the person who has the majority of voting rights or by coalitions of shareholders who, as a group, manage to acquire the controlling stake and decide on behalf of the company. Theoretical literature places the traditional majority voting break point at 50 percent of the voting rights. If full control is allowed below the 50 percent threshold, then there is the possibility of multiple control coalitions within a company[1]. This can lead to conflicts and disagreements. There is a research regarding dilution of power from a major shareholder to several others in order to increase cash flows and reduce costs showing that control was in existence only if there is a coalition with over fifty percent of the voting rights [2]. Additional, in the articles [3] and [4] argues that voting blockchains less than the majority can drive to reduced performance and motivation. On the contrast of above control threshold, a research on 27 wealth economies [5] revealed that a shareholder can partial control a company with a threshold of 20%. Of course, for this to happen, either he should belong to the founders of the company, or be a member of the board, or the remaining percentage is scattered, and dominant coalitions are very difficult to be formed. [6] following [7] showed that concentration of shares are characterized from centralization in a top holder with a cumulative percentage of almost 80 percent.

In groups, especially when there is a large number of companies with multiple networks between them, control is difficult to be established. Even when the calculation is seemingly easy or self-evident, the existence of indirect effects can lead to incorrect calculations. In literature, measurement of control is mainly obtained through matrixes, graphs, control or power indexes, and game-theory approaches. Matrixes were introduced in the article [8] by calculating the value of a pyramid group. Their perspective was followed by the articles [9], [10], and [11]. The former calculated ownership in two different countries while the latter made a deep analysis in voting rights of a pyramid structure.

[12], created a voting game theory to study different situations in majority determination. [13], [14] created indicators which are used extensively to measure power and influence of control in groups of companies. Indicators to measure control have been also used in the articles [15], [16], [17], [18] and [19]. Recently in the article [20] authors preview a new method with the introduction of a network power index. This new NPI allows to identify “hidden influencers”—including sovereign governments—.

Another significant issue in corporate groups is shares’ type issued by the companies. The “one share-one vote” majority rule is perhaps the most common corporate control mechanism. It is the most optimal type between the shareholders and can derive to fair rights between them [21]. However, when applicable by the law, companies can deviate from custom shares and issue weighted voting shares (dual shares, no-voting shares etc.). Companies issuing such shares benefit more as the voting share premium is much larger for the initial shareholders [22].

### 3. Group Structures

Companies can form groups with any combination, from very simple formations to complex ones with several combinations between companies. Nevertheless, we can distinguish three main categories as the base of each structure.

#### 3.1. Pyramid Structure

Pyramid structure consists of multiple layers of companies, whereas is characterized as a top-down chain of control. Parent company or group’s controller is placed on the top of the pyramid, while rest group’s companies spread out in the lower layers. Companies in middle layers are often called sub holding entities whilst those at the ending layers are the operational entities. Control is achieved mainly through indirect voting rights between the group.

In Figure 1 company M holds indirect rights in company H :

$$a_{m,h} = a_{m,a} * a_{a,c} * a_{c,h}$$

and same applies for every company in the group. It is the most classic form and allows the parent company to control affiliate companies in lower layers with very small or zero direct ownership percentages. In this structure no crossholdings exist between the companies, and the structure often leads to the dilution of minority shareholders.

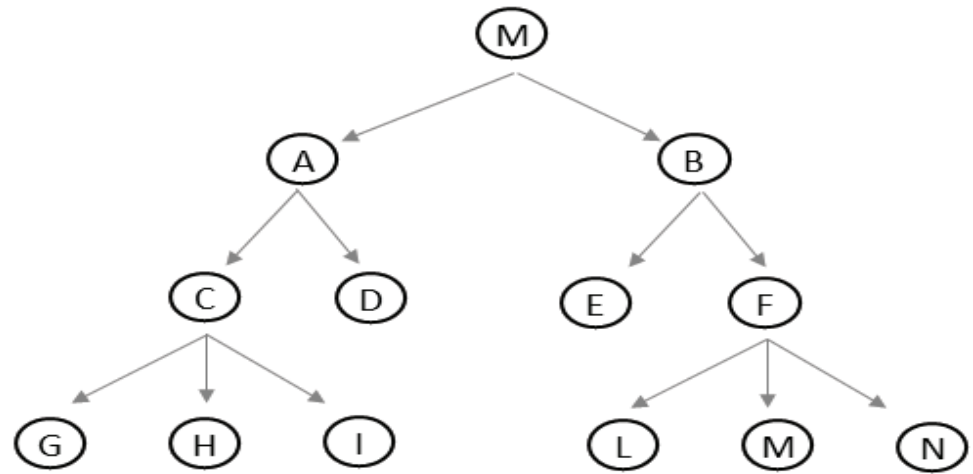


Figure 1: Pyramid Structure.

#### 4. Cycle Structure

A cyclical group structure is mainly characterized by crossholdings between companies. As (Lévy and Levy, 2007) documented, circular structure is essentially a loop of voting and cash flow rights, i.e. for n companies there is at least one where holds its own shares through intermediary companies.

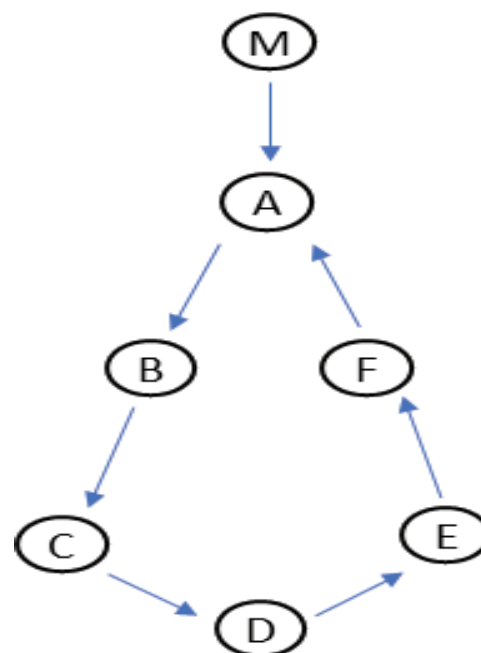


Figure 2: Cycle Structure.

In Figure 2 company A holds indirect rights in same :

$$a_{a,a} = a_{a,b} * a_{b,c} * a_{c,d} * a_{d,e} * a_{e,f} * a_{f,a}$$

Crossholdings are a sensitive situation, especially when intermediaries act in the name of the company where the loop is created. Under certain circumstances there are regulations and accounting rules that can create further changes in control & ownership rights within a group.

### 5. Mixed Structure

Mixed structures are a combination of the two above forms. In these business groups, we find both the multiple layers of the pyramid and crossholdings between the companies. This structure is characterized by high degree of complexity with multiple control networks between the companies. This implies that it is quite difficult to identify the actual controller of the group as there are a lot of parameters involved in the final calculation. Groups with above structure are often consist of multiple companies with different business activities and with several minor shareholders.

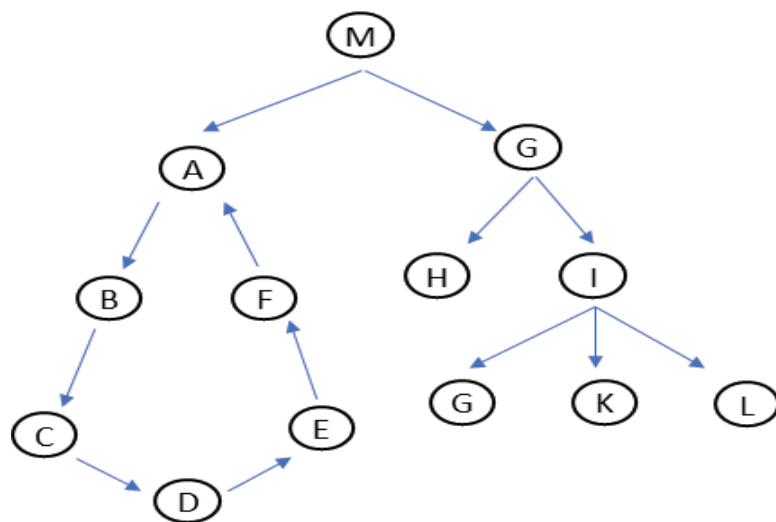


Figure 3: Mixed Structure.

### 6. Theoretical Approach

This is an addition of our previous work where we examined how a parent company can reallocate shareholdings within a group in order to reduce integrated control maintaining the effective management control of the group, and at the same time to increase

its capital inflow from the share trade. Our basic function is to minimize integrated control under certain specific constraints: the overall control of the group, and the cash available for share trade by the subsidiaries. That way the holding company can create additional cash inflows simply by reallocating group's shares to existing shareholders or outsiders[24]. [?] Below we list the key matrixes so that there is no misunderstanding in the flow of the present:

1.  $\Phi_{n \times n} = [\phi_{ij}]$  represents the Participation Between Subsidiaries (PBS)
2.  $A_{n \times 1} = [a_i]$  represent the Direct Participation of Holding Company (DPHC)
3.  $G_{n \times 1} = (I - \Phi^T)^{-1}_{n \times n} \times A_{n \times 1}$  represent the Effective Integrated Participation (EIP)
4.  $C_i = a_i + \sum_{i=1}^n \phi_{ij}$  the Effective Management Control (EMC)
5.  $M_{n \times 1} = U_{n \times 1} - G_{n \times 1} \Leftrightarrow [m_i] = [u_i] - [g_i]$  the Minority Interest
6.  $NA_{1 \times n} = [na_j]$  represents the Net Assets of the Group where  $n \alpha 0$  the net assets of parent company
7.  $[pn_0] = \sum_{i,j=1}^n (na_j \times \alpha_i)$  parent's participation value
8.  $[pn_j] = \sum_{j=1}^n (\sum_{i=1}^n (\phi_{ij} \times na_j))$ , subsidiaries' participation value
9.  $CE_{1 \times n} = [ce_j] = [p_j] \times [na_j]$  the Cash & Cash Equivalents (CCE-CE) for investment as Net Assets portion and  $P_{1 \times n} = [p_j]$  the portion value

Additional to the above in current we add some more insights regarding holding's own shares.

A company may, itself or through a third party acting on behalf of the company, acquire own shares already issued, but only with the approval of the general meeting. Moreover, a takeover shall be deemed to have been effected by the company itself, either through acquisition or by holding shares of another company where it holds, directly or indirectly, a majority of the voting rights or can exercise, directly or indirectly, a dominant influence.

In view of the above, holding's shares are reflected as per below :

**Own Shares**

Let  $O_{ih}$  the shares of subsidiary i in holding company and  $O_{hj} \in [0,1] \subseteq \mathbb{R}, \forall i=1,2...n$  subsidiaries

The Total Own Shares (TOS) that subsidiaries can hold in Holding company are :



$TOS = \sum_{i=1}^n O_{ih} \in [0,1] \subseteq \mathbb{R}, \forall i=1,2,\dots,n$  subsidiaries (1) The net integrated control of Holding company in subsidiaries is derived from the percentage of direct & indirect holdings minus the portion of its shares that the subsidiary holds. Thus, we derive to below distinction:

If  $G_{nx1}$  the integrated participation rate of Holding to the n companies of the group, then :

Holdings Portion on Own Shares (HPOS)

$$HPO_{ix1} = g_{i1} * o_{ih} \quad \forall i=1,2,\dots,n \text{ subsidiaries (2)}$$

Minorities Portion on Own Shares (MPOS)

$$MHPO_{ix1} = g_{i1} * (1 - o_{ih}) \quad \forall i=1,2,\dots,n \text{ subsidiaries (3)}$$

From above equations is obvious that part of the shareholdings in parent company are owned by investors outside of the group.

Moreover, the holdings of subsidiaries in Holding should be addressed, thus:

Subsidiaries Holdings Portion on Holding (SHPH)

$$SHPH_{ix1} = a_{ih} \quad \forall i=1,2,\dots,n \text{ subsidiaries (4)}$$

### 6.1. Maximizing Minority Interest Without Losing Control & Own Shares Trade Availability

Objective Function (OF) is minimizing the Effective Integrated Participation. What affects our model is the additional constraints of own shares trade availability.

Thus:

$$OF \rightarrow \min \text{ EIP or } G_{nx1} \text{ or } \min [ (I - \Phi^T)^{-1}_{n \times n} \times A_{n \times 1} ] \text{ (5)}$$

Restrictions that should be taken under consideration :

1. Restriction of Management Control: EMC or  $C_i = a_i + \sum_{i=1}^n \varphi_{ij} \geq 50\%$  (6)

$\forall i = 1, 2 \dots n$  subsidiaries in order the parent company to achieve the control of the Group.

1. *DPHC*  $A_{nx1} = [a_i] \geq x\%$  (7)

2. *PBS*  $\Phi_{n \times n} = [\varphi_{ij}] \geq y\%[?]$  (8)

3. *PV*  $PV_{1 \times n} = [dp_j] \leq [ce_j] \quad \forall j = 0, 1, 2 \dots n$  companies (9)

An additional restriction should be added regarding the size of parent company's own shares that can be traded in the group. Opinions on this restriction are divided.

The first states that the shares of the parent company that are directed to be acquired are the total shares acquired by its subsidiaries. The second view states that the total of these shares is only the shares owned by the majority, i.e., after deducting minority's share portion. Thus, here we have two restrictions, one for each case, but only one can be in force depending on the chosen case.

First Opinion:

If  $T\theta$  [?] the maximum percentage of parents' shares that can be traded inside the group, then:

$$5.1. THPO_{ix1} = \sum_{i=1}^n g_{i1} * (1 - o_{ih}) \leq T\theta \quad (10)$$

Second Opinion:

$$5.2. TOS = \sum_{i=1}^n O_{ih} \leq T\theta \quad (11)$$

## 7. Model

We assume a group of companies with given participation rates between them and given equity values (net assets). With these data the above matrixes as per our theoretical approach can be calculated.

Our target is to minimize integrated control, holding effective management control equal or above the majority threshold (50%) and increase holding's cash inflows. By adding constraint 5.1 or 5.2 and allowing holdings shares for trade, we test how the shareholdings are redistributed and whether the parent continues to control the group generating greater capital flows.

### 7.1. The static model

Our group consists of 10 companies. M is the parent or holding company whilst the rest its subsidiaries. M holds 50% in each subsidiary which give same integrated control in each. We assume that rest percentages are divided into multiple shareholders who cannot form a 50% percent coalition and oppose to M's control. The 1.00 percentage rate in M represents holding's shares which are concentrated in the ultimate beneficial owner/owners of the group.

We observed that incorporating holding's shares into our previous model did not give any changes in the group structure, i.e., no exchange of own shares took place. This may be the case for two reasons: First, imposing trade of holding's shares does

not have an effect on minimizing integrated control, i.e., there is no better combination in the group structure. Second, subsidiaries' cash are not sufficient to acquire own shares and improve the structure.

For this reason, we changed [p<sub>j</sub>] (CCE's portion on Net Assets) to increase cash and give greater trade volume. If it turns out that own shares do not have role in group's structure then the parent company should maintain 100% of its equity, otherwise it will fluctuate between the percentage rate that we have set.

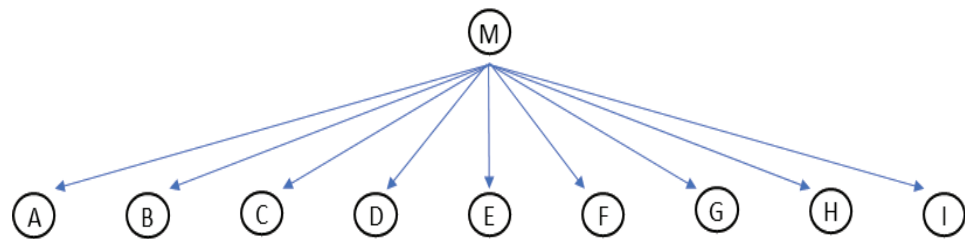


Figure 4: The Static Model Structure.

TABLE 1: The Static Model.

	M	A	B	C	D	E	F	G	H	I
M	1.00	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
A	0	0	0	0	0	0	0	0	0	0
B	0	0	0	0	0	0	0	0	0	0
C	0	0	0	0	0	0	0	0	0	0
D	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0
F	0	0	0	0	0	0	0	0	0	0
G	0	0	0	0	0	0	0	0	0	0
H	0	0	0	0	0	0	0	0	0	0
I	0	0	0	0	0	0	0	0	0	0
EMC		0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Source: Authors Calculations

The effective integrated participation is given by the equation :

$$G_{n \times 1} = (I - \Phi^T)^{-1}_{n \times n} * A_{n \times 1}$$

which results to :

We also assume a given Vector of Net Assets representing the value of each company:

TABLE 2: Effective Integrated Participation.

	M
A	0.5
B	0.5
C	0.5
D	0.5
E	0.5
F	0.5
G	0.5
H	0.5
I	0.5

Source: Authors Calculations

TABLE 3: Net Assets.

	M	A	B	C	D	E	F	G	H	I
NA	€ 60,000	€ 30,000	€ 25,000	€ 20,000	€ 15,000	€ 10,000	€ 5,000	€ 4,000	€ 2,500	€ 1,000

Source: Authors Assumptions

## 7.2. Models Applications

To evaluate our static model, we apply two constraint models according to 5.1 & 5.2 own shares constraint as presented in section 4.

Thus:

In order to have a clear understanding of above table we present below a briefly comment of its key elements.

1. Our goal is to find the minimum acceptable values of effective integrated participation – EIP under certain constraints. Thus, model’s goal is minEIP.
2. The cells we need to change are the ones holding the percentage rates of the group. Thus, through the optimization procedure we need to find the new participations rates between subsidiaries – PBS, the new direct participations of the holding company – DPHC, and the participation rates of subsidiaries on holding company– SHPH, if any.
3. The five bold constraints are those controlling the new participation rates that will be received in the adjustable sets. Actually, by changing these constructions we can instruct the structure of the company to follow certain requirements according to law or management/accounting policies.

Our models are structured as follows:

TABLE 4: Constraints.

Model 1			Model 2		
Objective Function			Objective Function		
<b>Cells to Optimize</b>	Cell	EIP	<b>Cells to Optimize</b>	Cell	EIP
<b>Type of Goal</b>		Minimum	<b>Type of Goal</b>		Minimum
<b>Adjustable Values (1)</b>	Cell	PBS	<b>Adjustable Values (1)</b>	Cell	PBS
<b>Adjustable Values (2)</b>	Cell	DPHC	<b>Adjustable Values (2)</b>	Cell	DPHC
<b>Adjustable Values (3)</b>	Cell	SHPH	<b>Adjustable Values (2)</b>	Cell	SHPH
<b>Constraints</b>		<b>Description</b>	<b>Constraints</b>		<b>Description</b>
<b>EMC</b>		$0,5 \leq EMC \leq 1$	<b>EMC</b>		$0,5 \leq EMC \leq 1$
<b>EIP</b>		$0,05 \leq EIP \leq 1$	<b>EIP</b>		$0,05 \leq EIP \leq 1$
<b>DPH<sub>1</sub></b>		$OR(a_i \geq 0.5) = TRUE$	<b>DPH<sub>1</sub></b>		$OR(a_i \geq 0.5) = TRUE$
<b>DPH<sub>2</sub></b>		$OR(a_i = 0, a_i \geq 0.05) = TRUE$	<b>DPH<sub>2</sub></b>		$OR(a_i = 0, a_i \geq 0.05) = TRUE$
<b>DPHC<sub>1</sub></b>		$sum(\varphi_{11} + \varphi_{22} + \varphi_{33} + \varphi_{44} + \varphi_{55} + \varphi_{66} + \varphi_{77} + \varphi_{88} + \varphi_{99}) = 0$	<b>DPHC<sub>1</sub></b>		$sum(\varphi_{11} + \varphi_{22} + \varphi_{33} + \varphi_{44} + \varphi_{55} + \varphi_{66} + \varphi_{77} + \varphi_{88} + \varphi_{99}) = 0$
<b>DPHC<sub>2</sub></b>		$OR(\varphi_{ij} = 0, \varphi_{ij} \geq 0.05) = TRUE$	<b>DPHC<sub>2</sub></b>		$OR(\varphi_{ij} = 0, \varphi_{ij} \geq 0.05) = TRUE$
<b>SHPH</b>		$0 \leq a_{ih} \leq 1$	<b>SHPH</b>		$0 \leq a_{ih} \leq 1$
<b>PV - CCE</b>		$pn_{1j} \leq ce_{1j}, pj = 0.4$	<b>PV - CCE</b>		$pn_{1j} \leq ce_{1j}, pj = 0.4$
<b>T<sub>θ</sub></b>		$HPO \leq 0.1$	<b>T<sub>θ</sub></b>		$TOS \leq 0.1$

Source: Authors Calculations

1. the parent company must hold the majority of the equity shares at least to one of its subsidiaries
2. the minimum DPHC participation must be 0.05, otherwise value is set to 0
3. the minimum value of PBS should be 0.05, otherwise value is set to 0
4. SHPH can take any value between 0 and 1
5. In order for the group to maintain control over a company, the effective management control – EMC should be equal or greater than 0.5 with a maximum value of 1.
6. The sum of a company's i participations in other companies should be equal or smaller to its available cash for investments. Available cash for the subsidiaries are 40% of their net assets, and 80% for the parent. Thus, the vector of CCE is formed as :
  1. Sum of own shares should be less than a Tθ threshold. This rate is given by country laws or accounting restrictions. In our case we used Greek threshold at the rate of 10% of holding's equity capital.

TABLE 5: Cash & Cash Equivalents.

	M	A	B	C	D	E	F	G	H	I
CE	€ 48,000	€ 12,000	€ 10,000	€ 8,000	€ 6,000	€ 4,000	€ 2,000	€ 1,600	€ 1,000	€ 400

Source: Authors Assumptions

## 8. Estimations

### 8.1. Outputs

Below we present the results after using the constraints in the static model. In figures 5 and 7 can be seen how the EIP is decreasing until it reaches a flat line. Figures 6 and 8 presents the new structure of the group for each of our models whilst in tables 6 and 7 present the new participation rates between the subsidiaries and the holding company.

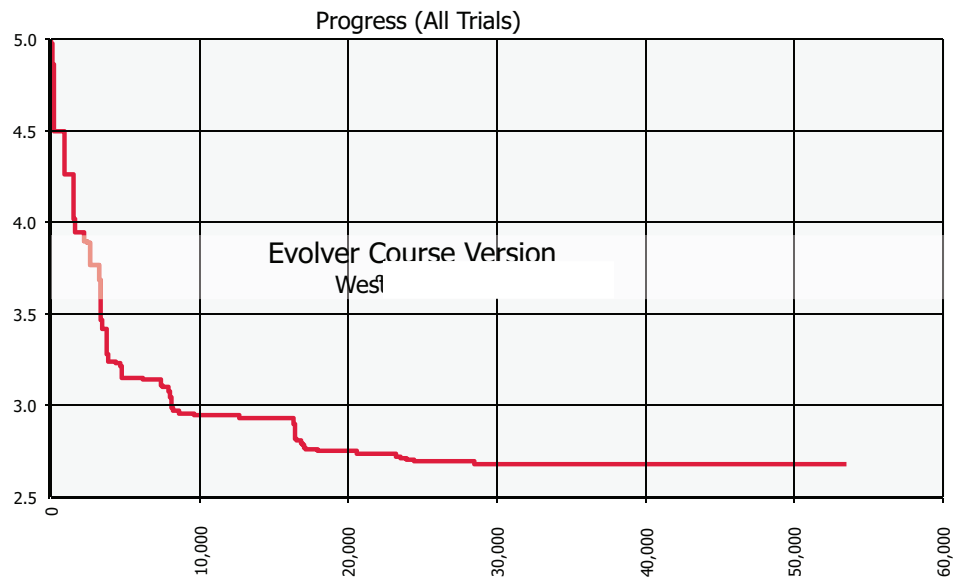


Figure 5: Evolver Model 1 - Sum of Own Shares.

Starting with a static group structure with given percentage rates, we applied our constraints but with the availability of own shares trade. Figures 7 and 8 present the new structure of the group under the given constraints and to increase cash inflows in holding company. Both models share the same constraints except with an exemption in the trade volume allowed in holding’s shares. The first follows option one as per our theoretical analysis and the second model option two respectively. In both new structures the holding company holds the majority percentage in the two larger subsidiaries, A and B. That is reasonable as both companies cash and cash

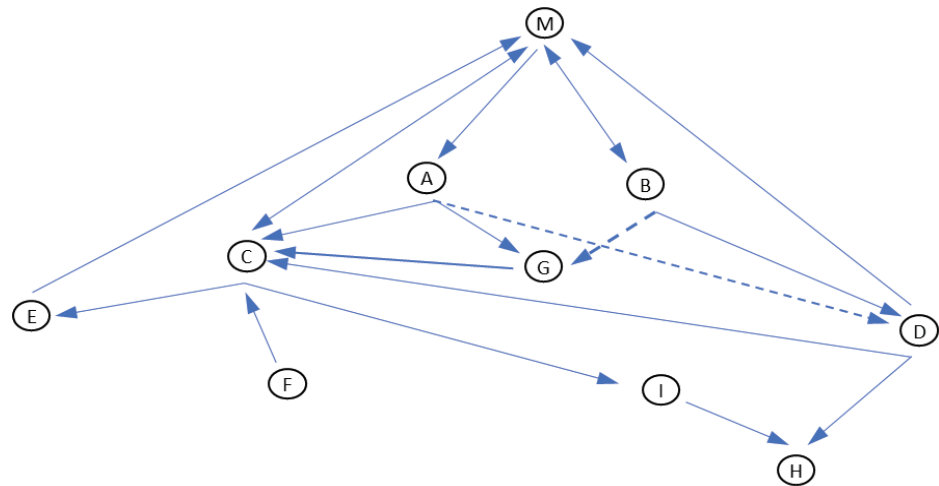


Figure 6: Model 1 Structure.

TABLE 6: New Structure - Model 1 - Sum of Own Shares.

	M	A	B	C	D	E	F	G	H	I
M	0.90	0.50	0.50	0.05	0.00	0.00	0.00	0.00	0.00	0.00
A	0.00	0.00	0.00	0.21	0.20	0.00	0.00	0.05	0.00	0.00
B	0.05	0.00	0.00	0.00	0.30	0.00	0.00	0.25	0.00	0.00
C	0.03	0.00	0.00	0.00	0.00	0.51	0.00	0.00	0.00	0.50
D	0.01	0.00	0.00	0.06	0.00	0.00	0.50	0.20	0.34	0.00
E	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00
G	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00
H	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.00
EMC	1.00	0.50	0.50	0.50	0.50	0.51	0.50	0.50	0.50	0.50

Source: Evolver Calculations

equivalent are adequate in order to proceed to the acquisition of a large amount of other companies equity and to guarantee groups' effective management control.

EMC remains stable in the lowest level for control purposes. In both models own shares are affecting the new structure as are being traded inside the group. In first model the traded volume of own shares is much lesser compared to the second due to the strict constrain that we applied. EIP is being decreased in the new structures, with a slightly best performance in model 2. Moreover, the greater trade of own shares in the second occasion leads the group to substantial growth in cash compared to the initial structure.

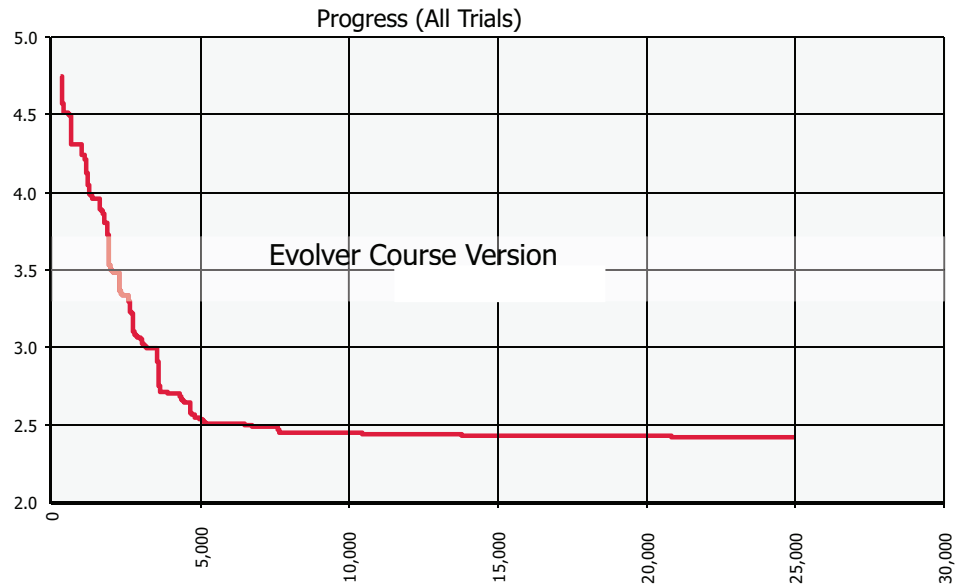


Figure 7: Evolver Model 2 - Sum of Majority's Own Shares.

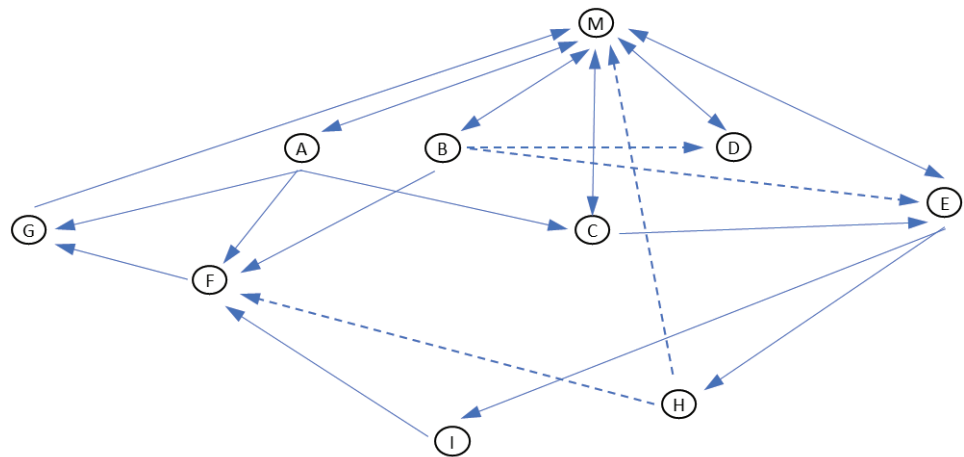


Figure 8: Model 2 New Structure.

The next three tables, 8, 9 and 10 display a comparison for the effective integrated participation, the non-controlling interest, and the net cash inflow respectively. The smaller the EIP i.e the group holds the minimum possible shares so as to maintain control, the greater the non-controlling interest that will be part of the subsidiaries.

In each restructure, the new EIP contains not only parent's direct & indirect control, but also the effect of the portion of own shares that end up under parent's control. This creates more complicated networks inside the group, with continuous loops. Table 8, above, presents the new effective integrated participation in each model. As can be



TABLE 7: New Structure Model 2 - Sum of Majority's Own Shares.

	M	A	B	C	D	E	F	G	H	I
M	0.62	0.50	0.50	0.12	0.29	0.00	0.05	0.00	0.00	0.00
A	0.05	0.00	0.00	0.38	0.00	0.00	0.24	0.05	0.00	0.00
B	0.07	0.00	0.00	0.00	0.21	0.24	0.05	0.00	0.00	0.00
C	0.09	0.00	0.00	0.00	0.00	0.26	0.00	0.00	0.00	0.00
D	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.35
F	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00	0.00
G	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15
H	0.01	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00
I	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00
EMC	1.00	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50

Source: Evolver Calculations

TABLE 8: Minimum EIP – Comparison.

Static Model		Model 1		Model 2	
Original Value	4.50	Minimum 1	1.93	Minimum 2	1.74
Original EIP		Optimal 1		Optimal 2	
M	1.00	M	0.93	M	0.72
A	0.50	A	0.47	A	0.36
B	0.50	B	0.47	B	0.36
C	0.50	C	0.19	C	0.22
D	0.50	D	0.23	D	0.28
E	0.50	E	0.09	E	0.14
F	0.50	F	0.12	F	0.15
G	0.50	G	0.19	G	0.09
H	0.50	H	0.09	H	0.07
I	0.50	I	0.09	I	0.06

Source: Authors Calculations

seen the EIP is relatively smaller in model 2 as there is a larger number of owned shares that are traded inside the group.

Table 9 presents the increase in non controlling interest inside the group. Actually, by minimizing the effective integrated participation and allowing for new investors to participate in subsidiaries of the group, we manage to take advantage of these investing in favour of the group. In the following table (table 10), we show the increase of the cash inflows for the holding company from these investments. The higher the number of own shares that are traded in the group, the greater the cash inflows for the holding company.

TABLE 9: Minimum NCI – Comparison.

Static Model		Model 1		Model 2	
Original Value	4.50	Maximum 1	7.07	Maximum 2	7.26
Original NCI		Max 1		Max 2	
A	0.50	A	0.53	A	0.64
B	0.50	B	0.53	B	0.64
C	0.50	C	0.81	C	0.78
D	0.50	D	0.77	D	0.72
E	0.50	E	0.91	E	0.86
F	0.50	F	0.88	F	0.85
G	0.50	G	0.81	G	0.91
H	0.50	H	0.91	H	0.93
I	0.50	I	0.91	I	0.94

Source: Authors Calculations

TABLE 10: New Cash Inflow.

Static Model		Model 1		Model 2	
Original EIP		Difference 1		Difference 2	
M	1.00	M	0.07	M	0.28
A	0.50	A	0.03	A	0.14
B	0.50	B	0.03	B	0.14
C	0.50	C	0.31	C	0.28
D	0.50	D	0.27	D	0.22
E	0.50	E	0.41	E	0.36
F	0.50	F	0.38	F	0.35
G	0.50	G	0.31	G	0.41
H	0.50	H	0.41	H	0.43
I	0.50	I	0.41	I	0.44
Cash Increase		€ 24,893.3		€ 41,737.3	

Source: Authors Calculations

The portion of own shares being controlled by the holding company is reflected on below tables.

## 8.2. Discussion

It is obvious that above numbers may differ depending on the strategy of the group. Moreover, maximum cash flows may occur with a greater effective integrated participation. As the number of the companies within a group increase, the model become more complex with multiple combinations among the participation rates of the companies. It should be highlighted that above method represents a strategic

TABLE 11: Majorities portion in Owned Shares.

Model 1		Model 2	
Majority	Minority	Majority	Minority
-	-	0.02	0.03
0.02	0.03	0.03	0.04
0.01	0.02	0.02	0.07
0.00	0.01	0.03	0.07
0.00	0.01	0.01	0.03
-	-	-	-
-	-	0.00	0.02
-	-	0.00	0.01
-	-	-	-
0.03	0.07	0.10	0.28

Source: Authors Calculations

management model. Groups’ managers can set the desire cash flows that is required in holding company and acquire it through shares trade inside the group while maintain the effective control. Further to above, they can still take advantage of the increased minorities rights and turn them to their favor.

This article is differentiated as tries to preview another aspect of group shares and voting rights. Most articles in according to corporates groups are trying to calculate and value the power of each shareholder in a company. Throughout the bibliography indexes are introduced to value and separate voting rights from control rights. In contrary, above case, through optimizing procedures is trying to recalculate these values in such a way that the ultimate beneficiary owner will absorb power from the other related parties.

## 9. CONCLUSION

When the shares of a parent company become traded between the companies of a group, the structure becomes even more complex than simple crossholdings between subsidiaries. The control loops that are created add greater differentiation between the actual control percentages. Thanks to the indirect sale of parent shares, the parent company manages to keep the control of a group with a much larger non-controlling interest.

An important fact is also the differentiation between the parent shares that are traded, and part of which are controlled by the parent company without actually owning them. The main conclusion of the above research is twofold. First, we have shown that with

the availability of own shares trade, a more optimal structure of a group, provided that control is still exercised over all subsidiaries, can be achieved by greatly increasing the rights of third parties. This is very important, as it creates increased capital flows within the group while the parent can exercise the group's strategy on each own benefit. Second, we have shown that when subsidiaries' available cash for trade increases, then the ratios between groups become even more differentiated as greater intra-group networks are created.

As can be seen, the above model leads to a reconfiguration of a group, a decrease in ownership percentages over the subsidiaries, and an increase in the parent's capital flows. In the real-world, every group is unique with its own peculiarities and data to be observed. The size of subsidiaries may vary substantially, and the portion of available cash is almost never the same. The model we have presented must be adapted to these data and to the decisions of the management of each group. Therefore, the optimal structure of each group is unique reflecting different strategies.

This paper tries to shed further light on the distribution of control in groups of companies. However, more elements should be taken under further consideration. If holding's target is to maximize cash inflows in holding by the distribution, a second optimization should be running simultaneously with the minimum of integrated control. In addition, more trials should be allowed in order to reach the optimal point, i.e., the one in which there are no better results with further distribution.

Finally, some further thoughts should be examined. Since the parent retains control, to what extent does it have the possibility of a corporate transformation inside the group to isolate third parties' rights and absorb even more capital? Furthermore, if third-party rights cannot avoid such a move, how can they be protected?

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