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Research article

Mathematical Proving Ability of Pre-service Teachers in Online and Blended Learning

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Abstract.
Mathematical proving is an important ability to learn. However, mathematics pre-service teachers often find this skill difficult. This research aimed to describe the mathematical proving ability of mathematics pre-service teachers in an online classroom and in the blended classroom. A descriptive mixed methods approach was used. A mathematical proving test was used to collect the data. The results showed that the average score of mathematical proving ability from the blended classroom was 82.6 (categorized as high). Meanwhile, the average score of mathematics pre-service teachers in the online classroom was 65.4 (categorized as intermediate). Given these findings, there is a significant need to improve the mathematical proving ability of the mathematical pre-service teachers in the online classroom by enhancing the conceptual understanding of group properties and ensuring the teachers are accustomed to practicing proving tasks.

Keywords: mathematical proving, online learning, blended learning, mathematics pre-service teachers, group theory

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1. Introduction

As part of university studies, group theory becomes one of the subjects that should be acquired in the teacher training institution in Indonesia [1]. Group theory course aims to develop the mathematical proving ability [2]. This skill becomes one of the essential aspects of learning abstract algebra [3]. Since group theory course is full of definitions and theorems which all require proof; therefore, the mathematics pre-service teachers required to understand each definition and theorems to organize the concept in proving activities. Mathematical proof also becomes the most crucial part to understand mathematics clearly [4-6]. However, many students encountered difficulties in constructing mathematical proof [4, 7-11]. Moreover, [12] and [13] found that most students still encountered difficulties in mathematical proving in higher education, especially in abstract algebra.

On the other hand, teaching and learning in the digital era have faced a challenge with online learning and blended learning. Online learning is defined as the use of

network technologies where the learners access learning content through learning materials rather than via teaching [14]. Therefore, the students could easily access the learning materials and attend the teaching and learning process by full online. Meanwhile, blended learning is defined as a mixed-mode of learning in which both face-to-face and online learning [15-16].

Even though, several studies focus on the identifying mathematical proving ability; however, it is focused on the face-to-face learning mode. There is a lack of study which identify how mathematical ability of mathematics pre-service teachers in group theory which is taught in the online and blended learning. Therefore, it is essential to know the mathematic pre-service teachers' mathematical proving ability in group theory in both online and blended learning.

2. Research Methods

This research used a mixed-method approach with descriptive quantitative – and qualitative respectively [17]. The descriptive quantitative analysis was used to describe the test results and the level of mathematical proving ability of the mathematics pre-service teachers in both online classrooms and blended classrooms. Meanwhile, the descriptive qualitative analysis was used to describe the proving procedures on each property. The study involved two groups of classes who enrolled in group theory courses in one of the teacher training institutions in Indonesia. The first group is called as online learning which all the teaching-learning activities are full in the online mode facilitated by the use of computers, phone cell, digital media, and networking. Meanwhile, the second group is called as the blended classroom which the teaching-learning activities are delivered through a combination of face-to-face and online delivery modes. The face-to-face activity in the blended learning facilitates the mathematics pre-service teachers to discuss physically. Meanwhile, the online part of the blended learning is facilitated by the lecturer to confirm the learning for the mathematics pre-service teachers who join the classroom through online mode. Face-to-face mode refers to a lecture-discussion method with discussion and presentation from the mathematics pre-service teachers who use the whiteboard and textbooks as teaching aids.

The instrument was that the written test consist of proving a group to obtain mathematical proving ability from the mathematics pre-service teachers in both groups of classes. The test results were analyzed by scoring 0 to 4 for each item with criteria as follows [4]. Score 0 was given if there is no proving process at all, score 1 was given if the students could make one approach but incorrect, score 2 was given if there is

substantial progress, score 3 was given if the solution is obtained with a minor fallacy, and score 4 if the students could make completion of proving process. The total score was then converted into 0 to 100 scale. The results were then analyzed descriptively and the mean them was categorized using a guideline presented in table 1 [6]. After that, the sample of answers was analyzed descriptively qualitatively to describe the mathematical proving procedures.

Table 1: Level category of mathematical proving ability.

Category	Mathematics Proving Ability Score Interval
High	$70 \leq X \leq 100$
Intermediate	$55 \leq X < 70$
Low	$0 \leq X < 55$

X: Mathematical proving ability score

3. Results

The data of this research were the mathematical proving ability in the group theory course. The data analysis is the descriptive statistical analysis to describe mathematical proving ability in online learning and in blended learning. The mathematical proving test results were analyzed by scoring 0 to 4 for each item according to the given criteria [4] then the total score was converted into 0 to 100 scale. The results were presented in Table 2.

Table 2: The mathematical proving ability test results.

Statistical Descriptive	Online Classroom	Blended Classroom
Mean	65.4	82.6
Median	75.0	100
Mode	75	100
Deviation Standard	20.9	25.6
Variance	436.4	589.9
Maximum Score	100	100
Minimum Score	25	25

From table 2, the average score of the mathematical proving ability from blended learning is 82.6 which means higher than the online learning score. It shows that the mathematical proving ability of the mathematics pre-service teachers in the blended learning is categorized at a high level in table 1. Meanwhile, the mathematical proving ability of the mathematics pre-service teachers in online learning is categorized at the intermediate level. The deviation standard of online and blended learning are

respectively 20.9 and 25.6. It is clear that the deviation standard of blended learning is higher than that of online learning. It indicates that the distribution of data on blended learning was quite far from the average. However, both groups of learning have the same maximum and minimum score respectively 100 and 25.

Table 3: Test score distribution based on mathematical proving ability category.

Category	Mathematics Proving Ability Score Interval	Online Learning (%)	Blended Learning (%)
High	$70 \leq X \leq 100$	53.8	81.4
Intermediate	$55 \leq X < 70$	38.5	0
Low	$\mathbf{0} \leq X < 55$	7.7	18.6

Table 4: Average score of proving each group properties of mathematical proving test.

Properties	**Online Learning**	**Blended Learning**
Closeness	100	100
Associative	92.31	88.37
Identity	53.85	81.40
Invers	15.38	60.47

Regarding the table 4 shows that all the mathematics pre-service teachers from both online and blended learning could prove the closeness properties from the test. Meanwhile, in the proving of identity and inverse properties the average score from blended learning is 81.40 and 60.47 respectively, which means higher than online learning only reach the average for identity and inverse properties are 53.85 and 15.38 respectively. However, in the proving associative properties, the mathematics pre-service teachers in the online learning have an average score 92.31 which is higher compared to the blended learning. It can be concluded that in online learning, the mathematics pre-service teachers could reach the high category of mathematical proving ability in proving closeness and associative properties. However, their mathematical proving ability on the existence of identity and inverse is in a low category. Meanwhile, the mathematical proving ability of mathematics pre-service teachers is a high category on the proving closeness, associative, and the existence of the identity. But, the proving of the existence of inverse elements is categorized at an intermediate level.

4. Discussions

Based on the results of this study, the mathematics pre-service teachers' mathematical proving ability in the online classroom is in the intermediate category. Meanwhile, the mathematical proving ability of the mathematics pre-service teachers in blended

learning is categorized at a high level. It means that the mathematical proving ability in blended learning is better than in online learning. It is in line with Ranjan [18] that the average achievement scores of the blended learning model were higher than the online learning mode since in the blended learning there is a potential to support better attainment and motivation. Moreover, the blended learning environment also lacks distraction [19]. Furthermore, a literature review from the previous studies [20-25] reveals that where the purely online groups were contrasted with the blended group but there was no significant difference.

Figure 1 and Figure 2 are examples of mathematics pre-service teachers' answers on proving the associative properties. Figure 1 shows that the associative properties could be proved elaborately. However, Figure 2 shows that the mathematics pre-service teachers could not change the algebraic form to become the intended solution. This is in line with the opinion of Hart [12], Weber [26], Weber [27], and Harel & Sowder [28] that writing proof is a common difficulty for students. Validating the proof is the essential skill needed to develop and evaluate the mathematical argument [29]. Therefore, the construction of the proving could proceed successfully. The findings were also in line with what was reported by Moore [13] that the difficulty of students in proving mathematics was unable to state the definition with their own words, too little intuition of understanding of a concept, inadequate concept images to write a proof, lack of understanding on how to use definitions to get the whole proof structure and how to start proving.

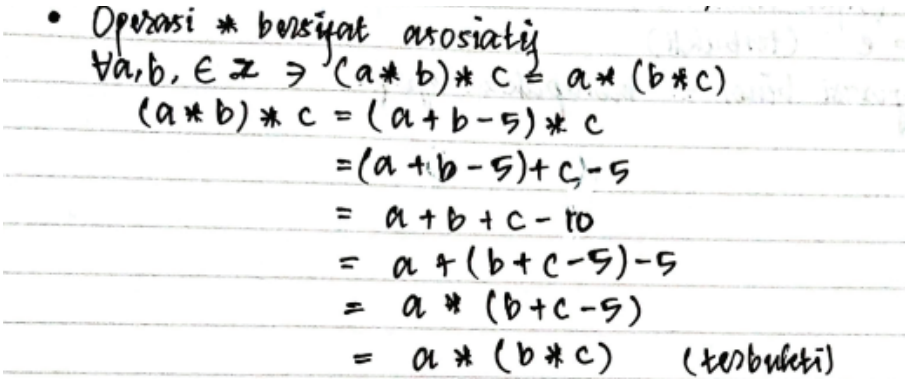

Figure 1: The sample of the correct answer of proving the associative property .

Figure 3 and Figure 4 are examples of mathematics pre-service teachers' answers on proving the existence of the identity element. Based on Figure 3, it shows that the mathematics pre-service teachers could determine the definition of the identity element and find the identity element correctly. Moreover, the procedures flow smoothly by showing each axiom at every step for determining the identity element. On the other

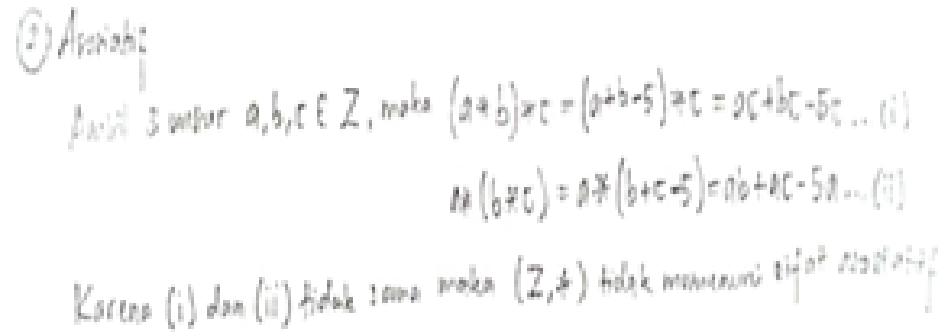

Figure 2: The sample of the incorrect answer on proving the existence of the inverse .

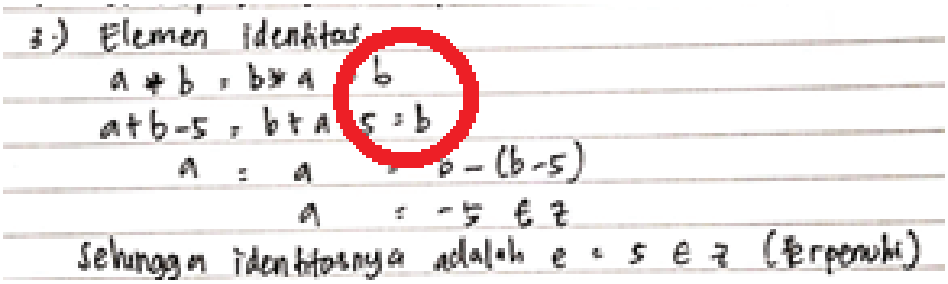

Figure 3: The sample of the correct answers proving the existence of the identity element.

hand, it could be found the sample of answers that shows the mathematics pre-service teachers could not define properly the existence of identity element and careless in calculating the algebraic operation. Therefore, it gains the wrong answer for the identity element.

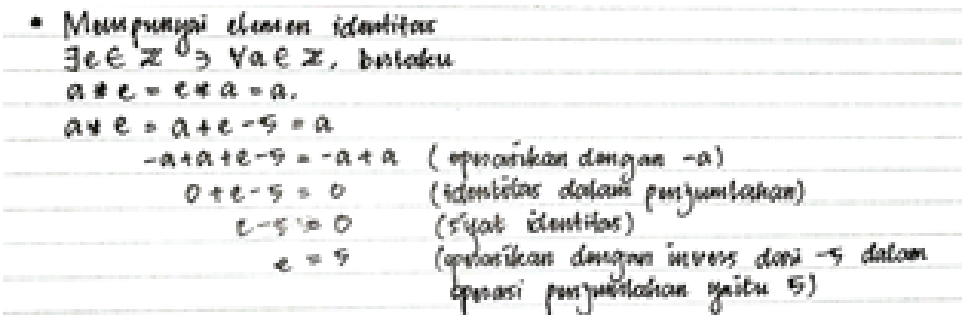

Figure 4: The sample of the incorrect answers for proving the existence of the identity element.

Figure 5 and Figure 6 are examples of mathematics pre-service teachers' answers on proving the existence of the inverse element. Figure 5 shows the correct answer where the mathematics pre-service teachers successfully elaborate the algebraic form of the inverse property definition and substitute the identity element that has been determined in the previous step. Meanwhile, Figure 6 shows the incorrect answer. The mathematics pre-service teacher wrote the identity element in that problem is 0. Whereas, it should

be written the first definition of the inverse element. The procedure is wrong from the beginning; therefore, the algebraic operation below could not determine the correct answer.

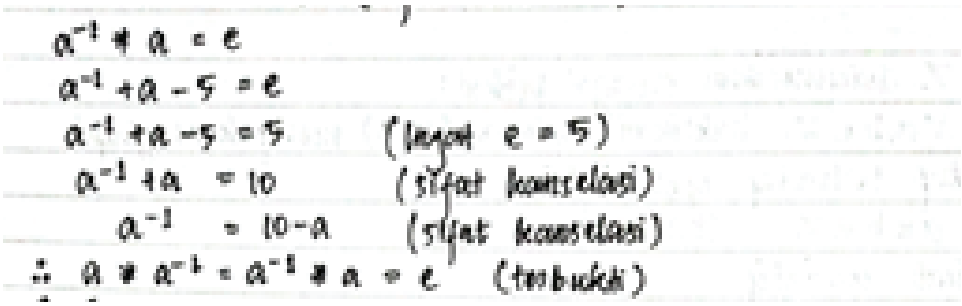

Figure 5: The sample of correct answer of proving the existence of the inverse.

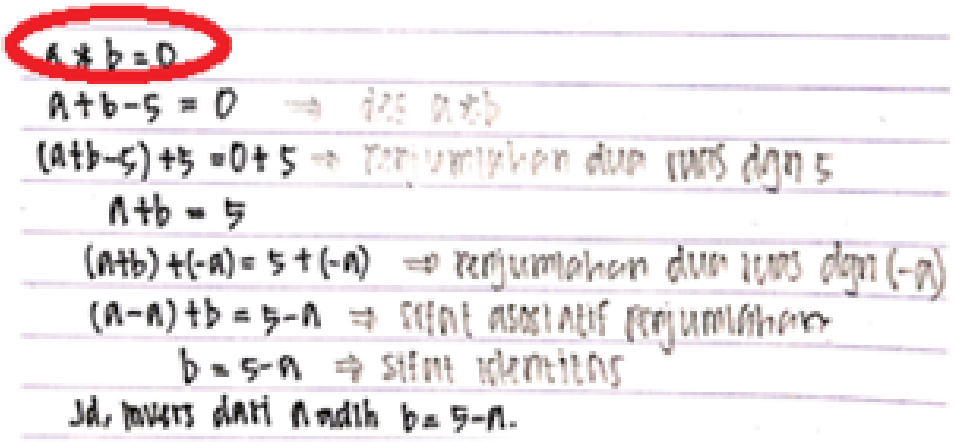

Figure 6: The sample of the incorrect answer of proving the existence of the inverse.

Regarding to the sample of the answers, it shows that the level of mathematics pre-service teachers' conceptual understanding affect the difficulty in proving the activity of group theory. This is in line with Pramasdyahsari and Rubowo [30] that mathematics pre-service teachers with higher mathematical ability can define and prove all the properties in detail. Therefore, it is necessary to make the mathematics pre-service teachers accustomed to practicing mathematical proving tasks of group theory in order to improve their level of thinking ability. Moreover, the teaching method in abstract algebra which accommodates full of proving activity in the proven theorem could encourage the students to think systematically [31]. This is in line with the purpose of the scripting task which was firstly introduced as a lesson play [32] that involves the mathematics pre-service teachers' mathematical thinking. Thus, the thinking activity involves the scripting task while the mathematics pre-service teachers connect the knowledge from abstract algebra to the school mathematics [33] it aligns to the mathematical thinking during proving a theorem. Since they have to connect the previous

knowledge such as to determine the inverse element we have to know the identity element. Moreover, the writing proof activity could also encourage the mathematics pre-service teachers to develop their critical thinking to link the mathematical connection concept-by-concept [34]. [35] stated that making the students accustomed to practicing mathematical proving tasks and making conjectures before doing them will enable the students to deal with the proving tasks. Also, [34] added that the learning process about mathematical proof should be displayed in the form of activities that facilitate the construction and reconstruction process of students' conceptual understanding. We also need to focus on strengthening the initial knowledge, because it greatly supports the students' understanding of learning mathematics as stated by Chamundeswari [37].

5. Conclusions

Based on the results of this study, it shows that the mathematics pre-service teachers in the blended learning have a better performance in mathematical proving ability compared to the online learning. The difficulties experienced by the mathematics pre-service teachers in the online learning in solving problems involving mathematical proof could also be identified, including the lack of ability to determine the form of an identity element and inverse element in the group. Therefore, it is necessary to make the mathematics pre-service teachers accustomed to practicing mathematical proving tasks of group theory in order to improve their level of thinking ability. Nevertheless, the mathematics pre-service teachers in both classrooms do not have any problem in proving closeness property. Furthermore, there is no significant problem for mathematics pre-service teachers in blended learning for solving the properties of the group. As online learning becomes a consideration of the teaching approach in the digital era; therefore, it is a significant need to improve the mathematics pre-service teachers' mathematical proving ability in the group theory course. Particularly for online learning mode, it needs to improve the conceptual understanding of group properties and accustom them to practice proving tasks.

Acknowledgments

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