

## Conference Paper

# Physical and Mechanical Properties of Sea Ice in Bending

S.V. Koshkin and N.A. Taranukha

Komsomolsk-on-Amur State Technical University, 681000 Komsomolsk-on-Amur, Russia

## Abstract

The work presents the study of changes in the physical and mechanical properties of sea ice in bending. Strength limit  $\sigma_B$  and deformation module  $E_B$  of ice under bending are conditional mechanical characteristics of sea ice. These characteristics do not reflect the actual stress state of ice destruction at the time. The ratio of the module to the strength limit  $E_B/\sigma_B$  determines the relative radius of curvature of neutral layer in the place of ice destruction. It is shown that this ratio increases with the increase of ice temperature. Dependencies for determining of  $E_B$  and  $\sigma_B$  of sea ice that generalize the experimental data are obtained.

**Keywords:** sea ice, bending, mechanical properties, elasticity, tensile strength

Corresponding Author: N.A.  
 Taranukha; email:  
 taranukha@knastu.ru

Received: 9 September 2016  
 Accepted: 19 September 2016  
 Published: 12 October 2016

Publishing services provided  
 by Knowledge E

© S.V. Koshkin and N.A.  
 Taranukha. This article is  
 distributed under the terms of  
 the [Creative Commons  
 Attribution License](#), which  
 permits unrestricted use and  
 redistribution provided that  
 the original author and source  
 are credited.

Selection and Peer-review  
 under the responsibility of the  
 ASRTU Conference  
 Committee.

## 1. Introduction

The Northern Sea Route (NSR) undergoes significant changes in the ice conditions every year. Drifting sea ice is characterized by the following parameters: age; thickness; dimensions; concentration; extent of failure, and other. Ice crystals consist of fresh ice, brine, insoluble salts, and air inclusions. Deformation and fracture of ice are due to its structure, temperature, salinity, and type of deformation. Bending failure of ice is typical for ships and hydraulic structures. The work presents the study of changes in the physical and mechanical properties of sea ice in bending.

## 2. Ice Physics

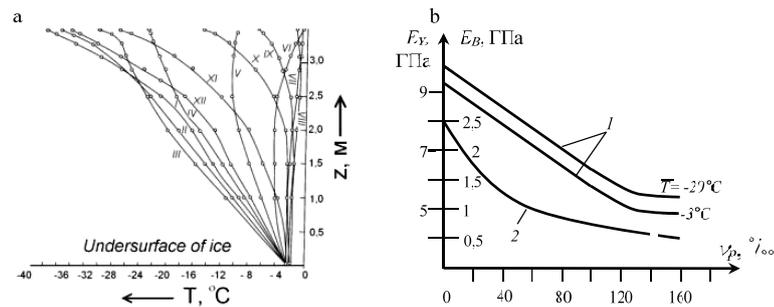
The annual change in the temperature of the ice thickness  $T(z)$  is shown in Fig. 1a. In summer, the ice temperature is practically constant being close to the average value  $\bar{T}$ :  $T(z) \approx \bar{T} = \text{const}$ . Z coordinate reports with the bottom of the ice surface (Fig. 1a) with a thickness H.

Ice is characterized by salinity:  $S(z) \approx \bar{S} = \text{const}$ .

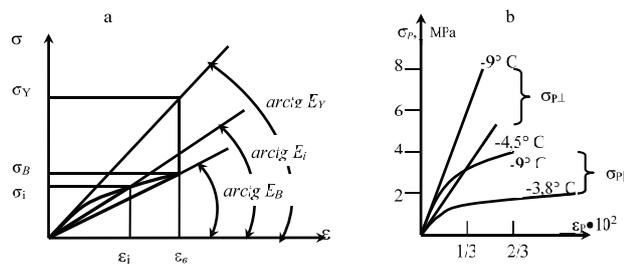
The average salinity  $\bar{S}$  for the seas (NSR) is defined by the following Ryvlin's expression [1]:

$$\bar{S} = 30 (-0,5H^{0,5}) + 4,5, \text{‰} \quad (1)$$

### OPEN ACCESS



**Figure 1:** Characteristics of sea ice: (a) temperature variation of sea ice  $T$  over the horizon  $z$  throughout the year (I...XII – months of the year); (b) changing the Young's modulus  $E_Y$  (1) and deformation modulus at a bend  $E_B$  (2) of sea ice by the brine content.



**Figure 2:** Ice tension diagrams: (a) schematic diagrams  $\sigma - \epsilon$  under bending of ice; (b) marine congelation ice in compression (small samples;  $\sigma_{B_{\perp/\parallel}}$  - loading direction relative to the horizon, the temperature indicated).

Multi-year ice has a salinity  $\bar{S} = 3 \dots 6 \text{ ‰}$ . Salinity is also determined by the relative content of the brine in the volume of ice  $v_p$  described by formula of Frankenstein and Garner [2]:

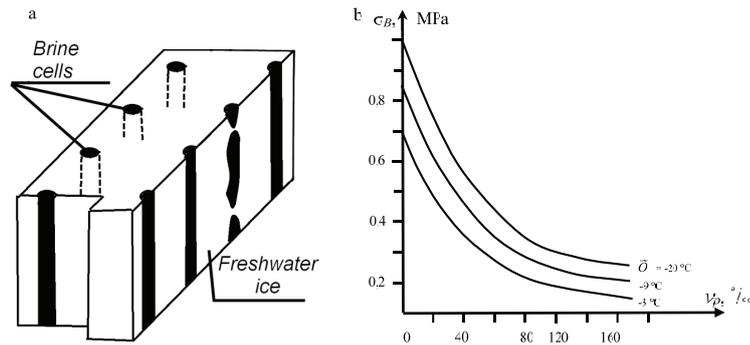
$$v_p = \bar{S} \cdot (0,532 - 48,19 \cdot \bar{T}^{-1}), \text{ ‰} \tag{2}$$

Ice is separated into two layers: the upper water-snow has a granular structure, while the lower core layer consists of congelation prisms or fibrillation.

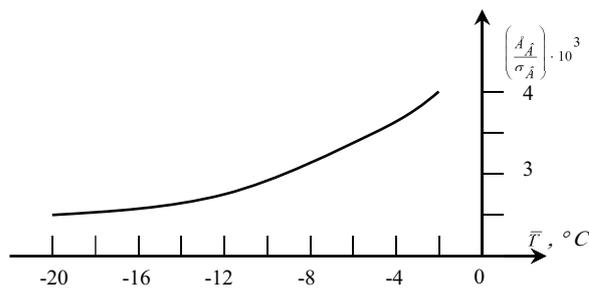
### 3. The Mechanical Properties of Ice Under Load

Deformation and fracture of ice is described in frame of kinetic and mechanical approaches. The kinetic one describes the development of microscopic damage (defects) basing on studies of Griffith, Irwin, and others. For example, this approach is applied for study of bending and compression of ice by Vershinin [2]. A more common approach uses mechanical stress state diagrams  $\sigma - \epsilon$  (Fig. 2a).

Ice deforms elastically at low  $\bar{T}$  and elastoplastically at increasing  $\bar{T}$  (Fig. 2b). In order to take it into account, a simple Hooke's law is replaced by more complex rheological models [3]. At the beginning of the ice deformation, its behavior is described by the elastic modulus (Young's)  $E_Y$ . With increasing load, secant modulus  $E_e$  is used (Fig. 2a). The extreme value  $E_i$  is deformation module  $E_B = E_{i \text{ min}}$  when ice is breaking. The relative value of  $E_B/E_Y \equiv \frac{\sigma_B}{\sigma_Y}$  characterizes the degree of ice elasticity. Here,  $\sigma_Y$  is the



**Figure 3:** The cellular model of sea ice and the tensile strength of: (a) ice model of the cellular structure (cell brine); (b) limit  $\sigma_B$  bending strength consoles.



**Figure 4:** Dependence of the relation of deformation module of sea ice at a bend to the limit strength depending on the temperature. Salinity of the ice is 3 ‰.

ultimate tensile stress of ice, when it is fully elastically deformed (Fig. 2a). Respectively,  $1 - E_B/E_Y = \frac{\sigma_Y - \sigma_B}{\sigma_Y}$  is the measure of ice plasticity.

Estimates made by authors show an elastic behavior of the ice at a bend near the neutral sheet and increase of the ice plasticity near the surface (Fig. 4).

### 4. The Elastic Characteristics of Ice in Bending

The initial deformation of ice is characterized by the elastic modulus  $E_Y$ . Reliable way to determine  $E_Y$  is a seismic dynamic method. In this case, the speed of bending waves is influenced by all features of ice construction [2]. These field experiments are approximated by the following dependence:

$$E_Y(\bar{S}, \bar{T}) = E_Y(0, \bar{T})(1 - 3,6 v_p) \tag{3}$$

Here, the Young’s modulus for the freshwater ice  $E_Y(0, \bar{T}) \approx 9,21(1 - 0,0015\bar{T})$  (GPa).

Simplified example in the study of the bending deformation is the equating of modules (intersecting modules at fracture point) in compression and tension  $E_p = E_1 = E_B$ . The closest case to the interaction of vessels with ice and sloped waterworks are experiments on the destruction of sawn consoles in ice. Under the destruction on a “top-down” the module  $E_B$  is defined by the following expression [4]:

$$E_B = 2,3 (1 - 11,55 \cdot v_p + 42,6 \cdot v_p^2), \text{ GPa} \quad (4)$$

Dependences of the elastic characteristics of the  $E_Y$  and  $E_B$  are shown in Fig. 1b.

## 5. Results and Discussion

When bending, it is believed that the break-up occurs when the upper limit of tensile strength layer  $\sigma_2$  is achieved. A direct test of the large ice samples tensile is technologically impossible. Therefore, this test is replaced by the destruction of the console afloat with the definition of conditional flexural strength  $\sigma_B = \sigma_2 = \sigma_p$  under the assumption of the equality of the deformation modules  $E_B = E_1 = E_2$ . Strength limit  $\sigma_B$  is determined using the results of the theory of bending beams [5]:

$$\sigma_B = E_B \cdot \rho^{-1} \cdot (0,5H), \quad (5)$$

where  $\rho$  - bending radius of neutral console layer.

Presentation of  $\sigma_B(\bar{S}, \bar{T})$  data is carried out using a cellular model of ice (Fig. 3a) [1]. Cells with brine of relative volume  $v_p$  pass through the volume of freshwater ice. Therefore, we obtain the following expression [1]:

$$\sigma_B = \sigma_B(0, \bar{T}) \cdot \left[ 1 - \left( \frac{v_p}{v_0} \right)^\kappa \right], \quad (6)$$

where  $\sigma_B(0, \bar{T})$  - strength of fresh ice,  $v_0$  - limiting amount of brine, containing which the ice has no strength ( $\sigma_B \rightarrow 0$ );  $\kappa$  - exponent, depending on the shape of the cell section.

Eq. (6) is universal for the entire range of  $v_p$ . For a small amount of brine, strength of the cellular structure is more accurately defined by the formula [6]:

$$\sigma_B = \sigma_B(0, \bar{T}) e^{-n \cdot v_p}, \quad (7)$$

where the exponent  $n$  is determined from experimental data.

Tensile strength of fresh ice, determined using results of Butyagin [7], Lavrov [8], and others, is (MPa):

$$\sigma_B(0, \bar{T}) = 0,42 \left[ 1 + 0,34(-\bar{T})^{0,5} \right], \quad (8)$$

The authors come to the following final formulas for the flexural strength:

for  $\gamma_p < 0,06 \dots 0,07$

$$\sigma_B(\bar{S}, \bar{T}) = \sigma_B(0, \bar{T}) e^{-15,2v_p}; \quad (9)$$

for  $\gamma_p > 0,07$

$$\sigma_B(\bar{S}, \bar{T}) = 0,46\sigma_B(0, \bar{T}) (1 - 3v_p), \quad (10)$$

	The average temperature of the ice $\bar{T}, ^\circ\text{C}$				
	-20	-15	-11	-7	-3
$\nu_p$ at $\bar{s} = 3\text{‰}$	8,97	11,43	15,01	22,68	50,78
$E_B(0,003; \bar{T}), \text{GPa}$	2,07	2	1,9	1,7	1,15
$\sigma_B(0,003; \bar{T}), \text{MPa}$	0,88	0,80(5)	0,76	0,57(5)	0,30(5)
$(E_B/\sigma_B) * 10^{-3}$	2,35	2,48	2,5	2,96	3,77

TABLE 1: Character of changes ( $E_B/\sigma_B$ ) in bending.

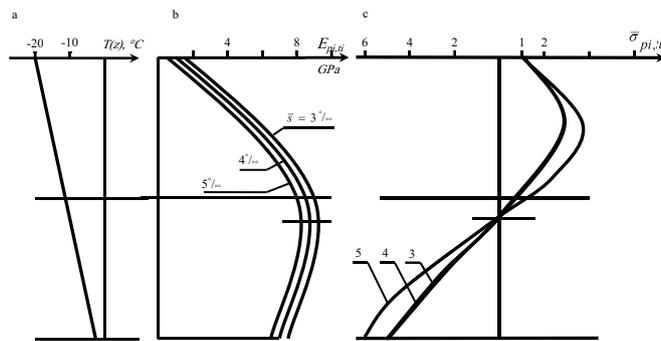


Figure 5: Dependence of the secant modulus (b) and the relative stress (c) sea-ice of congelation at  $\bar{\sigma} = -11^\circ\text{C}$  at a bend. (a) Change in temperature with the thickness.  $\bar{s}$ - salinity of the ice.

Results of  $\sigma_B(\bar{S}, \bar{T})$  calculations using (8-10) are shown in Fig. 3b.

Defined values of the physical and mechanical characteristics  $E_Y$ ,  $E_B$ , and  $\sigma_B$  of ice can be directly used in the calculation of the ice propulsion of ships and ice interaction with hydraulic structures. Despite the stability of the concept of modulus flexural  $E_B$ , it will allow to determine reliably the value of the rigidity of the cylindrical field of ice as a plate:

$$D = \frac{E_B H^3}{12(1 - \mu^2)}, \tag{11}$$

where  $\mu = 0,333 + 6,105 \cdot 10^{-2} \exp(0,182\bar{T})$  – Poisson coefficient of sea ice [2].

The ratio ( $E_B/\sigma_B$ ), in accordance with (5), is related to the radius of curvature of the ice neutral layer in the place of destruction by the following formula:

$$\rho(\bar{T})/0,5H = E_B(\bar{T})/\sigma_B(\bar{T}). \tag{12}$$

The nature of the changes of this ratio is shown in Fig. 4.

In the simulation of traffic movements and interactions of hydraulic engineering constructions with ice in the ice model basin, the fulfillment of the condition  $(E_B/\sigma_B) = idem$  is one of the criteria of dynamic similarity of the experiment [9]. The calculated value ( $E_B/\sigma_B$ ) indicates its growth with decreasing temperature  $\bar{T}$  (Fig. 4). This is contrary to the judgment of the opposite tendencies of its measurement [10]. Character of changes of ( $E_B/\sigma_B$ ) indicated in Fig. 4 is defined by the calculation in Table 1.

Numerically, it is explained by the fact that the gradient (negative)  $\frac{dE_B}{dv_p}$  is much less than the corresponding gradient  $\frac{d\sigma_B}{dv_p}$ . Studies of stressed state of ice in bending show that fracture occurs in the middle of ice stretching zone, rather than on the surface (Fig. 5). This makes the relation  $(E_B/\sigma_B)$  an artificial characteristic of the physical and mechanical properties of ice. It would be more correct to consider it as the relative value of the curvature radius of the neutral layer in the place of destruction in bending in accordance with (11).

## 6. Conclusion

Results of research lead to the following conclusions:

1. Strength limit  $\sigma_B$  and deformation module  $E_B$  of ice under bending are conditional mechanical characteristics of sea ice. These characteristics do not reflect the actual stress state of ice destruction at the time.
2. At the same time, module  $E_B$  allows estimating reliably the cylindrical stiffness of ice field.
3. The ratio of the module to the strength limit  $E_B/\sigma_B$  determines the relative radius of curvature of neutral layer in the place of ice destruction. It is shown that this ratio increases with the increase of ice temperature.
4. Dependencies for determining of  $E_B$  and  $\sigma_B$  of sea ice that generalize the experimental data are obtained.

## References

- [1] YP. Doronin and DE. Kheysin, (1975)., Gidrometeoizdat, .
- [2] SA. Vershinin, Impact of ice on marine structure shelf. Results of science and technology, in *Series Watercraft*, **13**, VINITI, Moscow, 1988.
- [3] SV. Koshkin, Generalized model of rheological behavior of ice under load, 238–240
- [4] N. A. Taranukha and S. V. Koshkin, Review and research of physical and mechanical properties for sea ice, 1093–1095
- [5] GS. Pisarenko, Yakovlev AP, Matveev VV: *Handbook of material resistance*, Naukova Dumka, Kiev, 1988.
- [6] S. Loset, KN. Shhiner, O. Gudmenstad, and K. Hoyland, *Effects of ice on marine and coastal structure*, Lan, St.Petersburg, 2010.
- [7] IP. Butyagin, *The strength of ice and ice cover*, Siberian Division, Novosibirsk, Nauka, 1966.
- [8] VV. Lavrov, *Deformation and strength of ice*, Gidrometeoizdat, Leningrad, 1969.
- [9] BP. Ionov, Gramuzov EM: Ice propulsion of ships. St, *Petersburg: Shipbuilding*, (2001).
- [10] KE. Sasonov, *Theoretical foundation of navigation in ice*. St.Peterburg: Krylov State Reserch Centre, Theoretical foundation of navigation in ice. St.Peterburg, Krylov State Reserch Centre, 2010.