

Conference Paper

An Optimal Inventory Model for a Retailer with Price Dependent Demand and Unavailability Supply

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Abstract

Today many retailers face high competition, therefore they have to operate in an efficient way. One aspect of efficiency is inventory. Many research on inventory is conducted intensively to get more realistic inventory model. In this paper, an inventory model was developed by considering pricing. Many retailers try to increase their profit by setting the best price for a single item, especially for some items that have high price-dependent demand. The customer demand depends on the price such household items. In the other side, some retailers face supply problems. Supplier often cannot supply products when the products needed on time. There is delay time between customer demand and products arrive at retailer warehouse. The retailer should determine the optimal price and replenishment time. There are some assumptions are used for the model. The first assumption, the demand is known and has constant elasticity. Second, there is stochastic replenishment period and demand that are not filled are lost sales. The model is developed mathematically and a numerical example is conducted to show how the model works. A sensitivity analysis is accomplished to get some management insight and some interesting result are derived.

Keywords: Deteriorating inventory model; genetic algorithm; stochastic time

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1. Introduction

Efficiency is one competitive advantage for some companies, however, it is not easy to achieve. The condition is more difficult for some products that are spoilage or decay gradually in some period. Some examples of these are fruits and vegetables. These kinds of items are called as deteriorating items. Some fruits and vegetables cannot be found during a year. There are some specific periods where the fruits or vegetables can be harvested. For the early harvest time number of fruits can be sold is not many, therefore the price is quite expensive and demand dependent with price. Another

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problem, sometimes difficult to get enough supply therefore there is possibility that order is not arrive when they are needed.

Research on deteriorating inventory with price dependent demand is conducted intensively. A deteriorating inventory model for product with stock and price dependent was developed by [1]. In their model number of goods displayed can attract customer and increase sales and profit. A deteriorating model with price dependent demand was developed by [2]. In their model a production lot size can be fulfilled by regular and over-time production and demand is assumed depleted due to deterioration and demand. In their model, supplier tries to increase demand by giving discounts and delayed payments to increase demand. Advance payment deteriorating inventory models since buyer need to pay in advance were considered by [3]. They also considered time-varying deterioration rate, allowed shortage and some percentage of the shortage items are backlogged. Deteriorating inventory models by considering pricing and discount for products with declined freshness in time. In their model, sales and profit can increase by setting pricing and discount were developed by [4]. A supplier-retailer supply chain model for deteriorating items by considering pricing was developed by [5]. Supplier and retailer try to optimize profit by setting optimal selling price and offering customers partial trade credits. A deteriorating inventory items model where the demand depends on three factors which are time, stock level, and price was developed by [6]. An economic production deteriorating inventory items dynamic pricing model with price dependent demand and stochastic process demand was developed by [7]. A model considering price and time demand function for deteriorating inventory model was developed by [8]. They also considering partial-backlogging and the price is periodically adjusted upward and downward. An integrating deteriorating inventory model where buyer is offered with quantity discount pricing by the vendor was developed by [9]. Quantity discount tempt the buyer to buy more and profit sharing is balanced between two players. An inventory model with demand depend on more than one parameter is developed by [10]. They developed an optimal inventory model where demand depend on price and time.

There is intensive research on deteriorating inventory model and optimization, however only a few considering price dependent demand and stochastic unavailability time. At the early harvest phase, only a few fruits can be harvested; therefore, the price tends to be high, and customer demand depends on the price offered by the supplier. The other characteristic is the supply is not always available. This paper contributes by developing deteriorating inventory models for price dependent demand and stochastic unavailability time. This paper is divided into four sections. Some relevant literature and

the contribution of the paper are introduced in the first section. Section 2 shows how some mathematical models are developed. In the last section, some conclusions are shown.

2. Model development

Notations:

I_t = Inventory level at t period

p = price rate

α = price constant rate

ε = price sensitivity rate

β = stock sensitivity rate

d = demand rate

θ = deteriorating rate

K = setup cost

H = holding cost

S = lost sales cost

TC = total inventory cost

T = total replenishment time

TP = total profit

In the price dependent inventory deteriorating model, the demand depends on the price rate. The inventory model can be modelled by Equation 1 and Equation 2 as follows:

$$\frac{dI}{dt} + \theta I_t = -\alpha p^{-\varepsilon}, \quad 0 \leq t \leq T \tag{1}$$

Since $I_{(0)} = Q$ when $t = 0$, then one has:

$$I_t = \frac{\alpha p^{-\varepsilon}}{\theta} (e^{-\theta(T-t)} - 1) \quad \text{for } 0 \leq t \leq T \tag{2}$$

Total inventory for $0 \leq t \leq T$ is shown in Equation 3

$$\int_0^T I(t)dt = \int_0^T \frac{\alpha p^{-\varepsilon}}{\theta} (e^{-\theta(T-t)} - 1) dt = -\frac{\alpha(e^{-\theta T} + \theta T - 1)p^{-\varepsilon}}{\theta^2 p^\varepsilon} \tag{3}$$

Total profit is equal to total revenue minus cost of placing an order, and cost of carrying inventory. The total profit is:

$TP = p(\alpha p^{-\epsilon}) - \frac{K}{T} - \frac{Hl_t}{T}$, substitute l_t from Equation 3 to this equation, one has Equation 4 as follows:

$$TP = p(\alpha p^{-\epsilon}) - \frac{K}{T} - \frac{H \left(-\frac{\alpha(e^{-\theta T} + \theta T - 1)p^{-\epsilon}}{\theta^2 p^\epsilon} \right)}{T} \tag{4}$$

The total profit is equal to total revenue minus setup cost, holding cost and loss of goodwill. Lost sales occur when supply unavailability period is bigger than the replenishment period. The total profit can be modeled as Equation 5

$$TP = p(\alpha p^{-\epsilon}) - \frac{K}{E(T)} - \frac{H \left(-\frac{\alpha(e^{-\theta T} + \theta T - 1)p^{-\epsilon}}{\theta^2 p^\epsilon} \right)}{E(T)} + S\alpha p^{-\epsilon} \frac{\int_{t=T}^{\infty} (t - T)f(t)dt}{E(T)} \tag{5}$$

Replenishment time is equal to replenishment period and supply unavailability time where supply unavailability is longer than replenishment period. The expected replenishment time can be modelled as Equation 6:

$$E(T) = T + E(T_s) = \left[T + \int_{t=T_d}^{\infty} (t - T)f(t)dt \right] \tag{6}$$

The total profit can be derived by substituting Equation 6 into Equation 5 as shown in Equation 7:

$$TP(p, T) = \frac{E \left[p(\alpha p^{-\epsilon}) - K - H \left(-\frac{\alpha(e^{-\theta T} + \theta T - 1)p^{-\epsilon}}{\theta^2 p^\epsilon} \right) + S\alpha p^{-\epsilon} \frac{\int_{t=T}^{\infty} (t - T)f(t)dt}{E(T)} \right]}{T + \int_{t=T}^{\infty} (t - T)f(t)dt} \tag{7}$$

In this case, it can be assumed that the unavailability time t is a random variable that is uniformly distributed over the interval $[0, b]$. The uniform probability density function, $f(t)$, is given as Equation 8:

$$f(t) = \begin{cases} 1/b, & 0 \leq t \leq b \\ 0, & otherwise \end{cases} \tag{8}$$

For uniform distribution, the value of T can be expressed as Equation 9:

$$E(T) = T + \frac{(b - T)^2}{2b} \tag{9}$$

Substitute uniform probability density function in Equation 8, one has Equation 10:

$$TCT(p, T) = \frac{\left[Kp(\alpha p^{-\epsilon}) - K - H \left(-\frac{\alpha(e^{-\theta T} + \theta T - 1)p^{-\epsilon}}{\theta^2 p^\epsilon} \right) + S\alpha p^{-\epsilon} \left(\frac{b - T}{b} \right) \right]}{T + \frac{(b - T)}{2T}} \tag{10}$$

Since the closed form solution cannot be found and there are two decision variables. The model is solved using a genetic algorithm. A simple genetic algorithm is used to solve the model and run in Matlab. The simple genetic algorithm scheme is derive as follows:

1. Chromosome

Chromosome represents the optimal price (p) and the optimal ordering time (T). The population type is bit string with 14 cells where seven cells are for the optimal price, and seven cells are for the optimal ordering time.

2. Initial solution and population

The initial solutions are generated randomly, and the population size is equal to 200.

3. Selection and reproduction

For each generation, the parent chromosomes are chosen using roulette wheel where chromosomes with the highest profit have a high probability to be chosen as parents. The reproduction using crossover and mutation. The crossover scheme is two points crossover, and the mutation scheme is uniform with probability 1%. Elitism scheme is using to guarantee the best chromosome for every generation will be a child in the next generation.

4. Stopping criteria

The genetic algorithm is stopped after 100 generations

3. A numerical example and sensitivity analysis

A numerical example is shown to illustrate the model. In this case. $K = 100$, $\theta = 0.05$, $\alpha = 1000$, $\varepsilon = 1.2$, $h = 1$, $S = 10$ and $b = 1$. The genetic algorithm is run five times, and the best solution is employed. The optimal price (p^*) that is obtained from the model is eight, the optimal ordering period (T^*) is 0.5078, and the optimal profit is 714.125. A sensitivity analysis is conducted to get some management insights. One parameter is changed and the other parameters are kept same. The result shows that the most sensitive parameter is price sensitivity parameters (Figure 1), therefore retailer is more difficult to get optimal price decision when price sensitivity parameters has high variation. The second most sensitive parameter for price decision is maximum unavailability time. Figure 2 shows sensitivity analysis for the optimal replenishment time. The optimal replenishment time decision mostly sensitive to price sensitivity parameter and maximum unavailability time. This sensitivity analysis result is the same as sensitivity analysis for the optimal price decision.

Figure 3 shows the sensitivity analysis for the optimal total profit. Similar with other sensitivity analysis result, the most sensitive parameter to the optimal total profit is

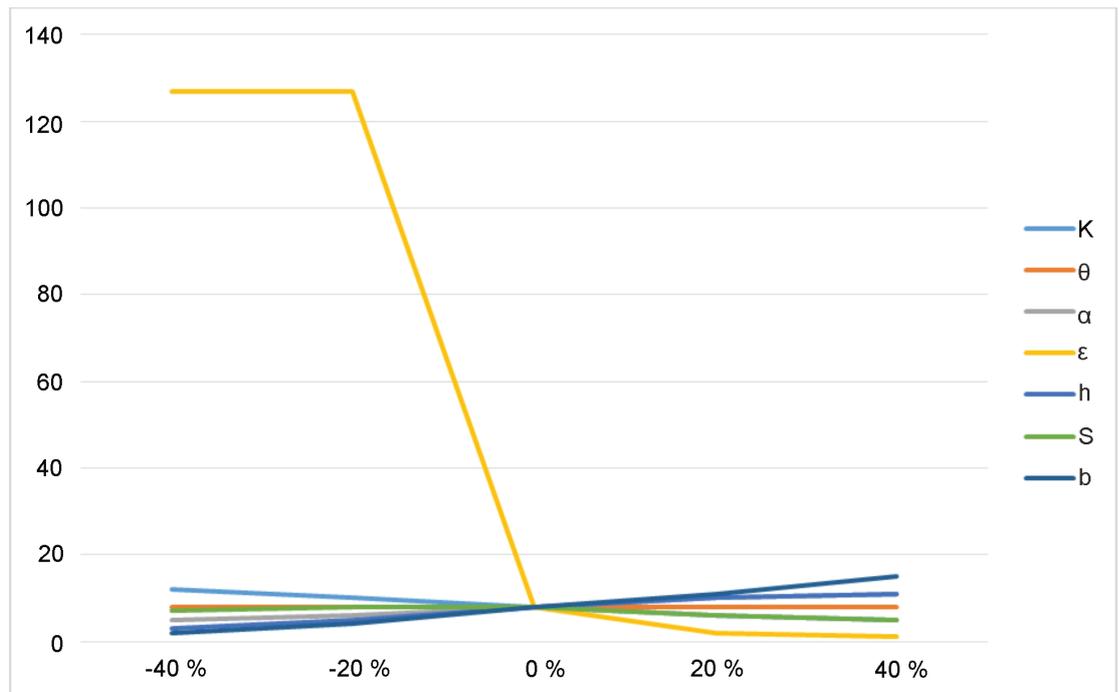


Figure 1: The sensitivity analysis for optimal price

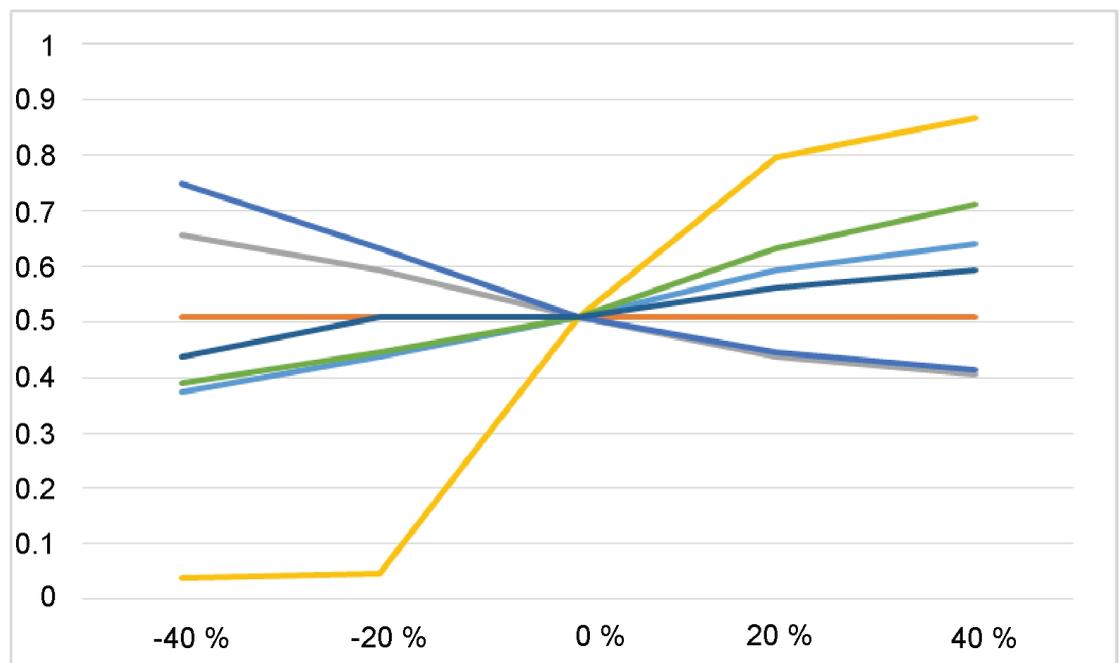


Figure 2: The sensitivity analysis for optimal replenishment time

the price sensitivity parameter. The profit decrease when the price becomes more sensitive. However, the price sensitivity parameter depends on the customer that cannot be managed by the supplier. The second parameter that has high sensitivity to the total profit is supply unavailability time. The total profit tends to decrease as the supply

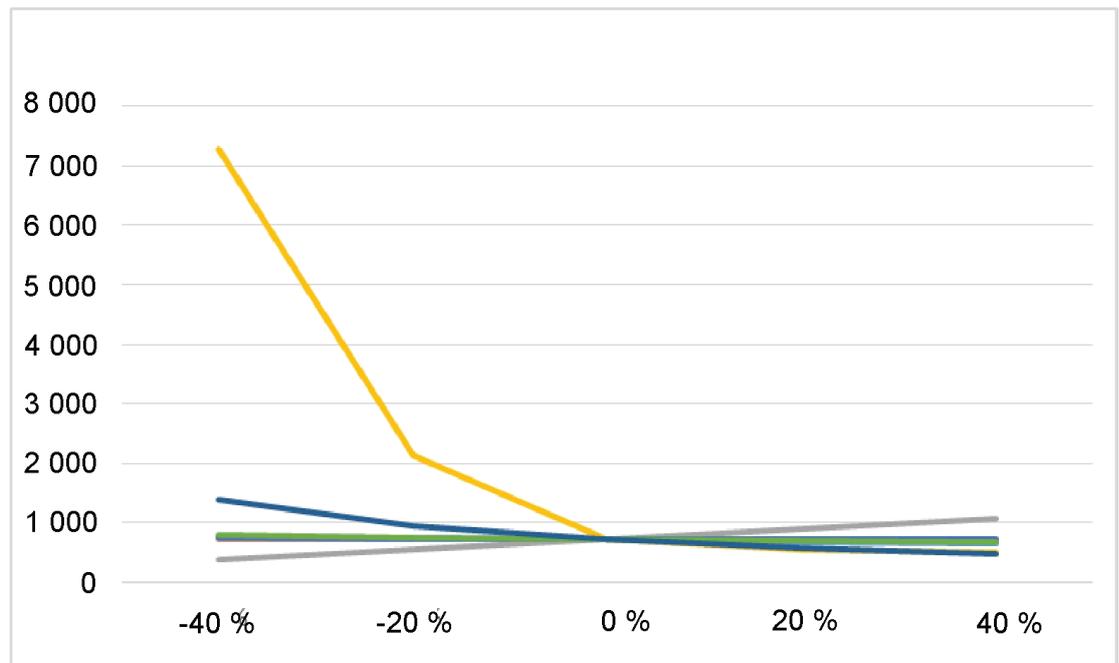


Figure 3: The sensitivity analysis for optimal total profit

unavailability time increase. Since the supplier can manage the parameter, the supplier can try to make the supply more reliable to keep the high profit.

4. Conclusion

In this paper a deteriorating inventory items by considering price-dependent demand and unreliable supply model. The model is relevant for some fruits that have specific harvest time. When the fruit is not always available constantly and the fruit demand depend on price. The model is solved using genetic algorithm since closed-form solution cannot be derived.

The sensitivity analysis shows that the most sensitive parameters to price decision is price sensitivity parameter. It is more challenging to find the best price where demand is very sensitive to price. The second most sensitive parameter to the price is supplied unavailability parameters. The supplier will increase his price to reduce cost because of supply unavailability. At the end of the supply period when demand depends on the stock, the total cost mostly sensitive in varies of ordering cost and unavailability supply. Therefore, it is essential for the supplier to keep ordering cost low since it is challenging to keep low unavailability supply sine the harvest period almost end. This research can be improved by considering some scenarios during first harvest period to the end of harvest period.

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