KnE Life Sciences
ICMScE
International Conference On Mathematics And Science Education
Volume 2024

Research Article

Input-Output Analysis on Pia Saronde Production Process Scheduling with Invariant Max-Plus Linear System

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Abstract.
Max-plus algebra is one of the analysis methods of discrete event systems which has many applications on systems theory and graph theory. Max-plus algebra is a set of real numbers R combined with ⊠=-∞ equipped with operations max (⊕) and plus (⊗), can be denoted [(R]_ε,⊕,⊗) with [(R]_ε=R⋃{ε}) . The production process of pia saronde is one of the problems that can be analyzed using max-plus algebra. The production process of this product is sequentially carried out by making skin dough, filling, baking, cooling, and packaging the pia. The max-plus algebra theory was used in this research to determine the optimal time in the production scheduling of pia saronde. Meanwhile, the Invarian Max-plus Linear System (IMLS), max-plus algebraic theory, and the Discrete Event System (DES) were used to solve the production-related problems. IMLS analysis produces eigenvalues that represent the optimum production time. The results obtained the max-plus algebra model of x(k+1)=A ⊠x(k), where A ⊠A⊕B⊗C and y=K⊗x_0⊕H⊗u for input-output IMLS analysis. From the matrix A ⊠ eigenvalue λ= 226 and eigenvector v=[278 278 278 279 299 302 324 356 488] were obtained. Furthermore, the value of λ describes the pia production schedule at a time span of 226 minutes.

Keywords: input-output analysis, pia saronde, scheduling, max-plus linear system

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Published: 27 March 2024

Publishing services provided by Knowledge E

Selection and Peer-review under the responsibility of the ICMScE Conference Committee.

1. INTRODUCTION

Gorontalo is a province in Indonesia with different culinary delights and exotic flavors [1] which are usually enjoyed by everyone including the residents, other Indonesians, and tourists from different countries. Pia cakes by Pia Saronde Bakery & Coffee which can be used as souvenirs by visitors. This pia is loved by people due to its good texture, taste, and attractive packaging [2]. The number of enthusiasts for the snack has increased the growth of the Pia Saronde Bakery & Coffee rapidly as indicated by the findings of Afiati [3] that consumer loyalty to the company increased by 20%. Moreover, the high demand for the company's products has increased its production time. An interview conducted with Fitriana Yusuf, the Human Resources Development (HRD), also showed that there is usually a large demand for products during the time of Eid, thereby, leading to an increase in the production as well as overtime or the addition of more working days. This overtime usually makes some employees feel uncomfortable because of the personal time they need to sacrifice for the company. Therefore, there is the need for a strategy to produce enough pia to meet consumer targets and demands at a more optimal time. One of the strategies required by the company is the optimization of the production time and this can be achieved through the application of mathematics such as max-plus algebraic theory to the scheduling process. This involves developing a mathematical model that can produce an optimal time which is to be subsequently applied to the production process in order to optimize the schedule.

Max-plus algebra was first discovered in the 1950s even though the associated theory was developed in the 1960s and has been used to model and analyze systems such as network problems, scheduling, combinatorics, and time optimization max-plus algebraically [4]. Moreover, the Linear Invariant System is a discrete event system with a time of activity and a deterministic sequence of events in max-plus algebra [5]. It is an max-plus algebraic science normally applied to determine the optimum value of the model to be developed for periodic scheduling.

Max-plus algebra was used in [6–9] to reduce the optimal time of production. The another max-plus application was on Image Steganography by [10]. The implementation of max-plus algebra also to study time disturbance propagation within a robustness improvement context [11]. Another implementation was in creating a production schedule for a Kok brand by [12]. It was also applied to modeling motion and control of lower limb exoskeleton [13]. The max-plus algebra used by [8] to make a model of inaportnet system ships service scheme.

The existence of applied science such as max-plus algebraic mathematics provides opportunities to over the problems associated with optimizing time and scheduling the production process of a product. This can also be applied to products from which one of the iconic companies in Gorontalo, namely Pia Saronde Bakery & Coffee considering the large number of production processes in the company. The production time can be made more effective through the development of a mathematical model to examine the maximum time being used in each stage of production. Therefore, this study develops a mathematical model on the scheduling problem in the production process based on the theory and observation from previous research. As for what has been developed from previous studies, a mathematical model will be created that has 4 inputs and 1 output and was implemented using the Linear Invariant System to obtain the maximum time for each production stage after which the eigenvalues were used to represent the optimal time to schedule the pia production process was obtained and analysis was conducted at several conditions. So that the author can contribute by implementing max-plus algebra at the Pia Saronde Coffee and Bakery company.

2. RESEARCH METHOD

This is applied research which involves the application of the max-plus algebraic theory in determining the optimal time to prepare the production schedule for pia. The stages involved include 1) review of previous studies on the max-plus algebraic theory and concepts in producing pia saronde, 2) collection of time data and steps in producing pia, 3) making assumptions based on the theory and data obtained, 3) formulation of a diagram of activity in the pia production process, 4) creation of a mathematical model, 5) development of a matrix based on the model, 6) determination of the λ to be used as the optimal production time, 7) arrangement of the production time based on the eigenvalues, and 8) analyzing of the model and design of a schedule. It is important to note that data were collected through direct interviews conducted with the HRD of Pia Saronde Bakery & Coffee.

3. RESULT AND DISCUSSION

3.1. Assumptions in the Pia Production Process

The initial observations and theoretical studies showed that the pia production time can be modelled based on the following assumptions:

1. The input unit is defined as"$u(k)$", the processing unit as "P_i" where i indicates the process sequence, the output unit as "$y(k)$", and the material transfer time between processes as "t_j" where j indicates the time sequence at the time of material transfer, and "d_m" is the time required by the processing unit to process the material, where m is the sequence of time used in each processing unit. Meanwhile, "k" is an iteration of the production process.

2. The production time was calculated continuously such that when "k" is completed, it is immediately followed by the "$k + 1$" process and so on.

3. Material preparation time at each input unit u_n, for $n = 1,\ 2,\ 3\ ,4$ is negligible or equal to zero $\left(u_1 = 0, u_2 = 0, u_3 = 0, u_4 = 0\right)$, and n indicates the type of input. The process starts counting at $k + 1$.
4. The input process is simultaneous.
5. The production time of a processor is initiated when the previous production process has been completed.
6. The production start time is carried out according to the company's schedule.
7. Every processor runs smoothly during the production process and there are no defects.
8. The time to transfer materials from one room to another is 1 - 2 minutes while the transfer of materials in the same room is 0 minutes.
9. The model was simulated at an initial state conducted under several conditions.
10. The input-output analysis process was conducted using any vector u and any vector y.

3.2. Pia Production Activity Diagram

The existing assumptions were used to design the pia production process in the chart presented in Fig. 1**.**

Description:

$u_i\,(k + 1\ )$: Input units with $i = 1, 2, 3, 4$;

P_i : Processing unit at Pia Saronde Production, $i\ =\ 1, 2, 3, ...9$;

t_j : Time at which the processed material moves, $j\ =\ 1, 2, 3, ...10$;

d_1: Time to transfer ready-made cheese to the container

d_2: Time to transfer ready-to-use chocolate to a container

d_3: Time to cut the green bean dough to ensure it is ready to use

d_4: Time to mix the ingredients for the pie layer dough

d_5: Time to transfer materials from the mixer to the container

d_6: Time to unify each filling with the dough layer of pia

d_7: Time for the pia roasting process

d_8: Time for the pia cooling process

d_9: Time for the pia packaging process

DOI 10.18502/kls.v8i1.15545

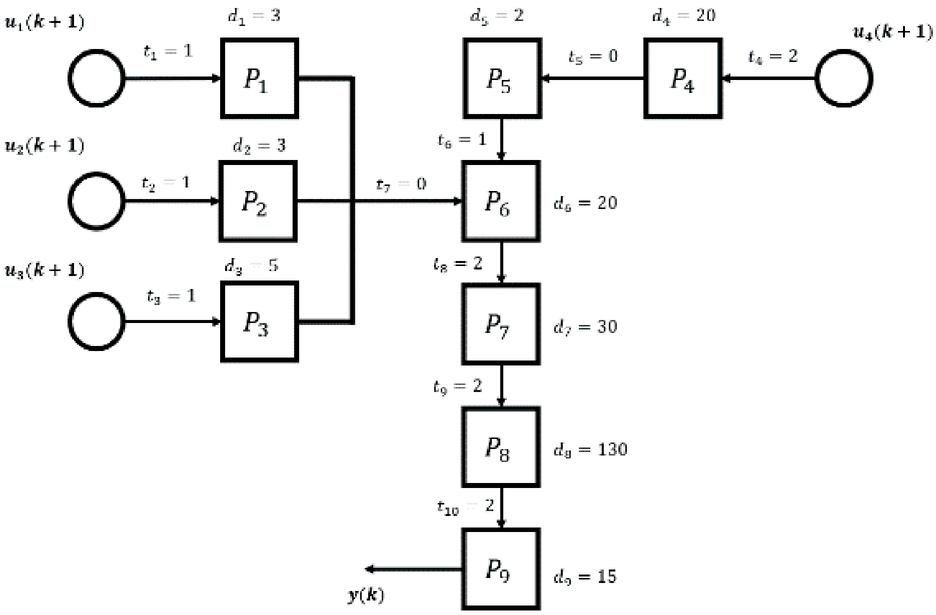

Figure 1: Pia production activity diagram.

$y(k)$: Unit *output*.

3.3. Pia Production Process Modeling

The results of the interviews were used to determine the production process at each processing unit P_i where $i = 1, 2, 3..9$ and production time d_i where $i = 1, 2, 3..10$ as indicated in the following Table 1.

TABLE 1: Pia Production Process Time.

P_i	**Process**	d_i**(Minute)**
P_1	Transferring the cheese to the container	3
P_2	Transfer of chocolate to container	3
P_3	Green bean dough cutting	5
P_4	Pie layer dough mixing	20
P_5	Transferring the dough to the container	2
P_6	The union of the pie filling and dough layers	20
P_7	Pie roasting	30
P_8	Cooling	120
P_9	Packaging	15

Therefore, Fig. 1 and Table 1 show that the pia production process can be defined as follows:

$u_i(K+1)$: the time when the layer materials and contents of pia saronde enter the processing unit for processing$-(k+1)$;

$x_i(k)$: the time when each$-i$th processor starts working for the $-k$th process.

$y(k)$: the time when the pia product on the $-k$th process finishes and leaves the system

With using the basic operations of max-plus algebra which include maximum (denoted by the symbol $\oplus$) and added (denoted by the symbol $\otimes$), these definitions were used the develop the mathematical model for the production process of pia as follows

$$x_1(k+1) = 3 \otimes x_1(k) \oplus 1 \otimes u_1(k+1) \quad (1)$$

$$x_2(k+1) = 3 \otimes x_2(k) \oplus 1 \otimes u_2(k+1) \quad (2)$$

$$x_3(k+1) = 5 \otimes x_3(k) \oplus 1 \otimes u_3(k+1) \quad (3)$$

$$\vdots$$

$$y(k) = 15 \otimes x_9(k) \quad (4)$$

3.4. Input-Output Analysis in Optimizing the Pia Production Process

The model in the linear system formed was used to rewrite the equation in matrix form as follows

$$x(k+1) = A \otimes x(k) \oplus B \otimes u_i(k+1), \; i = 1,2,3,4 \quad (5)$$

$$y(k) = C \otimes x(k) \quad (6)$$

Where, matrix A shows the ongoing pia production process, matrix B indicates the input time, and matrix C represents the end time.

$$x(k+1) = A \otimes x(k) \oplus B \otimes u_i(k+1), \; i = 1,2,3,4 \quad (7)$$

$$y(k) = [\,\varepsilon \;\; \varepsilon \;\; \varepsilon \;\; \varepsilon \;\; \varepsilon \;\; \varepsilon \;\; \varepsilon \;\; \varepsilon \;\; 15\,] \otimes x(k) \quad (8)$$

The production process was assumed to be conducted continuously and this means $u(k) = y(k)$, therefore, the evolution of the system state was found to be:

$$x(k+1) = \underline{A} \otimes x(k) \quad (9)$$

Where,

$$\underline{A} = A \oplus B \otimes C \quad (10)$$

Therefore, the value of

$$\underline{A} = [3\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ 16\ \varepsilon\ 3\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ 16$$

$$\varepsilon\ \varepsilon\ 5\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ 16\ \varepsilon\ \varepsilon\ \varepsilon\ 20\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ 17\ \varepsilon\ \varepsilon\ \varepsilon\ 40\ 2\ \varepsilon\ \varepsilon\ \varepsilon$$

$$37\ 6\ 6\ 10\ 43\ 5\ 20\ \varepsilon\ \varepsilon\ 40\ 28\ 28\ 32\ 65\ 27\ 42\ 30$$

$$\varepsilon\ 62\ 60\ 60\ 64\ 97\ 59\ 74\ 62\ 130\ 94\ 192\ 192\ 196\ 229\ 191\ 206\ 194\ 262\ 226\ ] \quad (11)$$

The eigenvalues and eigenvectors were determined based on [14] assisted by Scilab software and those for matrix $\underline{A}$ were obtained as follows. Assume the initial state $x(0) = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ]^T$ was determined by the evolution of state $(0), x(1), x(3), x(4), \ldots..$. It was discovered that $x(3) = 226 \otimes x(2)$, where $p = 3, q = 2$, and $c = 226$, therefore, the eigenvalues of matrix $\underline{A}$ are $\lambda = 226$. The eigenvectors are stated as follows

$$\upsilon = [278\ 278\ 278\ 279\ 299\ 302\ 324\ 356\ 488\ ] \quad (12)$$

A simulation of several initial conditions was conducted to form a periodic system and prove the values of λ and υ using Scilab Software. System evolution for $k = 1, 2, 3, 4, 5$ shows the initial state $x(0) = [0\ 0\ 0\ 1\ 21\ 24\ 46\ 78\ 210\ ]$ reached periodically since the first time.

3.5. Pia Production Process Scheduling

A pia production schedule was formulated after the eigenvalues and eigenvectors had been determined. According to (9), the start time of the system is normally selected from the minimum non-negative eigenvector which, in this case, was obtained by changing the smallest eigenvector to zero. In matrix $\underline{A}$, the eigenvector produced $\upsilon = [0\ 0\ 0\ 1\ 21\ 24\ 46\ 78\ 210\ ]^T$. Based on the assumptions made, the production start time is 08.00 am and minute $-0th$ was used as the time when the machine starts working exactly at 08.00 am in this case. This, therefore, led to the design of the pia production schedule based on this production time in Table 2.

Table 2 shows that Pia Saronde Bakery & Coffee can conduct two production processes in a day with the first starting at 08.00 am and finishing at 11.30 am and the second from 11.46 am to 15.16 am.

The optimal timing of the input-output production process was also analyzed using [15] to determine the fastest and longest time to start and complete the production.

Input-Output IMLS (A, B, C, x_0). This section focuses on conducting the input-output analysis of max-plus invariant linear system at an initial state and at a predetermined

TABLE 2: Pia Production Process Schedule.

P_i	**Iteration 1**	$Iteration\ 2$
P_1	08.00 am	11.46 am
P_2	08.00 am	11.46 am
P_3	08.00 am	11.46 am
P_4	08.01 am	11.47 am
P_5	08.21 am	12.07 pm
P_6	08.24 am	12.10 pm
P_7	08.46 am	12.32 pm
P_8	09.18 am	01.04 pm
P_9	11.30 am	03.16 pm

time. For example, the initial conditions in the production process of pia are provided as follows: $x_0 = [0\ 1\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ \varepsilon\ ]^T$. The data were analyzed to obtain the output value y which is defined as $y = [y(1),\ y(2),\ y(3),\ ...,\ y(9)]^T$ to produce

$y = K \otimes x_0 \oplus H \otimes u$, it is obtained $y = [226\ 358\ 486\ 616\ 746\ 876\ 1006\ 1136\ 1266]$. This means the production process is complete and leaves the system at time y.

Input-Output IMLS (A, B, C, ε). An input-output analysis was also conducted under different conditions such that $x_0 = \varepsilon$ was set as the initial condition and the input value $u = [0\ 20\ 40\ 60\ 80\ 100\ 120\ 140\ 160\ ]^T$ to produce the output value.In this case, $y = [y(1), y(2), y(3),\ \ldots,\ y(9)]^T$ is defined as $y = H \otimes u$ and obtained $y = [226\ 356\ 486\ 616\ 746\ 876\ 1($ This means there is no processing unit for the raw or semi-finished materials at the initial conditions of the production process.

According to [16] and [17], the input time for the production process can be determined by knowing the largest sub-completion $\hat{u}$. For example, the vector y used to determine the completion of the production process was defined as follows $y = [230\ 256\ 486\ 630\ 771\ 899\ 1025\ 1153\ 1273]$ so that the Pia Saronde Bakery & Coffee can determine the optimal time to start the production process using the formula.

$$\hat{u} = H^T \otimes (-y) \quad (13)$$

$$\hat{y} = H \otimes \hat{u} \quad (14)$$

$$\tilde{u} = \hat{u} \otimes \frac{\delta}{2} \quad (15)$$

$$\tilde{y} = H \otimes \tilde{u} \quad (16)$$

So that obtined

$$\hat{u} = [0\ 130\ 260\ 397\ 527\ 657\ 787\ 917\ 2047]^T \quad (17)$$

$$\hat{y} = [226\ 356\ 486\ 623\ 753\ 883\ 1013\ 1143\ 1273]^T \quad (18)$$

$$\tilde{u} = [9\ 139\ 269\ 406\ 536\ 666\ 796\ 926\ 1056]^T \quad (19)$$

$$\tilde{y} = [235\ 365\ 495\ 632\ 762\ 892\ 1022\ 1152\ 1282]^T \quad (20)$$

This calculation shows that $\hat{u}$ and $\tilde{u}$ are the largest sub-completion and optimal time to start the production process.

4. CONCLUSION

The pia production process time at Pia Saronde Bakery & Coffee can be modeled by using max-plus algebra to determine the maximum time required by each process $(x_i(k+1))$ and the maximum time for output $(y(k))$ and transform the findings into IMLS form by changing each equation $(x_i(k+1))$ and $(y(k))$ using basic operations of max-plus algebra. The mathematical model was used to determine matrix $\underline{A}$ which produced the eigenvalue and eigenvectors to be $\lambda = 226$ and $v = [278\ 278\ 278\ 279\ 299\ 302\ 324\ 256\ 488\ ]^T$ respectively. The production time which was initially designed to start at 08.00 am and end at 04.00 pm eventually finishing at 03.16 pm. This means the max-plus algebraic theory applied to the pia production process solved the scheduling problem by ensuring the production process can be completed 44 minutes faster than the required production time. With this paper, the author hopes that further research can use different assumptions to model the production process and suggest using a more varied vector value in performing input-output analysis of an the IMLS.

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