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Research Article

Detecting Quantum Topologic Phase Transitions Through The C-Function

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Abstract.
Topological Quantum Field Theory or TQFT is a quantum field theory that calculates topological invariance in measurement theory and mathematical physics. In recent years, several attempts have been made to find efficient observations to determine the TQFT of quasiparticle properties. In this paper, we propose a different and very effective way to detect the critical points of TQFT by considering the system functions. We suggest the C-Function as a novel probe that is accurate for detecting the location of critical points on topological quantum. The C-function uses a holographic model to show a topological quantum phase transition between a simple topological isolation phase and a gapless Weyl semimetal. The quantum tipping point displays a strong Lifshitz-like anisotropy in the spatial direction, and a quantum phase transition that does not follow the standard Landau paradigm. The C-function precisely shows the global features of quantum criticality and distinguishes very accurately between two separate zero-temperature phases. Considering the C-function relationship with entanglement entropy can detect quantum phase transitions and can be applied outside the holographic framework.

Keywords: quantum topologic, phase transitions, c-function

1. INTRODUCTION

The effect of phase transitions provides a rapid development in physics, and its relation to quantum theory [1]. the quantum phase occurs at absolute zero [2], the effect resulting in a condensed matter paradigm shift, thus generally not accepting the Ginzburg-Landau description [3]. The effect is often difficult to explain, as is the pattern of spontaneous symmetry breaking. This case also occurs in transition metal insulators [4].

In the area of topological quantum phase transition (TQFT), in quantum space systems display exotic features such as fractional statistics and topological degeneration [5]. This parameter serves to sort, calculate the local quantity indicating the location of the TQFT, also has an impact on the development of quantum physics. Parameters serve to sort, calculate local quantities that indicate the location of TQFT [6]. C-functions can detect phase transitions. Reorganizing degrees of freedom along key transitions understands the two distinct phases involved. In relativistic theory, to find the effective number of degrees of freedom provided by the C-function [7].

Monotone nature of the degrees of freedom along with the normalized group flow, the C-function theorem guarantees. The dof number decreases monotonically towards lower energies and its validity is closely related to the energy state of absolute zero temperature [8]. At each point, it will coincide with the central charge of the system, and prove its connection with the theory of degrees of freedom..

Weyl semimetals (WS) is a band structure characterized by a point singularity, the two bands at contact produce a linearly spreading cone [9]. The low energy descriptions at these points display relativistic symmetry, and are described by chiral Weyl spinors that always appear in pairs [10]. WS exhibits exotic transport properties which are a direct consequence of anomalous quantum field theory [11]. In order to understand the basic dynamics of WS and TQFT it is necessary to consider the weak and simple paired field theory of the fermionic lagrangian [12].

2. RESEARCH METHOD

We propose the C-Function as a novel probe that is accurate for detecting the location of critical points in a topological quantum. The C-function uses a holographic model to demonstrate the topological quantum phase transitions between a simple topological isolating phase and a gapless Weyl semimetal. The quantum critical point displays a strong Lifshitz-like anisotropy in the spatial direction, and a quantum phase transition that does not follow Landau's standard paradigm. The C-function pinpoints the global feature of quantum criticality and distinguishes it very accurately between two separate zero-temperature phases. Considering the relationship of C-functions to entanglement entropy can detect quantum phase transitions and can be applied outside the holographic framework.

3. RESULT AND DISCUSSION

To reveal a critical point in the quantum, we must first obtain the numerical background, while maintaining a dimensionless temperature $\hat{T}$. We are particularly interested in times of zero and very low temperatures in ($\hat{T} \simeq 0.005$). Nevertheless, we have checked directly that similar results are obtained for slightly larger values ($\hat{T} \simeq 0.05; 0.1$).

To find the quantum tipping point, we compute a C-function corresponding to the entangled surface, with a boundary length large enough to extend itinto the mass. This type of entangled territory to probe into IR, and provides good accuracy in finding phase transitions. The C-function has the advantage that its values are valid and well defined even for the anisotropic phase. We can calculate this feature throughout the full phase diagram by taking advantage of this feature $\left(\underline{M}, \underline{T}\right)$.

Practically speaking, the external parameters $\hat{M}$, in the range of which can cover three different theoretical phases, are; on insulators which are trivial $\left(\hat{M} > 0.744\right)$, critical points ($\hat{M} \sim 0.744$), and the semimetal Weyl $\left(\hat{M} < 0.744\right)$.

The C-function also develops a clear pattern. As we approach the quantum tipping point, it increases and reaches a maximum value, right at the quantum tipping point with Lifshitz-like symmetry. In this sense, the C-function acts as a very accurate probe to find the tipping point of the quantum topology.

When $T - 0$ the geometry is near the horizon, the region in the background is anisotropic space-time like Lifshitz. can also be calculated analytically in conceptual geometry, this type maintains anisotropy along the flowRG. By direct application of the band formulas, we also obtain an independent cut-off term, which is as follows:

$$S_{\perp} \simeq -N^2 H_{\perp} H_{\parallel} \frac{\beta_{\perp}{}^{-1}}{l_x^{1+\frac{1}{x}}}, \tag{1}$$

$$S_{\parallel} \simeq -N^2 H_{\perp}^2 \frac{\beta_{\parallel}{}^{-1}}{l_y^{2z}}, \tag{2}$$

The dotted line in the figure above shows the position of the quantum critical point $Mc \sim 0.744$, Both functions have normalized magnitudes for presentational reasons.

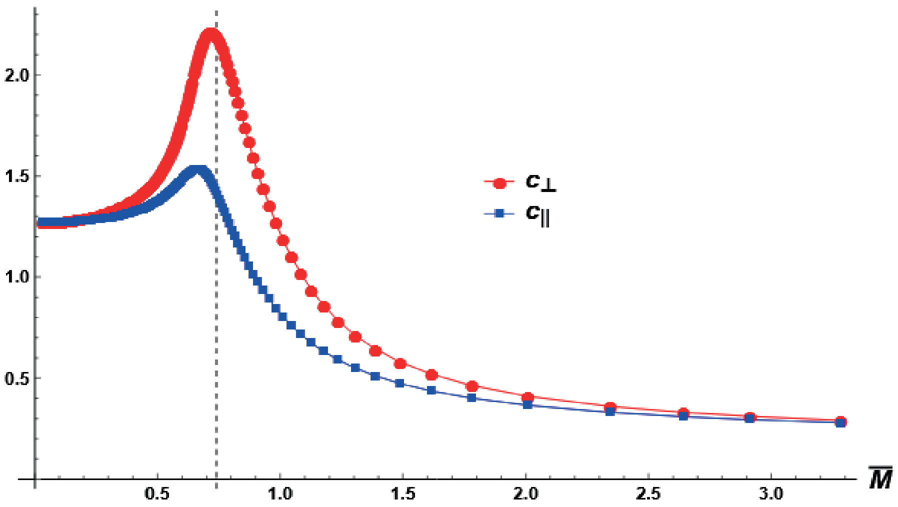


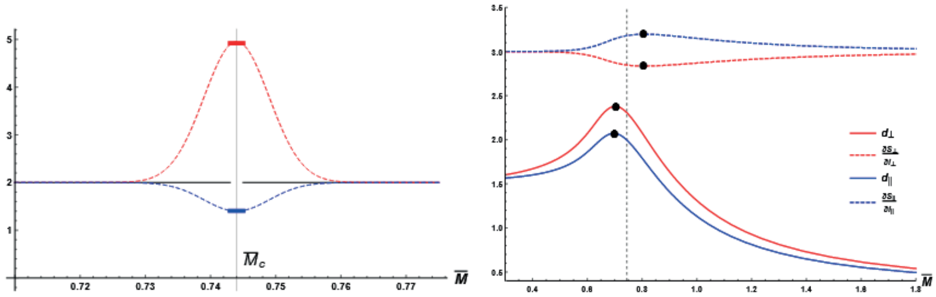
Figure 1: Parallel and transverse C-functions when $\hat{T} = 0,005$ in external parameter functions M.

With $\beta_\parallel$ and and $\beta_\perp$ is a constant that depends on the effective dimension, which goes through the gamma function.

Since we are focusing on the suitable surface with a large winding distance, we can use for the purpose of obtaining analytical results, approx. for equations (1) and (2), and the effective dimensions are determined for the region IR as in the numerical method. By extracting the C-function from the entropy of the windings, we get $c_\perp - \beta_\perp{}^{-1}(1 + \frac{1}{z})$ and $c_\parallel - \beta_\parallel{}^{-1}2z$with proper normalization. Two functions converge to $z - 1$ be a single value because $\beta_\parallel - \beta_\perp$. Far from the quantum tipping point, the constant reflects the AdS property of a fixed point for $\hat{M} \gtrless \hat{M}_c$. At the quantum tipping point M_c, the system's symmetry changes, resulting in a splitting of the C-function where $c_\perp - c_\parallel$in and its discontinuous jump in value into a new constant, each depending on the corresponding effective dimension ($d_\perp, d_\parallel) - \left(2 + \frac{1}{z}1/z,\ 1 + 2z\right)$.

This discontinuity is a characteristic signal of the quantum phase transition at zero temperature through the C-function. At this low temperature, which is also referred to in the quantum critical region. this discontinuous feature is smoothed by thermal effects, with the task of identifying the C-function crossover point. For large entangled surfaces, both C-functions detect very accurately the position of the anisotropic quantum critical point. Although the temperature effect is limited, it is a common feature reminiscent of the so-called quantum critical region, which may make experimental measurements (which are definitely impossible at zero temperature) more feasible.

To identify the adequate dimensions of numerical data. We isolate the critical exponent by calculating the derivative of the logarithmic ratio g_{11}/g_{33}, at a fixed radial distance and by considering that we can always rescale the transverse metric element to have a known isotropic scaling.



(a) (b)

Figure 2: (a) The normal value of the C-function on $T = 0$is of M, (b) The effective dimension $d_{\perp}$, and the derivative of EE $\partial S_{\parallel,\perp}/\partial l_{\perp}$ in a function of the external parameter M, around QCP.

The normal value of the C-function at time $T = 0$, is from M. The black line indicates the value of adS $c/\beta\,(d) = 2$, which is normalized by . In the figure below, the comparison scheme used here is for a constant winding length as M varies and, to normalize the magnitudes are set as presentation grounds. The red and blue lines show the value of $c_{\perp}/\beta_{\perp}(d_{\perp}), c_{\parallel}/\beta_{\parallel}\left(d_{\parallel}\right)$ at the quantum critical point where the C-function is clearly disconnected. we compare and involve shifts in the parameter M, we normalize the rest of the scale. in principle we want to investigate the theory at a certain energy for the change in M. Most of the qualitative details of function C are weak in comparison schemes, whereas its derivatives depend heavily on entanglement.

The turning point of the surface of the holographic winding varies with M. and can also normalize the length of the entangled surface with respect to the scale energy of the theory of gravity and maintain this dimensionless quantity constant. While the turning points and surface lengths vary. for M, the turning point of the holographic surface is kept constant across the black hole horizon. we obtain clear signals of phase transitions for large winding surfaces, due to deeply probing IR. The transverse C function develops maximally when it is at a critical point. On parallel units will be more sensitive to the schematic, especially for small entangled surfaces, which also still indicate the location of the critical point.

The C-function contains accurate information for signaling the inner phase transition. In fact, the phase transition information belongs to the effective dimension $d_{\parallel}$ and $d_{\perp}$, because the critical point has a Lifshitz scaling anisotropy, so it is different from the AdS phase. We show the behavior of the effective dimension and the EE derivative in the M function. The exact combination of the two, shows the position of the quantum critical point with the greatest accuracy. the larger winding surface implies to probe deeper into the IR structure. the saddle point derivative of the winding entropy gets closer to the phase transition point. For the entangled region to be larger at the boundary, the entanglement entropy receives a contribution from the thermal entropy.

The anomalous conductivity at low temperature with respect to its quartic coupling, this dashed line indicates the position of the QCP. In a small plot the points correspond to QCP which is also in relation to the quartic coupling.

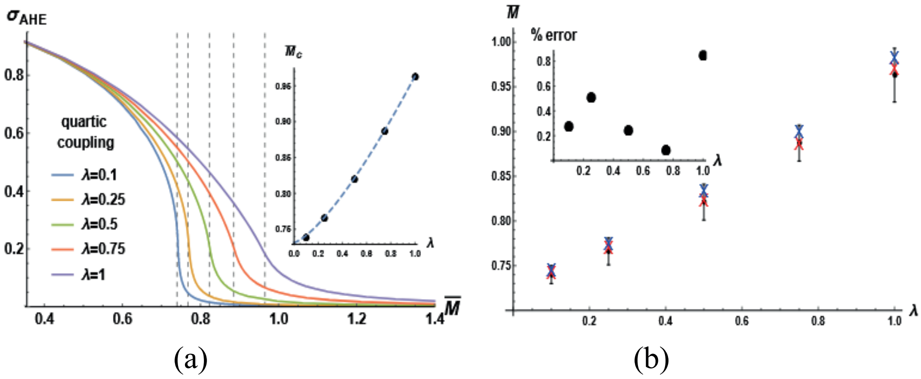


Figure 3: (a)Anomaly conductivity, (b) The X represents the QCP calculated by functions $(c_\perp, c_\parallel)$ for various quartic couplings.

The dots are the QCP system, and in the error bars there is support, due to thermal effects. for the conductivity space anomaly error is close to the benchmark value. C function can determine QCP with high accuracy. It is shown that the thermal contribution to the winding entropy does not affect the C-function for finding the probe. Thermal entropy will provide clues about phase transitions in a temperature-limited system. this approach is still valid for smaller entangled surfaces, which invariably develop a saddle point in the critical regime, developing a conductivity anomaly-like behavior, and a phase transition signal then arrives with a peak whose derivative develops. moment on a large entangled surface, the two C functions accurately signify QCP.

To better confirm the ability of the C-function to detect the quantum critical point, we performed a more detailed analysis of the theoretical parameters by changing the quartic coupling of the model λ. By increasing it, the tipping point M_c. becomes larger as shown in Figure 3, Then we calculate the saddle point of the C-function. We show the ratio between the critical point locations for various criticals and the saddle point positions of the C-function. The C-function is able to find the quantum critical point with <1% error.

Acknowledgments

With uncertainly known phase transitions, it would be interesting to investigate deeper and more subtle probes, to distinguish topological information from critical points. It is possible to consider what is meant by topological entropy, in holography. It is very important to understand beyond the qualitative argument of dof calculation, what is the origin of the fundamental c function, its peaks and, whether or not they are related to the features of the transport coefficients.

References

[1] X.-G. Wen, *Quantum field theory of many-body systems: From the origin of sound to an origin of light and electrons. Oxford University Press on Demand*, 2004.

[2] Sachdev S. “Quantum phase transitions.,” In: *Handbook of Magnetism and Advanced Magnetic Materials* (1999). https://doi.org/10.1088/2058-7058/12/4/23.

[3] Grossman B. Hierarchy of soliton solutions to the gauged nonlinear Schrödinger equation on the plane. Phys Rev Lett. 1990 Dec;65(26):3230–2.

[4] Imada M, Fujimori A, Tokura Y. Metal-insulator transitions. Rev Mod Phys. 1998;70(4):1039–263.

[5] Laughlin RB. Anomalous quantum hall effect: an incompressible quantum fluid with fractionally charged excitations. Phys Rev Lett. 1983;50(18):1395–8.

[6] J. Jacobsen, S. Ouvry, V. Pasquier, D. Serban, and L. Cugliandolo, *Exact Methods in low-dimensional statistical physics and quantum computing: Lecture notes of the les houches summer school: Volume 89, July 2008. OUP Oxford*, 2010.

[7] Casini H, Huerta M. A c-theorem for entanglement entropy. J Phys A Math Theor. 2007;40(25):7031–6.

[8] Myers RC, Sinha A. Seeing a c-theorem with holography. Phys Rev D Part Fields Gravit Cosmol. 2010;82(4):46006.

[9] Volkov B, Pankratov O. Two-dimensional massless electrons in an inverted contact. Soviet Journal of Experimental and Theoretical Physics Letters. 1985;42:178.

[10] Nielsen HB, Ninomiya M. The adler-bell-jackiw anomaly and weyl fermions in a crystal. Phys Lett B. 1983;130(6):389–96.

[11] Landsteiner K. Notes on anomaly induced transport. Acta Phys Pol B. 2016;47(12):2617.

[12] Colladay D, Kostelecký VA. Lorentz-violating extension of the standard model. Phys Rev D Part Fields. 1998;58(11):116002.