



Conference Paper

Critical Buckling Load for Lattice Column Elements with Variable Dimensions

Ahmad Alghamdi*, Martin Leary, Ma Qian, Wei Xu, and Milan Brandt

RMIT Centre for Additive Manufacture, RMIT University

Abstract

Lattice structures are used in a variety of high-value engineering applications; for example, in automobile, aerospace and biomedical applications, due to their light weight, high specific strength, stiffness, heat transfer control and energy absorption. Additive Manufacturing (AM) technologies, such as Selective Laser Melting (SLM), offer radical net-shape manufacturing solutions for metallic structures directly from digital data. The prediction of AM lattice structure mechanical properties prior to manufacture is both cost and time-consuming; particularly as existing models do not readily accommodate the effects of manufacturing defects and lattice node geometry on column buckling. The critical buckling load of columns was algebraically and numerically simulated for a full Design of Experiments (DOE) of independent variables, including column length, column radius, node radius and material type. This simulation data quantifies the effect of independent variables on critical buckling load and demonstrates the limitations of algebraically prediction. This research can be extended to allow the simulation of the load carrying capacity of entire lattice structures; and to accommodate the effect of manufacturing variation on the associated load carrying capacity of AM lattice structures.

Keywords: Column buckling, Numerical simulation of column, Analytical analysis of column, Selective Laser Melting (SLM), Electron Beam Melting (EBM), Ti-6Al-4V

Corresponding Author:
Ahmad Alghamdi; email:
destech@deakin.edu.au

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1 Introduction

According to the Additive Manufacturing Technology Standard F2792-10, additive manufacturing is the “process of joining materials to make objects from 3D model data, usually layer upon layer, as opposed to subtractive manufacturing methodologies, such as traditional machining” (1). Additive Manufacturing (AM) provides various industrial benefits, including the ability to manufacture lightweight lattice structures of high complexity with low lead time and low environmental impact (2).

In this work, lattice structures consist of numerous cylindrical elements connected by spherical nodes (Figure 1). A column is a cylindrical compression member, which is unable to support the applied compressive load when the critical buckling load, P_{cr} , is exceeded (3). Buckling is an abrupt, large deformation failure, where that column had exhibited minimal deformation before the compressive load was increased (4).

Columns can be divided into three classifications: short columns, intermediate columns and slender columns. These classifications are dependent on the column slenderness ratio, S_r , which is a relationship between the column length, L , and radius of gyration, r (Equation 1) (7). The classification of columns indicates whether structure fails due to material reasons, buckling or both (3).

$$S_r = \frac{L}{r} [\text{mm}/\text{mm}], \quad r = \sqrt{\frac{I}{A}} [\text{mm}^4/\text{mm}^2] \quad (1)$$

Short columns fail materially by stress exceeding the yield strength of the material (5, 6); this occurs when column length is very small as compared to radius of gyration. An intermediate column has a slenderness ratio between that of a short or slender column, and fails by a combination of buckling and yield (5). A slender column fails structurally by elastic buckling at stress values significantly lower than the yield strength (5, 6).

The prediction of critical buckling load P_{cr} , for columns with variable dimensions is essential to economical design of AM lattice structures, and can be predicted numerically or analytically. This paper will compare both numerical and analytical predictions of P_{cr} to provide a preliminary step towards the automated analysis of the mechanical properties of AM lattice structures. It presents the critical buckling load of columns for sizes relevant to AM lattice design. Specifically, this research provides insight into the relative merit of analytic and numerical approaches for estimating critical buckling loads, as well as providing design data for the critical buckling load for range of cases relevant to AM design.

2 Methodology

2.1 Numerical Simulation

A full factorial design of experiments (DOE) based on geometry relevant to AM lattice design has enabled six hundred columns to be simulated with varying column length, sphere radius and column radius (Figure 3). This data is used to evaluate the difference in P_{cr} prediction for analytical and numerical results; as well as providing data for AM lattice design.

A 3D numerical column model, with cylindrical structure and spherical nodes was designed to simulate a column in response to simple loading (Figure 2). A load was applied into the centre of the upper sphere to measure critical buckling load of the column using free rotation end constraints (Table 2). A custom Matlab script was developed to allow automated simulation (within Abaqus) of P_{cr} for dimensions of column radius, column length and sphere radius, that vary according to the DOE (Table 1). The material used for all simulations was Ti-6Al-4V.

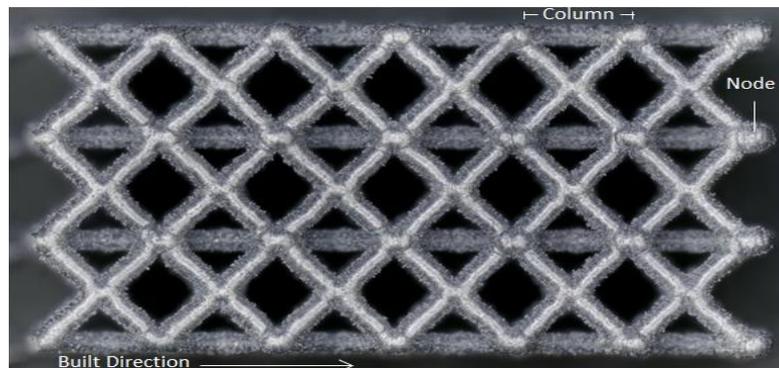


Figure 1: Image of SLM manufactured AlSi₁₂Mg Lattice Structure (10).

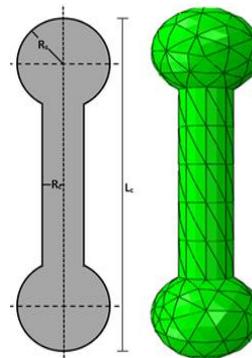


Figure 2: A 3D COLUMN SAMPLE WITH TWO SPHERES, WHERE L_c = LENGTH OF COLUMN, R_s = RADIUS OF SPHERE AND R_c = RADIUS OF COLUMN.

2.2 Analytical prediction of P_{cr}

The Euler buckling formula predicts P_{cr} , for a slender compression member (Equation 2); based on the Modulus of elasticity, E , second moment of area, I , column length, L , and length factor, k (7). The length factor of a column varies according to the boundary conditions: both ends fixed, both ends pinned, fixed and pinned, and fixed and free (Table 1) (9).

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} \quad (2)$$

Euler provides rapid prediction of P_{cr} , however there exist several challenges to the use of Euler in AM lattice structural design. Euler Buckling is limited to scenarios with straight columns with length that is significantly larger than the column radius (8). However, practically manufactured AM column structures often include manufacturing effects that mean columns are not perfectly straight or are intentionally in the short or intermediate range. Furthermore, the connection between columns is achieved by spherical nodes, which introduce stress concentration and change in effective length that conflict with the assumptions of Euler and affect predictions of P_{cr} . This research provides insight into the relative merit of analytic and numerical approaches for estimating critical buckling loads for geometries relevant to AM design.

Parameter	Range		Step
	Min	Max	
Length of Column (L_c)	0.5 mm	10 mm	0.5 mm
Radius of Sphere (R_s)	0.125 mm	$\frac{L_c}{4}$ mm	0.2 mm
Radius of Column (R_c)	0.125 mm	$\frac{L_c}{4}$ mm	0.2mm

TABLE 1: SUMMARY OF ALLOWABLE RANGES FOR VARYING L_c, R_s, R_c . NOTE: $R_s \geq R_c$.

Buckled Shape						
Theoretical Effective Length	0.5	0.7	1.0	2.0	2.0	1.0
Recommended	0.65	0.8	1.0	2.0	2.10	1.2
Symbols For End Conditions	= Rotation fixed, translation fixed = Rotation free, translation fixed			= Rotation free, translation fixed = Rotation fixed, translation fixed		

TABLE 2: Effective length of a single column with different end boundaries (7).

3 Results

Numerical simulation and analytical prediction of P_{cr} was completed for the DOE of (Table 1). Data for 5mm column length is presented in Figure 3, and indicates that the results are divided into three sections: low, medium and high error percentage.

Low error percentage: The numerical and algebraic prediction of P_{cr} show low error when the column radius and sphere radius are equal (Figure 3a). The error percentage increases from approximately 0.2% to 2.0% as the column radius increases from the lowest to highest radius. This increasing error corresponds to a reduction in slenderness ratio, which is contradictory to the assumptions of Euler buckling model.

Medium error percentage: A medium error percentage of approximately 2% to 30% occurs when the sphere radius exceeds the column radius, and is less than approx-

imately fifteen percent of the column length (Figure 3b). This increase is due to the effect of the node spheres introducing stress concentration and increasing local resistance to buckling, thereby contradicting the assumptions of the Euler buckling model.

High error percentage: The largest error percentage occurs when the sphere radius larger than column radius and exceeds approximately fifteen percent of the column length (Figure 3c). The error percentages increase from approximately 30% to over 300% as sphere radius increased.

4 Summary and Conclusion

Notwithstanding the research that has been undertaken of the limitations of analytical prediction of P_{cr} , the limitations documented in this research occurred when minimum column radius and maximum sphere radius applied. This limitation produces a large error percentage between numerical analysis and analytical analysis when sphere radius is large. The prediction of mechanical properties of columns with variable column radius, sphere radius and column length before fabrication, is attempting to reduce the cost and time consumption of fabrication. This research is first to allow simulation of entire Lattice Structures. In summary:

- Simulation of columns under varying geometric conditions is important to enable the robust manufacture of lattice structures, reduce time consumption and cost.
- The numerical simulation model of columns was a cylinder column with two spherical nodes generated into vertex and bottom of column.
- The numerical simulation was running a varying range of spherical node radius, column radius and column length automatically (Table 1).
- Analytical analysis was used to validate the results of numerical simulation models with set dimensions.
- The error percentage between numerical and analytical estimates of P_{cr} show three classifications: low error percentage (approximately less than 2%), medium error percentage (approximately from 2% to 30%), and high error percentage (from 30% to 300%).
- These errors are consistent with geometric effects (such as stress concentrations, local stiffening and low slenderness) that are contradictory to the fundamental assumptions of the Euler buckling model.
- This outcome provides insight on the applicability of analytic and numerical approaches for estimating critical buckling loads within AM lattice structures, as well as providing design data for the critical buckling load for range of cases relevant to AM design.

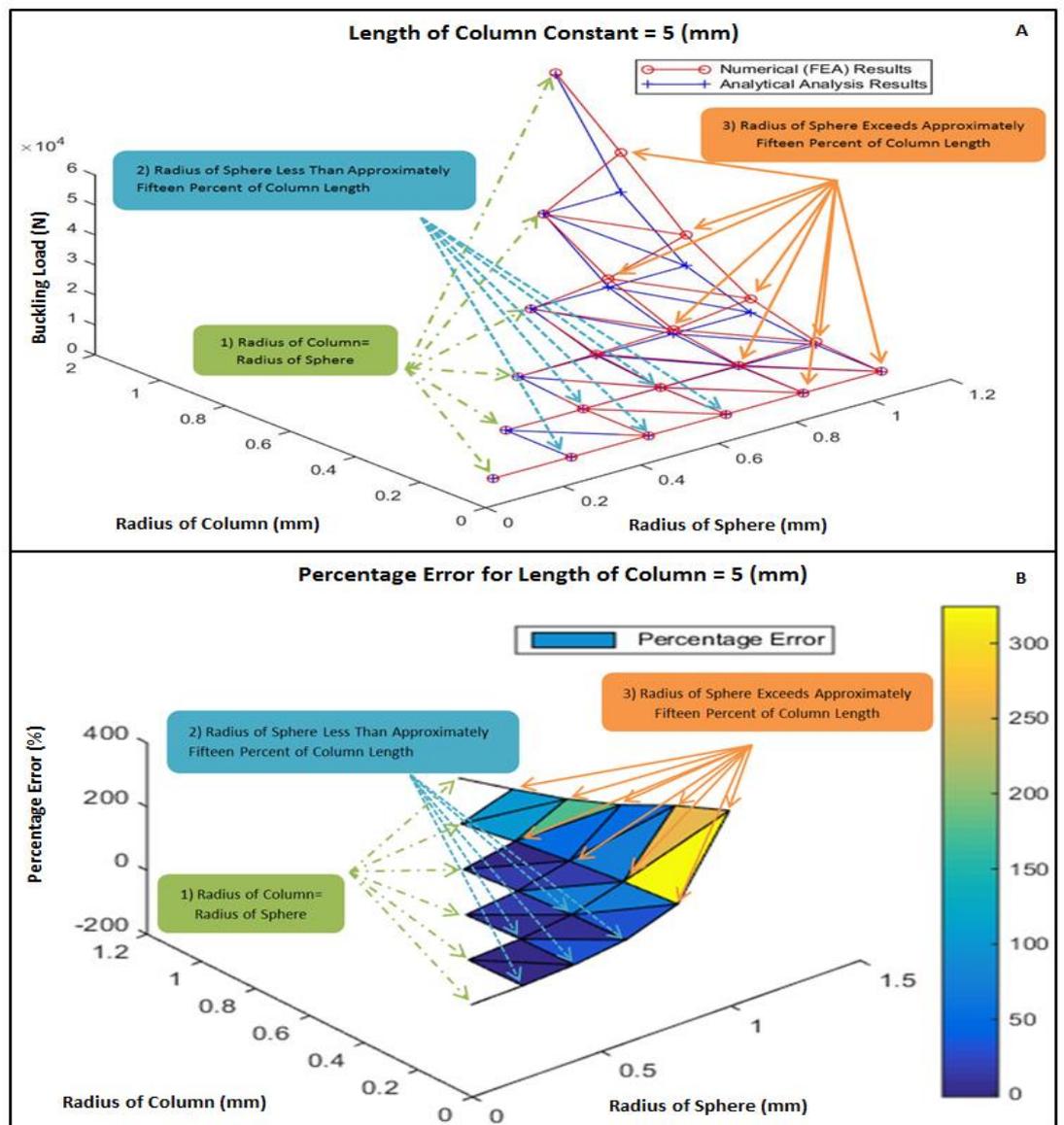


Figure 3: a) Data of numerical and analytical predation of P_{cr} ; b) percentage error between numerical and analytical predation of P_{cr} . Both a and b have column length of 5 mm. (1) Low error percentage: radius of column equal to radius of sphere; (2) Medium error percentage: radius of sphere less than approximately fifteen percent of column length and radius of column variable; (3) High error percentage: radius of sphere exceeds approximately fifteen percent of column length and radius of column variable.

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