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# Binary Composite Fiber Elasticity Using Spring-Mass and Non-Interacting Parallel Sub-Fiber Model

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## Abstract

Composite materials have been investigated elsewhere. Most of the studies are based on experimental results. This paper reports a numerical study of elasticity modulus of binary fiber composite materials. In this study, we use binary fiber composite materials model which consists of materials of types A and B. The composite is simplified into compound of non-interacting parallel sub-fibers. Each sub-fiber is modeled as $N_s$ point of masses in series configuration. Two adjacent point of mass is connected with spring constant $k$ (related and proportional to Young Modulus $E$), where it could be $k_{AA}$, $k_{AB}$, or $k_{BB}$ depend on material type of the two point of masses. Three possible combinations of spring constant are investigated: (a) [$k_{AB}$ < min($k_{AA}$, $k_{BB}$)], (b) [min($k_{AA}$, $k_{BB}$) < $k_{AB}$ < max($k_{AA}$, $k_{BB}$)], and (c) [max($k_{AA}$, $kBB$) < $k_{AB}$]. The combinations are labeled as composite type I, II, and III, respectively. It is observed that only type II fits most the region limited by Voight and Reuss formulas.

**Keywords:** composite materials, one-dimension, spring, combination, sequence

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## 1. Introduction

One of the important things in studies of composite materials is the relationship between mechanical properties of the composites with that of its components. Although many studies on composite materials are based on experimental results, but theoretical or numerical studies on mechanical properties of composites are also important.

This study is a numerical study of elasticity modulus of binary fiber composite materials, i.e. binary composite materials. In the binary composite model used, two adjacent point of mass is connected with spring constant $k$ (related and proportional to Young Modulus $E$). This study focuses on the relationship between mechanical properties of the composites with that of its components through analysis on spring constant $k$.

Previously, binary composite materials have been discussed in 3-d experiment [1] and 2-d simulated based on granular particles [2], where the mixture could be homogenously dispersed [1] or the components can still be differed [3]. A model of binary composite materials are proposed in this work and compared to the Voight and Reuss formulas, which has been common as benchmark in studying composite models [4].

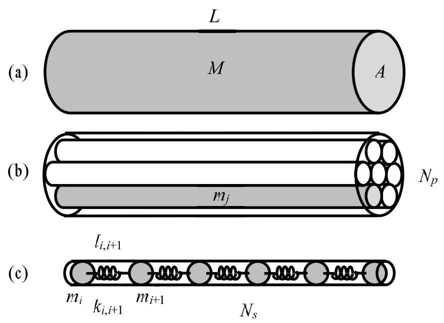



**Figure** 1: (a) A composite fiber with cross section $A$, mass $M$, and length $L$. (b) A bundle of $N_p$ paralel non-interacting sub fibers. (c) Each sub-fiber consists of $N_s$ point masses and $N_s$−1 springs.

## 2. Theory

A composite fiber can be modeled as a bundle of $N_p$ parallel non-interacting sub-fibers as shown in Figure 1, where each sub-fiber consists of serial arrangements of point masse and spring systems.

In sub-fiber $j$ with mass $m_j$ there are $N_s$ point masses and $N_s$−1 springs. Spring with spring constant $k_{i,i+1}$ and length $l_{i,i+1}$ connects point masses $m_i$ and $m_{i+1}$.

Mass of the whole composite fiber is

$$M = \sum_{j=1}^{N_p} m_j = \sum_{j=1}^{N_p} \sum_{i=1}^{N_s} m_{ji} \tag{1}$$

with $m_{ij}$ is point mass $i$ in sub-fiber $j$.

Supposed that the composite fiber of material X has elastic modulus $E_X$ with relation between stress $F$, strain $\Delta L$, initial length $L$, and cross section $A$.

$$\frac{F}{A} = E_{\mathrm{X}} \frac{\Delta L}{L} \tag{2}$$

and Hooke's law for elastic system

$$F = k_{\mathrm{X}} \Delta L, \tag{3}$$

then a relation can be derived

$$k_{\mathrm{X}} = E_{\mathrm{X}} \frac{A}{L}. \tag{4}$$

Spring constant of fiber of material X is constructed from parallel arrangement of sub-fibers with spring constant $k_j$

$$k_{\mathrm{X}} = \sum_{j=1}^{N_p} k_j = N_p k_j \tag{5}$$

| Material | Elastic modulus | Spring constant | Type of connected point masses | | Lennard-Jones (12,6) constants |
|---|---|---|---|---|---|
| | | | $m_i$ | $m_{i+1}$ | |
| A | $E_A$ | $k_{AA}$ | A | A | $\varepsilon_{A-A}$, $\sigma_{A-A}$ |
| B | $E_B$ | $k_{BB}$ | B | B | $\varepsilon_{B-B}$, $\sigma_{B-B}$ |
| Composite | $E_{comp}$ | $k_{AB}$ = $k_{BA}$ | A | B | $\varepsilon_{A-B}$, $\sigma_{A-B}$ |
| | | | B | A | |

Table 1: Spring constants connecting two adjacent point masses, the elastic modulus, and Lennard-Jones (12,6) constants.

or

$$k_j = \frac{k_\mathrm{X}}{N_p}. \tag{6}$$

This $k_j$ in a sub-fiber $j$ is built of serial arrangement of spring constants $k_{i,i+1}$ which connects point masses $i$ and $i$+1

$$\frac{1}{k_j} = \sum_{i=1}^{N_s-1} \frac{1}{k_{i,i+1}} = \frac{N_s - 1}{k_{i,i+1}} \tag{7}$$

or

$$k_{i,i+1} = \left(N_s - 1\right) k_j. \tag{8}$$

Substituting Equation 7 into Equation 9 will give

$$k_{i,i+1} = \frac{\left(N_s - 1\right)}{N_p} k_\mathrm{X}. \tag{9}$$

In this work only binary composite is considered, e.g. materials A and B. Three types of spring constants would be sufficient for the binary composite system $k_{AA}$, $k_{AB}$, and $k_{BB}$, where each spring constant connects point masses of types A-A, A-B (or B-A), and B-B, respectively. Table 1 give clearer picture of relation between spring constants $k_{i,i+1}$ and materials composing the composite fiber.

The spring constant $k_{i,i+1}$ can be considered as approximation of Lennard-Jones (12,6) potential [Jones, 1924] about its separation distance $r_m$ which produces minimum potential. The potential is commonly expressed as

$$V_\mathrm{LJ} = 4\varepsilon \left[ \left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6} \right] = \varepsilon \left[ \left(\frac{r_m}{r}\right)^{12} - 2\left(\frac{r_m}{r}\right)^{6} \right], \tag{10}$$

where $\varepsilon$ is the depth of potential well, $\sigma$ is the finite distance at which the inter-particle potential is zero, and $r$ is distance between a pair of atoms or molecules. Equation 10 can be approximated using

$$V_{\mathrm{LJ}} \approx V_{\mathrm{LJ}}\Big|_{r=r_m} + \frac{dV_{\mathrm{LJ}}}{dr}\Big|_{r=r_m}\left(r-r_m\right) + \frac{d^2V_{\mathrm{LJ}}}{dr^2}\Big|_{r=r_m}\frac{\left(r-r_m\right)^2}{2!}, \tag{11}$$

which produces

$$V_{\mathrm{LJ}} \approx -\varepsilon + \frac{36\varepsilon}{r_m^2}\left(r-r_m\right)^2. \tag{12}$$

Position $r = r_m$ can be chosen as local coordinate for equilibrium position and by shifting potential reference, so that Equation 12 can be adjusted to

$$V = \frac{36\varepsilon}{r_m^2} r^2 = \frac{1}{2} k_{i,i+1} r^2 \tag{13}$$

with $r_m = 2^{1/6}\sigma$. Then, it can be obtained that

$$k_{i,i+1} = \frac{72\varepsilon}{r_m^2} = \frac{72\varepsilon}{2^{1/3}\sigma^2}. \tag{14}$$

Value of $\varepsilon$ and $\sigma$ for pair of atoms, molecules, or cluster of molecules are already common [5-7]. Equations 10-14 are shown only for showing that it is possible to obtain $k_{AA}$, $k_{AB}$, and $k_{BB}$ based on materials molecular interactions. In this work values of $k_{AA}$ and $k_{BB}$ will be proportional to $E_A$ and $E_B$ according to Equation 4 and $k_{AB}$ will be a adjustable parameter.

Concentration of composite materials is defined as

$$c = \frac{N_{\mathrm{B}}}{N_{\mathrm{A}} + N_{\mathrm{B}}}, \tag{15}$$

which means that $c$ = 0 is for pure material of type A, while $c$ = 1 is for pure material of type B.

## 3. Results and Discussion

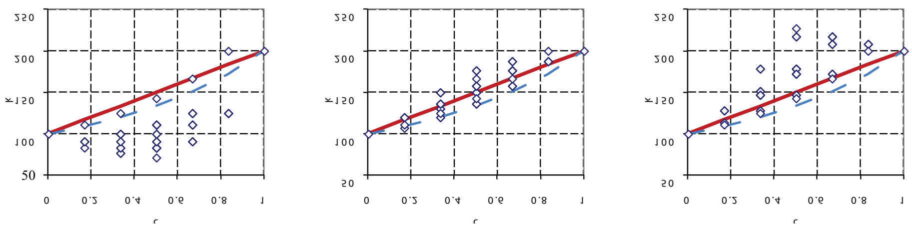
Configurations of composite materials with $N_p$ = 1 and $N_s$ = 2–4 are given in Table 2, which shows that higher value of $N_s$ will give smoother values of various concentration $c$. Probable occurring sequences $S$ for each value of $N_s$ are given. For $N_p > 1$ value of $k_{i,i+1}$ for certain concentration $c$ will be fallen between minimum and maximum values of the $k$'s of the all sequences at the concentration $c$.

Prediction of the well-known rules of mixture from Voight and Reuss [4] is shown in Figure 2 as solid (red) and dashed (blue) lines for isostrain and isostress conditions, respectively. The rules give upper and lower bounds for composite materials elasticity. The proposed model in this work, which is calculated using a spreadsheet-software, can go beyond these bounds by adjusting the parameter $k_{AB}$. Some sequences $S$ are still between those two bounds, especially most in the type II. Materials of type I represents adhesive force is less than cohesive force, while type III represent adhesive force is more than cohesive force.

In the future, a rule how to select possible occurring sequence $S$ should be defined, i.e. why some composite materials could be type I, II, or III.

| $N_s$ | $c$ | $S$ | $k_{i,i+1}$ | | $N_s$ | $c$ | $S$ | $k_{i,i+1}$ |
|---|---|---|---|---|---|---|---|---|
| 2 | 0 | AA | $k_{AA}$ | | 4 | 0 | AAAA | $k_{AA}$ $k_{AA}$ $k_{AA}$ |
| | 0.5 | AB | $k_{AB}$ | | | 0.25 | AAAB | $k_{AA}$ $k_{AA}$ $k_{AB}$ |
| | 0.5 | BA | $k_{AB}$ | | | | AABA | $k_{AA}$ $k_{AB}$ $k_{AB}$ |
| | 1 | BB | $k_{BB}$ | | | | ABAA | $k_{AB}$ $k_{AB}$ $k_{AA}$ |
| | | | | | | | BAAA | $k_{AB}$ $k_{AA}$ $k_{AA}$ |
| | | | | | | 0.5 | AABB | $k_{AA}$ $k_{AB}$ $k_{AA}$ |
| $N_s$ | $c$ | $S$ | $k_{i,i+1}$ | | | | ABAB | $k_{AB}$ $k_{AB}$ $k_{AB}$ |
| 3 | 0 | AAA | $k_{AA}$ $k_{AA}$ | | | | ABBA | $k_{AB}$ $k_{BB}$ $k_{AB}$ |
| | 0.33 | AAB | $k_{AA}$ $k_{AB}$ | | | | BAAB | $k_{AB}$ $k_{AA}$ $k_{AB}$ |
| | | ABA | $k_{AB}$ $k_{AB}$ | | | | BABA | $k_{AB}$ $k_{AB}$ $k_{AB}$ |
| | | BAA | $k_{AB}$ $k_{AA}$ | | | | BBAA | $k_{BB}$ $k_{AB}$ $k_{AA}$ |
| | 0.67 | ABB | $k_{AB}$ $k_{BB}$ | | | 0.75 | ABBB | $k_{AB}$ $k_{BB}$ $k_{BB}$ |
| | | BAB | $k_{AB}$ $k_{AB}$ | | | | BABB | $k_{AB}$ $k_{AB}$ $k_{BB}$ |
| | | BAA | $k_{AB}$ $k_{AA}$ | | | | BBAB | $k_{BB}$ $k_{AB}$ $k_{AB}$ |
| | 1 | BBB | $k_{BB}$ $k_{BB}$ | | | | BBBA | $k_{BB}$ $k_{BB}$ $k_{AB}$ |
| | | | | | | 1 | BBBB | $k_{BB}$ $k_{BB}$ $k_{BB}$ |

TABLE 2: Possible sequences $S$ of a sub-fiber for $N_s$ = 2–4 and its spring constant $k_{i,i+1}$ types.



(a) (b) (c)

**Figure** 2: Composite spring constant $k$ ($N_p$ = 1, $N_s$ = 4) as function of concentration $c$ for: (a) type I with $k_{AA}$ = 100, $k_{AB}$ = 50, $k_{BB}$ = 200, (b) type II with $k_{AA}$ = 100, $k_{AB}$ = 150, $k_{BB}$ = 200, and (c) type III with $k_{AA}$ = 100, $k_{AB}$ = 250, $k_{BB}$ = 200, with lower (dashed line) and upper (solid line) bounds.

## 4. Conclusions

A model to predict elasticity of composite materials based on spring-mass system has been conducted in this work. It can give bounds beyond upper and lower bounds predicted from Voight and Reuss formulas. Composite material of type II [min($k_{AA}$, $k_{BB}$) < $k_{AB}$ < max($k_{AA}$, $k_{BB}$)] is the most fitted to the formulas.

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