Conference Paper

Bayesian Estimation of Spatial Regression Models with Skew-normally Covariates Measured with Errors: Evidence from Monte Carlo Simulations

Mohammad Masjkur[1] and Henk Folmer[2]

[1]Bogor Agricultural University, Faculty of Mathematics and Natural Sciences, Department of Statistics, Kampus IPB Darmaga, Bogor, 16680, Indonesia
[2]University of Groningen, Faculty of Spatial Sciences, The Netherlands

Abstract

Spatial data are susceptible to covariates measured with errors. However, the error-prone covariates and the random errors are usually assumed to be symmetrically, normally distribution. The purpose of this paper is to analyze Bayesian inference of spatial regression models with a covariate measured with Skew-normal error by way of Monte Carlo simulation. We consider the spatial regression models with different degree of spatial correlation in the covariate of interest and measurement error variance. The simulation examines the performance of Bayesian estimators in the case of (i) Naive models without measurement error correction; (ii) Normal distribution for the error-prone covariate and random errors; (iii) Skew-normal distribution (SN) for the error-prone covariate and normal distribution for random errors. We use the relative bias (RelBias) and Root Mean Squared Error (RMSE) as valuation criteria. The main result is that the Skew-normal prior estimator outperform the normal, symmetrical prior distribution and the Naive models without measurement error correction.

Keywords: Spatial regression, measurement error, Bayesian analysis, Skew-normal distribution

Corresponding Author:
Mohammad Masjkur
masjkur@apps.ipb.ac.id

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1. Introduction

The spatial data are typically collected from points or regions located in space and thus tend to be spatially dependent. Ignoring the violation of spatial independence between observations will produce estimates that are biased, inconsistent or inefficient. A large variety of spatial models to take spatial dependence among observations into account have been developed [1-3].

Measurement errors in the spatially lagged explanatory variables is are not routinely accounted for, in spite of the fact that their consequences are serious. The estimator of the coefficients spatially lagged exogenous variables are attenuated, while the estimator of the variance components are inflated, if covariate measurement error is ignored [4]. However, the amount of attenuation depends on the degree of spatial correlation in both the true covariates and the random error term of the regression model [5].

Several approaches to correct for measurement error in spatially lagged exogenous regressors have been proposed in literature. The Maximum Likelihood (ML) based on an Expectation-Maximization EM algorithm correct the biases in the estimators of the naive estimator, i.e. the estimator that ignore the measurement error, but are associated with larger variances [4]. Another approaches adjusting the estimates by means of an estimated attenuation factor obtained by the method of moments, or using an appropriate transformation of the error prone covariate [5]. Additionally, a semiparametric approach i.e. penalized least squares to obtain a bias-corrected estimator of the parameters could be as an alternative [6].

The error-prone covariates and the random errors are usually assumed to be symmetrically, normally distribution [4-6]. However, the assumption of normality may be too restrictive in many applications [7, 8]. The linear models with Skew-normal measurement error models perform better when there is evidence of departure from symmetry or normality [7]. Furthermore, the Skew-normal linear mixed measurement error outperform the normal mixed measurement error model when the actual covariate distribution has a Skew-normal [8].

Among several approaches to correct for measurement error, Bayesian methods provide the most flexible framework. The advantage of Bayesian approaches is that prior knowledge, and in particular prior uncertainty of error variance can be incorporated in the model. While frequentist approaches require fixing the regression coefficients and the variance components parameters to guarantee identifiability, the Bayesian setting allows to represent uncertainty with suitable prior distributions [9].

The purpose of this paper is to analyze Bayesian inference of spatial regression models with covariate measured with Skew-normal error by way of Monte Carlo simulation.

2. Materials and Methods

2.1. The spatial linear model with measurement error

A spatial regression model defined as follows:

Let x_i represents the error prone true covariate for spatial unit i, i=1, ...,n, and is related to the response y_i corresponding to a linear model:

$$y_i = \beta_0 + \beta_x x_i + \varepsilon_i \tag{1}$$

where $\varepsilon = (\varepsilon_1, \dots \dots ., \varepsilon_n)^T \sim N(0, \Sigma_\varepsilon)$ and Σ_ε is a covariance matrix with a spatial structure. Suppose q_i the observed error prone covariate for spatial unit i related to the true covariate x_i according to a classical measurement error model:

$$q_i = x_i + u_i \tag{2}$$

where $u = (u_1, \dots \dots ., u_n)^T \sim N(0, \Sigma_u)$. When x is also a normally distributed (say with mean μ_x and covariance Σ_x), then $y = (y_1, \dots \dots ., y_n)^T$ and $q = (q_1, \dots \dots ., q_n)^T$ have a multivariate normal distribution,

$$\begin{pmatrix} y \\ q \end{pmatrix} \sim MVN \left(\begin{pmatrix} \left(\beta_0 + \beta_x \mu_x\right) 1 \\ \mu_x 1 \end{pmatrix}, \begin{pmatrix} \Sigma_\varepsilon + \beta_x^2 \Sigma_X & \beta_x \Sigma_X \\ \beta_x \Sigma_X & \Sigma_X + \Sigma_U \end{pmatrix} \right)$$

where 1 is an $n \times 1$ vector of 1's. The $(y \mid q)$ is normally distributed with conditional mean

$$E\left(y \mid q\right) = \beta_0 1 + \beta_x \left(I - \Lambda\right) \mu_x + \beta_x \Lambda q \tag{3}$$

and conditional variance

$$\text{Var}\left(y \mid q\right) = \Sigma_\varepsilon + \beta_x^2 (I - \Lambda) \Sigma_X$$

where

$$\Lambda = \Sigma_X \left(\Sigma_X + \Sigma_U\right)^{-1} \tag{4}$$

These results indicate that the regression coefficients obtained by regressing the response y on the observed, but measured with error, covariate q are biased. The same holds for the conditional variance [5].

2.2. Bayesian analysis of measurement error

The joint density of all relevant variables of measurement error model (1) can be factored as

$$f\left(y, x, q \mid \theta_R, \theta_M, \theta_E\right) = f\left(y \mid x, \theta_R\right) f\left(q \mid x, y, \theta_M\right) f(x \mid \theta_E) \tag{5}$$

where $\theta = (\theta_R, \theta_M, \theta_E)$ is the vector of the model parameters. The first term on the right hand side of (5) known as the outcome model, represents the relationship between the response y and the true covariate x. The vector, θ_R is the regression parameters in the outcome model. The second term is the measurement error model, and the third term is the covariate (exposure) model.

In the presence of measurement error, we observe (y, q) instead of (y, x), hence

$$f\left(y, q \mid \theta_R, \theta_M, \theta_E\right) = \int f\left(y, x, q \mid \theta_R, \theta_M, \theta_E\right) dx \tag{6}$$

is required to form the likelihood. In some cases, this integral does not have a closed form. However, the Bayes MCMC approach can be applied with (5) and works with the integral in (6) only implicitly [10].

2.2.1. Posterior distribution

Furthermore the equation (5) can be written as

$$f(y, x, q, \theta) = \prod_{i=1}^{n} f\left(y_i \mid x_i, \theta_R\right) f\left(q \mid x_i, \theta_M\right) f\left(x_i \mid \theta_E\right) \times \pi(\theta_R, \theta_M, \theta_E) \tag{7}$$

where $\pi(\theta_R, \theta_M, \theta_E)$ is the prior distribution of the model parameters. The joint posterior density for the unknown θ and x conditional on the observed response data and surrogate covariate values (y, q) is given by

$$f\left(x, \theta \mid y, q\right) \propto \left[\prod_{i=1}^{n} f\left(y_i \mid x_i, \theta_R\right) f\left(q \mid x_i, \theta_M\right) f\left(x_i \mid \theta_E\right)\right] \times \pi(\theta_R, \theta_M, \theta_E) \tag{8}$$

Given the joint posterior distribution, it is straightforward to derive the full posterior conditional for each unobserved quantity given the observed quantities and the remaining unobserved quantities. The Bayesian inference can then be carried out based on the posterior conditionals by applying appropriate MCMC algorithms [10].

2.2.2. Skew-normal covariate model

In this paper we extend the above measurement error model (2) by considering that the covariate follow a Skew-normal distribution. The univariate Skew-normal distribution with location parameter μ, scale parameter σ^2 and skewness parameter γ is defined as:

$$f\left(x; \mu, \sigma^2, \gamma\right) = 2\phi\left(\frac{x-\mu}{\sigma}\right)\Phi\left(\gamma\frac{x-\mu}{\sigma}\right), x, \mu, \gamma \epsilon R, \sigma > 0 \tag{9}$$

where $\phi(.)$ and $\Phi(.)$ denote the probability density function and cumulative distribution function of the normal distribution, respectively. The distribution is denoted as $SN(\mu, \sigma^2, \gamma)$. A random variable $Z = \frac{x-\mu}{\sigma}$ following a standard Skew-normal distribution with μ=0 and $\sigma^2 = 1$, which is denoted as SN(γ) [11].

The Skew-normal distribution has the following properties,

1. $E(X) = \mu + \sqrt{\frac{2}{\pi}\frac{\gamma}{\sqrt{1+\gamma^2}}}$,
2. $Var(X) = \left(1 - \frac{2\gamma}{\pi(1+\gamma^2)}\right)\sigma^2$,
3. $\upsilon = \frac{1}{2}(4-\pi)\left(\frac{E^2(X)}{Var(X)}\right)^{\frac{3}{2}}$ and $\kappa = 2(\pi-3)\left(\frac{E^2(X)}{Var(X)}\right)^2$ where υ and κ are asymmetry and kurtosis indexes, respectively.
4. If $\gamma = 0$ then $X \sim N(\mu,\ \sigma^2)$,
5. If $Z \sim\ SN(\gamma)$ then $Z \overset{d}{\Leftrightarrow} \frac{\gamma}{\sqrt{1+\gamma^2}}\left|Z_0\right| + \frac{1}{\sqrt{1+\gamma^2}}Z_1$

where Z_0 and Z_1 are $iidN\ (0,\ 1)$ random variables and $\overset{d}{\Leftrightarrow}$ means "distributed as" [7, 8].

2.3. Simulation

We consider the spatial regression model as follows,

$$Y = \alpha + X\beta + \varepsilon \tag{10}$$

with Y the response; α the intercept, X the single true covariates with coefficients β, and ε the error term. The unobserved true covariate X was generated spatially autocorrelated by means of spatial weight matrix W, i.e., X = λWX + ϵ, where the weight w_{ij} is 1 if areas i and j are neighbors and 0 otherwise, λ the spatial dependence parameter [12].

We assume that

$$Q = X + U \tag{11}$$

where Q is the observed covariates related to the true covariates X according to a classical measurement error model with $U \sim N\left(0,\ \sigma_U^2\right)$. We assume $X \sim SN\left(\mu_x,\ \sigma_x^2, \gamma_x\right)$ with $\mu_x = 0, \sigma_x^2 = 1$, and $\gamma_x = 3$.

We take the data to be on a regular grid with the grid size to be 7 $(n = 7x7)$, $10(n = 10x10)$ and $20(n = 20x20)$ representing small, medium and large sample sizes. The weights matrix W is row normalized. We allow three different values for λ, namely 0.3, 0.6, and 0.9 for a weak, medium, and strong spatial dependence [13]. The observed error-prone covariate Q is generated by adding Gaussian noise with variance $\sigma_U^2 = 0.1, 0.3$ and 0.7 to X. Outcome data, Y are then generated with slope and intercept parameters set at $(\alpha,\ \beta)^T = (1,\ 2)^T$. We further take ε $\sim N\left(0,\ \sigma_\varepsilon^2\right)$ with $\sigma_\varepsilon^2 = 1$.

For each sample size (T), λ and σ_U^2, we generate 100 Monte Carlo simulation datasets. For each generated dataset, the Spatial Regression Models are estimated under the assumption of

1. Naive models without measurement error correction
2. Normal distribution for the error-prone covariate $X \sim N\left(\mu_x,\ \sigma_x^2\right)$ and random errors, $\varepsilon\ \sim N\left(0,\ \sigma_\varepsilon^2\right)$.
3. Skew-normal distribution for the error-prone covariate $X \sim SN\left(\mu_x,\ \sigma_x^2, \gamma_x\ \right)$ and Normal distribution for random errors, $\varepsilon\ \sim N\left(0,\ \sigma_\varepsilon^2\right)$.

The following independent priors were considered to perform the Gibbs sampler, $\alpha,\ \beta\ \sim N\,(0,\ \ 100)\,, \sigma_\varepsilon^2 \sim\ IG\,(0.01,\ 0.01)\,, \sigma_U^2 \sim\ IG\,(0.01,\ 0.01)\,, \mu_x \sim\ N\,(0,\ 1000)\,, \sigma_x^2 \sim IG\,(0.01,\ 0.01)$. For these prior densities, we generated three parallel independent runs of the Gibbs sampler chain of size 25 000 for each parameter. We disregarded the first 5 000 iterations to eliminate the effect of the initial value. We assessed chain convergence using the Brooks-Gelman-Rubin scale reduction factor ($\hat{R}$). The $\hat{R}$ approximately 1 indicates convergence [14]. We estimate the models using the R2jags package available in R [15].

For each simulation, we compute the relative bias (RelBias) and the Root Mean Square Error (RMSE) for each parameter estimate over 100 samples. These statistics are defined as

$$RelBias\ (\beta) = \frac{1}{k}\sum_{j=1}^{k}\left(\frac{\widehat{\beta}_j}{\beta} - 1\right), \quad RMSE\ (\beta) = \sqrt{\frac{1}{k}\sum_{j=1}^{k}(\widehat{\beta}_j - \beta)^2}$$

where $\widehat{\beta}_j$ is the estimate of β for the jth sample and k=100.

We also compare the models based on the expected Akaike information criterion (EAIC) and the expected Bayesian information criterion (EBIC). The EAIC and EBIC can be estimated using MCMC output as follows

$$\widehat{EAIC} = \overline{\mathscr{D}} + 2p, \quad \widehat{EBIC} = \overline{\mathscr{D}} + p\log\,(T)$$

where $\overline{\mathscr{D}}$ is the posterior mean of the deviance, p the number of parameters in the model, T the total number of observations [16].

3. Results and Discussion

Tables 1, 2 and 3 show that for the Spatial regression model and Skew-normal data, the average RelBias (in absolute value) and the average RMSE for all T, λ_X, three measurement error variance and the coefficient β_x of the normal prior (N-N) are quiet similar to the Skew normal prior (SN-N). However, for the Naive model are larger than for the normal (N-N) and Skew normal prior (SN-N).

Table 1: RelBias and RMSE of the Naïve, Normal (N-N), and Skew Normal (SN-N) prior for the Spatial regression model with measurement error variance 0.1.

		Prior					
T	λ_X	Naive		N-N		SN-N	
		RelBias	RMSE	RelBias	RMSE	RelBias	RMSE
49	0.3	-0.185	0.4411	-0.0088	0.2317	-0.0087	0.2323
	0.6	-0.1495	0.3542	-0.0049	0.1626	-0.005	0.1628
	0.9	-0.0488	0.169	0.0149	0.1236	0.0149	0.1224
100	0.3	-0.1957	0.4249	-0.0136	0.1509	-0.0133	0.1508
	0.6	-0.1351	0.3051	0.0036	0.1334	0.0034	0.1335
	0.9	-0.0637	0.1669	-0.0059	0.0891	-0.0058	0.0895
400	0.3	-0.1804	0.3699	-0.0032	0.073	-0.0034	0.0731
	0.6	-0.1403	0.2885	0.0001	0.053	0.0002	0.0528
	0.9	-0.0531	0.1154	0.0011	0.0365	0.001	0.0364
Average		-0.1280	0.2928	-0.0019	0.1171	-0.0019	0.1171

Table 2: RelBias and RMSE of the Naïve, Normal (N-N), and Skew Normal (SN-N) prior for the Spatial regression model with measurement error variance 0.3.

		Prior					
T	λ_X	Naive		N-N		SN-N	
		RelBias	RMSE	RelBias	RMSE	RelBias	RMSE
49	0.3	-0.3888	0.8043	0.0215	0.181	0.022	0.1811
	0.6	-0.33	0.6997	-0.0012	0.186	-0.0011	0.1859
	0.9	-0.1743	0.4071	0.0041	0.1517	0.0049	0.1507
100	0.3	-0.4028	0.8212	0.0038	0.1605	0.0038	0.161
	0.6	-0.3301	0.6773	-0.0127	0.1312	-0.0127	0.1311
	0.9	-0.1483	0.326	0.003	0.0813	0.003	0.0803
400	0.3	-0.3876	0.7786	0.0056	0.0759	0.0056	0.0757
	0.6	-0.3316	0.6669	-0.0001	0.0591	0	0.0591
	0.9	-0.1468	0.3026	-0.0004	0.0395	-0.0005	0.0394
Average		-0.2934	0.6093	0.0026	0.1185	0.0028	0.1183

We observed that the naïve estimate of the regression coefficient β_x is attenuated toward zero. Additionally, the values of RelBias and RMSE of the coefficient β_x for the three estimators increase with the measurement error variance σ_U^2, but decrease with

TABLE 3: RelBias and RMSE of the Naïve, Normal (N-N), and Skew Normal (SN-N) prior for the Spatial regression model with measurement error variance 0.7.

Prior							
T	λ_X	Naive		N-N		SN-N	
		RelBias	RMSE	RelBias	RMSE	RelBias	RMSE
49	0.3	-0.6074	1.2298	-0.01	0.2305	-0.0102	0.23
	0.6	-0.5369	1.0998	0.0023	0.1809	0.0029	0.1811
	0.9	-0.3363	0.7162	0.0003	0.1236	0.0003	0.1245
100	0.3	-0.5966	1.1996	-0.0026	0.1495	-0.0026	0.1495
	0.6	-0.5396	1.0879	0.0034	0.1254	0.0035	0.1256
	0.9	-0.2982	0.6205	-0.0035	0.0782	-0.0034	0.0786
400	0.3	-0.6048	1.2117	0	0.0743	0	0.0745
	0.6	-0.5409	1.0845	0.0006	0.0566	0.0007	0.0562
	0.9	-0.2874	0.5814	-0.0027	0.0375	-0.003	0.0375
Average		-0.4831	0.9813	-0.0014	0.1174	-0.0013	0.1175

TABLE 4: EAIC and EBIC of the Naïve, Normal (N-N), and Skew Normal (SN-N) prior for the Spatial regression model with measurement error variance 0.1.

Prior					
T	λ_X	Parameter	Naive	N-N	SN-N
49	0.3	EAIC	159.2988	274.4879	208.6433
		EBIC	158.0596	272.629	206.4747
	0.6	EAIC	161.8768	289.3854	224.1755
		EBIC	160.6376	287.5266	222.0068
	0.9	EAIC	163.3675	341.1097	267.4752
		EBIC	162.1283	339.2509	265.3066
100	0.3	EAIC	320.3707	551.9545	418.4622
		EBIC	320.3707	551.9545	418.4622
	0.6	EAIC	322.7067	584.9334	473.0221
		EBIC	322.7067	584.9334	473.0221
	0.9	EAIC	324.1289	681.606	584.0127
		EBIC	324.1289	681.606	584.0127
400	0.3	EAIC	1253.649	2187.9461	1614.2875
		EBIC	1256.0572	2191.5584	1618.5019
	0.6	EAIC	1260.2144	2304.397	1828.0391
		EBIC	1262.6226	2308.0094	1832.2536
	0.9	EAIC	1272.5743	2737.4345	2469.2567
		EBIC	1274.9826	2741.0468	2473.4711

Table 5: EAIC and EBIC of the Naïve, Normal (N-N), and Skew Normal (SN-N) prior for the Spatial regression model with measurement error variance 0.3..

			Prior		
T	λ_X	Parameter	Naive	N-N	SN-N
49	0.3	EAIC	174.4723	329.846	260.8013
		EBIC	173.2331	327.9872	258.6326
	0.6	EAIC	176.6635	346.5873	286.8002
		EBIC	175.4243	344.7285	284.6315
	0.9	EAIC	180.0722	385.6472	316.7996
		EBIC	178.833	383.7883	314.631
100	0.3	EAIC	346.6795	665.5882	533.479
		EBIC	346.6795	665.5882	533.479
	0.6	EAIC	350.8364	698.126	593.5298
		EBIC	350.8364	698.126	593.5298
	0.9	EAIC	358.7151	792.5358	685.0883
		EBIC	358.7151	792.5358	685.0883
400	0.3	EAIC	1367.4139	2631.9434	2079.6499
		EBIC	1369.8221	2635.5557	2083.8643
	0.6	EAIC	1377.4296	2729.3906	2300.703
		EBIC	1379.8378	2733.003	2304.9174
	0.9	EAIC	1426.5864	3165.1932	2892.4515
		EBIC	1428.9947	3168.8056	2896.6659

Table 6: EAIC and EBIC of the Naïve, Normal (N-N), and Skew Normal (SN-N) prior for the Spatial regression model with measurement error variance 0.7.

			Prior		
T	λ_X	Parameter	Naive	N-N	SN-N
49	0.3	EAIC	183.9067	376.9027	306.301
		EBIC	182.6675	375.0439	304.1324
	0.6	EAIC	186.2684	383.4209	319.1254
		EBIC	185.0291	381.562	316.9568
	0.9	EAIC	199.8853	432.0987	365.3364
		EBIC	198.6461	430.2398	363.1678
100	0.3	EAIC	364.5025	748.9659	618.5672
		EBIC	364.5025	748.9659	618.5672
	0.6	EAIC	377.1581	779.0555	655.9786
		EBIC	377.1581	779.0555	655.9786
		EAIC	398.2557	878.7195	779.1036
	0.9	EBIC	398.2557	878.7195	779.1036
400	0.3	EAIC	1442.9183	2972.8864	2411.4332
		EBIC	1445.3266	2976.4987	2415.6477
	0.6	EAIC	1477.3687	3074.5225	2589.6749
		EBIC	1479.7769	3078.1348	2593.8893
	0.9	EAIC	1582.8831	3508.7772	3244.6129
		EBIC	1585.2913	3512.3896	3248.8273

the spatial dependence parameter λ_X. According to [4] that the stronger dependence implies that neighbor areas can provide more information, and hence the estimates are more resistant to the effect of measurement error.

Note also that the RelBias and RMSE of β_x in the case of the normal and Skew-normal prior with the measurement error variance $\sigma_U^2 = 0.7$ are smaller than $\sigma_U^2 = 0.3$. Moreover, for the measurement error variance $\sigma_U^2 = 0.1$ the RelBias of β_x with the spatial dependence parameter $\lambda_X = 0.9$ are larger than $\lambda_X = 0.6$, but for the RMSE the opposite holds.

Tables 4, 5 and 6 show the overall fit statistics for the Spatial measurement error model. Compare to the normal model, the EAIC and EBIC all tend to favor the Skew-normal model for all sample sizes (T), the three dependence parameter λ_X, and the three measurement error variance σ_U^2. Note that the Naive model have the smallest EAIC and EBIC values, but this model does not account for the measurement error. Therefore, the above results show that the Skew-normal prior outperform the normal, symmetrical prior and the Naive model without measurement error correction.

4. Concluding Remarks

This paper analyzed by way of Monte Carlo simulation Bayesian inference of Spatial Regression models with a Skew-normally spatially lagged covariate measured with errors. The simulation examines the performance of Bayesian estimators in the case of (i) Naive models without measurement error correction; (ii) Normal distribution for the error-prone covariate and random errors; (iii) Skew-normal distribution (SN) for the error-prone covariate and normal distribution for random errors.

The simulation results show that the Skew-normal prior estimator outperforms the normal, symmetrical prior and the Naive models without measurement error correction.

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