Conference Paper

Coulomb Corrections in Photoelectron Spectra in the Adiabatic Limit

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Abstract

A momentum-dependent Coulomb correction to the probability of nonlinear ionization in a strong low-frequency laser field is derived analytically in the adiabatic limit, when the quasi-static tunneling model applies. Obtained formulas show that the Coulomb modification of photoelectron spectra can be significant both in linearly and circularly polarized fields. For linear polarization, it leads to a relative enhancement of the ionization probability for photoelectron energies of the order of the ponderomotive energy. This Coulomb effect is expected to be most significant for atomic species with relatively low ionization potentials, such as alkali atoms.

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1. Introduction

Ionization of atoms in intense laser fields remains in the scope of theoretical and experimental studies for already several decades (see [1–4] for review). The dominant part of experiments on strong field ionization is performed using high-power infrared and optical lasers with photon energies considerably smaller than characteristic ionization potential of atoms and positively charged ions. As a result, strong-field ionization usually proceeds via a highly nonlinear interaction when dozens and even hundreds of laser photons are absorbed in a single ionization event. Such regime is usually referred in the literature as multiphoton or tunneling ionization, depending on the value of the adiabaticity parameter introduced in the fundamental work by Keldysh [5]

$$\gamma = \frac{\sqrt{2mI}\omega}{eE_0}. \tag{1}$$

Here, I is the ionization potential of the atomic or molecular level, E_0 and ω are the electric field amplitude and the frequency of the laser field correspondingly and m and e are the electron mass and the elementary charge. The parameter (1) can be

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physically interpreted as a ratio of the characteristic atomic momentum in a bound state with the ionization potential *I* to the quiver momentum of a free electron in the respective laser field. When $\gamma < 1$, ionization dynamics proceeds in the regime of tunneling in the sense that the potential barrier created by the laser field is oscillating slowly, so that the electron dynamics is essentially similar to that in a static electric field. This regime of ionization is also known as adiabatic limit. For infrared lasers with wavelengths $1 \div 4 \mu m$ and intensities close to 10^{14}W/cm^2 the Keldysh parameter (1) is below unity for the most of atomic ground states making the adiabatic regime routine for strong filed ionization experiments.

Among analytic approaches to the description of nonlinear ionization dynamics, the Keldysh theory [5] or the Strong Field Approximation [6, 7] are known as particularly efficient theories, which combine relative technical simplicity to an exceptional predictive power [8]. In their original formulation, these approaches discard the Coulomb interaction between the photoelectron and the parent atomic core, assuming that the interaction with the laser field dominates the ionization dynamics. This approximation makes the theory simple to implement reducing at the same time its qualitative accuracy. In order to improve the precision of theoretical predictions, an approximate technique of the Coulomb corrections has been developed [8–12] basing on the Imaginary Time Method [13] and on the representation of the ionization amplitude in terms of complex quantum trajectories [14]. During the past decade a number of Coulomb effects in photoelectron spectra have been successfully investigated with the help of this technique (see [8] for review).

In this work, we present simple analytic formulas for the momentum-dependent Coulomb factor in the amplitude of ionization in a strong low-frequency laser field assuming that the adiabatic regime is achieved. These expressions are valid for arbitrary polarized laser radiation and could be helpful for a quantitative analysis of experimental data.

2. Basic Equations

Since there are no exact explicit expressions for the amplitude of nonlinear ionization of atoms, such amplitudes are usually calculated for a model system bound by short-range forces with a subsequent semi-classical account of the Coulomb interaction. In the limit of low laser frequencies the general idea for the analytic account of the Coulomb interaction was suggested by Perelomov and Popov [9]. Their calculation led to the celebrated Coulomb-corrected quasi-static ionization rate. Recently, this approach was generalized to arbitrary photoelectron momenta and laser frequencies (see [8] for a review). This generalization encountered two problems – topological and analytical. Both are connected with the analytic properties of the Coulomb-free complex-time photoelectron trajectories in the laser field, $r(t)$. The complex-valued function, $r^2(t)$, may have first- and second-order zeros, generating branch points and poles of the Coulomb potential energy $-Z/\sqrt{r^2(t)}$ [15–17] (here and later, we use atomic units $e = m = \hbar = 1$). The topology of these points imposes certain constraints on the integration contour used to calculate the Coulomb corrections. The analytic part of the problem consists in the calculation of the integral

$$S_C = -Z \int \frac{dt}{\sqrt{r^2(t)}} \tag{2}$$

that determines the Coulomb contribution to the complex photoelectron action. This integral is divergent and requires regularization (see [4, 8] for details).

The analytical properties of the Coulomb-free trajectories are determined by the time-dependence of a laser pulse. Usually, further approximations are required to find analytically the positions of the branch points and the poles. For nonlinear ionization in intense mid-infrared laser fields, the adiabatic approximation can be applied. The latter suggests that the imaginary parts of the complex saddle points – the initial time instants for the photoelectron motion in the laser field – are small compared to the laser period. Equations determining these points are discussed in detail in [1–5]. In this case, the major contribution to the Coulomb integral is given by small intervals of time near the saddle point.

Under such assumption, we obtained an approximate expression for $r^2(t)$ and analyzed the topology of the poles and the branch points. Figure 1 shows an example of the branch points position and evolution and a possible integration contour in Equation (2).

Expanding the trajectory in (2) in series with respect to $\gamma \ll 1$ and integrating the Coulomb potential energy along this contour we obtained for the Coulomb factor in the ionization amplitude

$$Q = exp\left\{iS_C\right\}, \tag{3}$$

a simple analytic expression which is valid for an arbitrary pulse shape:

$$Q(p) = Q_{stat}[E(p)] \cdot R(p), \tag{4}$$

where

$$Q_{stat} = \left(\frac{2\kappa}{E(p)}\right)^{\frac{Z}{\kappa}} \tag{5}$$

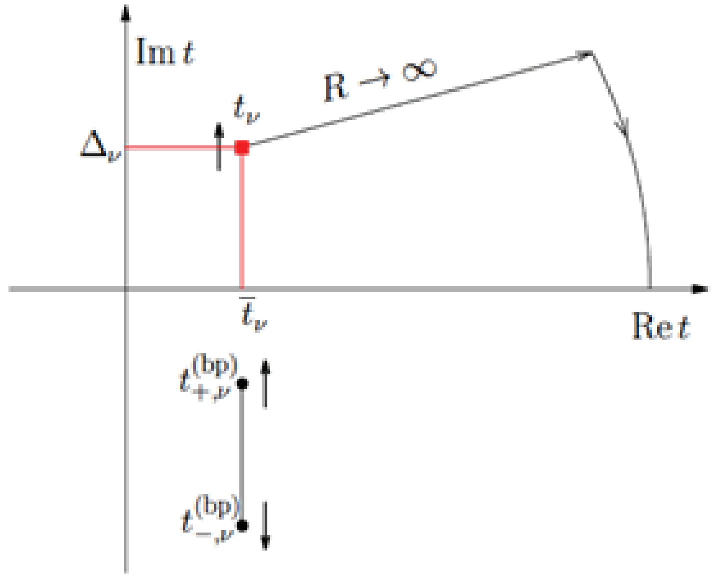

Figure 1: Map of the saddle points t_ν and of the branch points $t^{(bp)}_{\pm,\nu}$ in the complex time plane t. The vertical segment with endpoints $t^{(bp)}_{\pm,\nu}$ is a cut in the Riemann surface. Arrows show the directions in which the saddle and the branch points move with increasing of the initial velocity V_ν. The arc shown by a thin solid line represents a possible integration path. Index ν numerates stationary points.

is the static Coulomb factor [9], $E(p)$ is the absolute value of the laser field at the corresponding saddle point and $\kappa = \sqrt{2I}$ is the characteristic momentum of the bound state. The dynamical factor $R(p)$ can be expressed via the field $E(p)$, the electron velocity $V = p + A(t)$ (with $A(t)$ being the laser field vector potential) taken at the real part of the stationary point and the values

$$\mathrm{K} = \sqrt{\kappa^2 + V^2}, \qquad \mathrm{E} = \sqrt{E^2 - \dot{E}\cdot V}.$$

It has the form similar to (5) although with different arguments:

$$R(p) = \left(\frac{2E(p)}{\mathrm{E}\left[\sqrt{1 + (V/\mathrm{K})^2} + \frac{2}{\sqrt{3}}\sqrt{1 - (E(p)/2\mathrm{E})^2}\right]} \right)^{z/\kappa}. \tag{6}$$

Equations (4–6) present the main result of our work. They describe the momentum-dependent Coulomb distortion of photoelectron spectra in the limit of long laser wavelengths.

3. Discussion

We illustrate the effect of the Coulomb factor (6) calculating it along with the dimensionless imaginary part of the stationary point ω (a complex stationary point is denoted as $t = t' + i$) versus the photoelectron momentum for two commonly considered cases of linear (Figure 2) and circular (Figure 3) polarizations assuming ionization of hydrogen by a field with intensity 10^{14}W/cm^2 and wavelength $1.6\mu m$. Under these conditions, the Keldysh parameter $\gamma = 0.53$. As is seen from the plots, the Coulomb correction can increase the differential probability of ionization (note that the probability is proportional to the value (4) modulus squared) by the factor of two. Such effect could in principal be seen in experimental data recorded with high precision in a wide range of photoelectron energies. The most straightforward way to verify the correction experimentally consists in a precise measurement of the logarithmic slope of photoelectron spectra in a wide range of energies.

In a field with linear polarization, the factors shown in Figure 2 should lead to a small increase in the (negative) slope of photoelectron spectra. This change in the slope is dictated in particular by the factor Z/κ, which is equal to unity for hydrogen but can be larger for atoms with relatively small ionization potentials. For example, for cesium with I = 3.89eV, it is close to 2, considerably enhancing the slope variation.

The same technique of calculation of adiabatic Coulomb factors can applied for description of the process of high-order harmonics generation in intense laser fields [18]. In this case, a preliminary analysis shows that in linearly polarized fields the Coulomb factors only weakly depend on the harmonic number. Instead, in so-called bicircular laser fields, which consist of two counter-rotating laser pulses with circular polarization and an integer frequency ratio this dependence could be significant. This task can be motivated by the growing interest both from the side of experiment and theory [19, 20], in spectra of high harmonics produced in bicircular fields.

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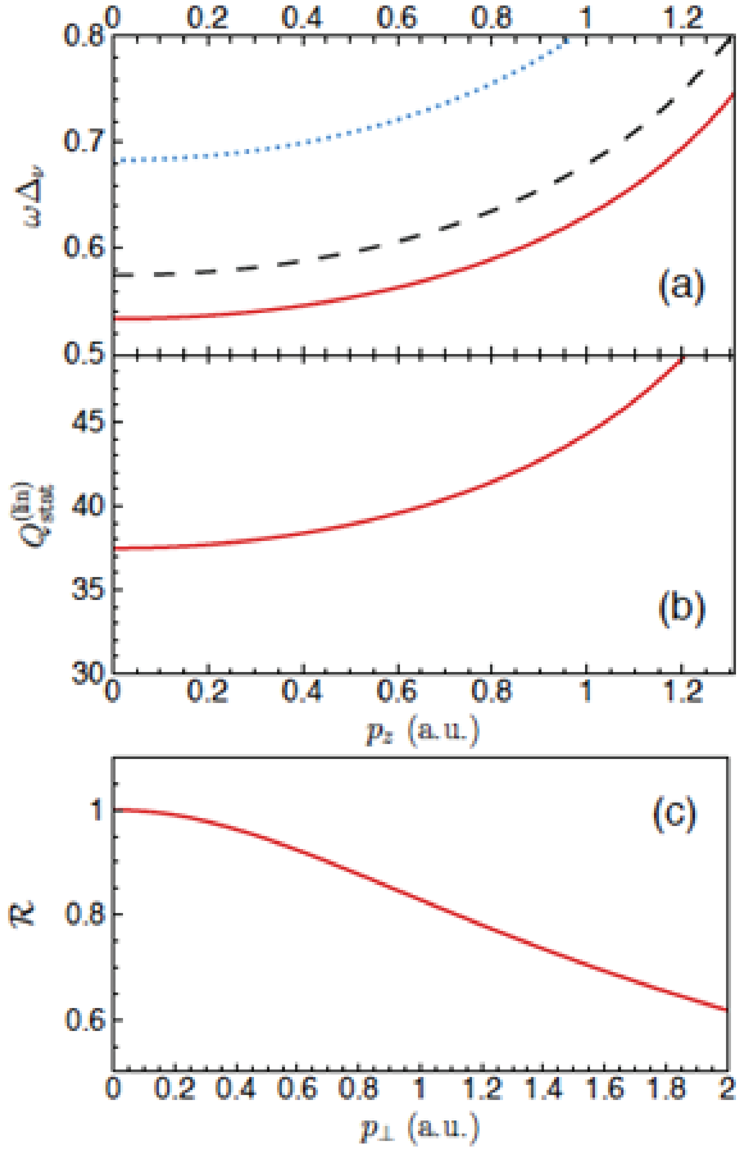

Figure 2: (color online): Momentum dependence of the imaginary part of the stationary point and of the Coulomb factors (5) and (6) for a monochromatic linearly polarized field. Panel (a): dependence of ω on the momentum component p_z along the laser polarization for three different values of the lateral momentum equal to 0 (solid red line), 0.4κ (dashed black line) and 0.8κ (dotted blue line). Panel (b): dependence of the static Coulomb factor (5) on the longitudinal momentum at zero lateral momentum. Panel (c): dependence of the dynamic Coulomb factor (6) on the lateral momentum at zero longitudinal momentum.

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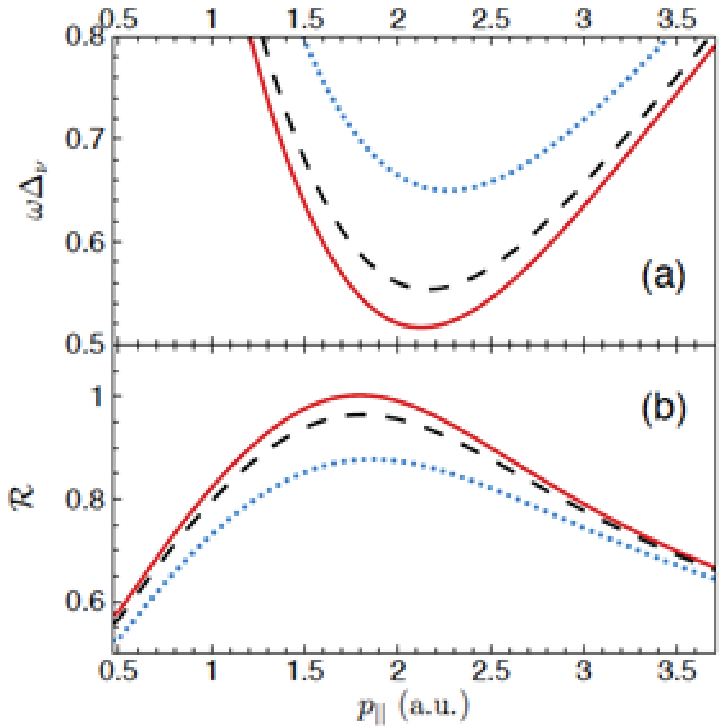

Figure 3: (color online): Same as in Figure 2 but for a monochromatic circularly polarized field. Due to the axial symmetry of the circularly polarized field the longitudinal momentum can be chosen in arbitrary direction in the polarization plane. Panel (a): dependence of ω on the momentum component in the polarization plane; panel (b) – the same for the dynamic factor (6). The lateral momentum is equal to 0 (solid red line), 0.4κ (dashed black line) and 0.8κ (dotted blue line).

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