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Analytical Model for the Performance Curves of a Family of Propellers Based on Wind Tunnel Tests

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Abstract

Propeller aircraft performance is greatly influenced by the performance of the propeller it uses. Thus, proper selection of a propeller for a given aircraft design at the early stages of the design process is fundamental. During the design of a new aircraft, simple yet accurate performance models are required to properly optimize the design. Significant experimental performance data of low speed, small propellers is available. The main objective of this work is to create and validate an analytical model for the performance curves of a family of propellers tested at low Reynolds numbers, which can be used in selecting a propeller for a given existing aircraft design or during its design optimization process. This kind of propellers is more commonly used in Unmanned Aerial Vehicles (UAVs). The model is designed in MATLAB® using a variety of regression techniques, such as the Least Squares Method (LSQ), applied to experimental data acquired at University of Illinois at Urbana-Champaign (UIUC), for seventeen APC Thin Electric propellers, and at the Department of Aerospace Sciences (DCA) of University of Beira Interior (UBI), for ten more APC Thin Electric propellers. The analytical model predicts propeller power coefficient and propulsive efficiency accurately for the family of propellers tested and can also be used for the propellers with dimensions close to those used for its development. The propeller performance data obtained during the experimental tests are made available the community to further increase the documentation on propellers tested at low Reynolds numbers.

Keywords: Propeller, Low Reynolds propeller performance, Propeller tests, Wind tunnel

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1. Introduction

With the Unmanned Aerial Vehicle (UAV) industry becoming larger in modern times, the design of UAVs becomes more important every day. An efficient airplane requires a rigorous design, and this includes its propulsive systems. Most UAVs' propulsive systems use propellers to generate the thrust they need to fly. The propeller's performance can be evaluated by measuring the thrust and power coefficients and propulsive efficiency. Good predictions of these performance curves will be a great asset during UAV preliminary design when numerical optimization tools are used.

There are several readily available performance calculators that can estimate the parameters of propeller performance. Examples of such programs are: PropSelector [1], but the credibility of this calculator is questionable [2]; JBLADE, developed at Universidade da Beira Interior (UBI) by Silvestre et al [3]; QPROP, developed by Drela [4].

A study conducted by Brandt and Selig [5] shows that propeller efficiencies vary greatly depending on the propeller geometry and size, thus, making an accurate prediction of propeller performance curves can greatly improve overall UAV design productivity and optimization procedures. This selection requires many tests in a wind tunnel to acquire enough data to analyze the propeller's performance. Therewith the construction of an analytical propeller performance model will enable to rapidly select different propeller designs by reducing the number of propeller samples tested in a wind tunnel. The researcher shall input all the desired propellers' diameter and pitch into the model and obtain an estimation of the results thus allowing to choose a smaller group of propellers. Ultimately a reduced number of propellers will be selected to analyze in a wind tunnel, thus obtaining results without the need of testing an initially large quantity of propellers.

The goal of this work is to develop a mathematical propeller performance model for a family of propellers, namely the Thin Electric Propellers of the brand APC. The propeller nomenclature for these propellers is a set of two numbers separated by an 'X', where the first number is the propeller diameter in inches and the second number is the propeller pitch in inches per revolution [6]. To achieve this, the work is divided into two phases: the first is to construct the model using data provided by UIUC [7]; the second phase includes tests conducted at UBI in order to collect experimental propeller performance data to further improve the analytical model.

2. Methodology

The work is divided into two parts. First, performance data points of several propellers are experimentally obtained in a wind tunnel. Then, the obtained data is used to create an analytical representation of the power coefficient and the propulsive efficiency for all propellers.

The experimental setup was created by Alves [8]. It consists of three subsystems, the Propeller Balance (Figure 1), Signal Conditioners and the Data Acquisitions System. More details regarding the wind tunnel located in the Aerodynamics Laboratory of Department of Aerospace Sciences at UBI are described in [9] and [10].

The test procedure (Figure 1) is as explained in [8], the only difference being the convergence criteria. In this work, the convergence criteria for propeller speed and freestream velocity have been increased to the values shown in Table 1.

For the dynamic tests, where advance ratio is greater than zero, the propeller rotational speed is fixed, and the wind tunnel's freestream velocity is increased from 4 m/s to 28 m/s in 1 m/s increments. The automatic test must be stopped once the thrust value reaches zero to avoid windmill break state.

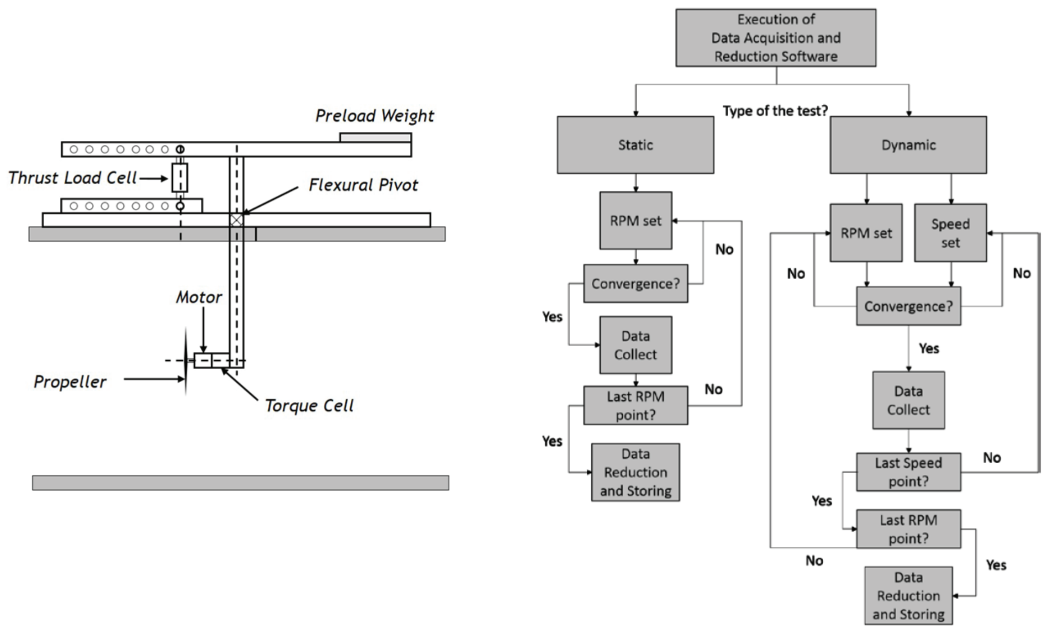

Figure 1: (left) T-shaped pendulum thrust balance concept and (right) flowchart of the test methodology. (Source: [9])

Table 1: Convergence criteria to achieve wind tunnel steady freestream speed and propeller's rotational speed.

Criteria
$\lvert$RPM-RPM$_{target}\rvert \leq$ 10 rpm
$\lvert$V-V$_{target}\rvert \leq$ 0.20 m/s

The data points obtained are the measured variables of thrust, T, torque, Q, freestream velocity, V, rotational speed in revolutions per second, n, static pressure, atmospheric pressure and temperature. With the acquired measurements, the derived performance parameters can be obtained.

Advance ratio, J, is calculated with V and n:

$$J = \frac{V}{nD} \tag{1}$$

Thrust coefficient, C_T, and power coefficient, C_P, are calculated with the respective parameters T and power, $P = 2\pi nQ$:

$$C_T = \frac{T}{\rho n^2 D^4} \tag{2}$$

$$C_P = \frac{P}{\rho n^3 D^5} \tag{3}$$

where D is the propeller diameter and ρ is the air density. Finally, the propeller efficiency is calculated with the variables of C_T and C_P:

$$\eta = \frac{J C_T}{C_P} \tag{4}$$

After the calculation of each individual parameter, C_P and η are plotted against J. Upon observing the different behaviors of all dispersions, all the power coefficient and propeller efficiency points are divided by the natural logarithm of the respective propeller rotational speed at which they were tested, therefore implicitly including the Reynolds number, *Re*, effects into the performance parameters since it varies with propeller rotational speed. To simplify the terminology, the following parameters are defined for the reduced power coefficient and propulsive efficiency:

$$C_{P_r} = \frac{C_P}{ln(N)} \tag{5}$$

$$\eta_r = \frac{\eta}{ln(N)} \tag{6}$$

where $N = n/60$ and subscript *r* stands for "reduced".

The result of this reduction is illustrated in Figure 2, where an example is shown for the APC Thin Electric 10x7 propeller where the data shows less dispersion when divided by $ln(N)$. This effect is observable in all the tested propellers' performance curves. The data reduction is repeated for all the tested propellers.

Since the curves have different sizes, but similar shapes, in order to compare these curves, to come up with a single model that can approximate the values of C_P and η accurately, the next step is to produce scaled curves. To do this, the value of C_{P_r} must be divided by its value at $J = 0$, which is called C_{Pr0}, and the η_r must be divided by its maximum value, η_{rmax}. This procedure is done using the polynomial parametric fitting in the *Curve Fitting Tool* in MATLAB® to create a fitting polynomial function for $C_{P_r} \times J$ and $\eta_r \times J$. Since this tool also displays the coefficients of those curve fits, it is possible to calculate the values of C_{Pr0} and η_{rmax}. The maximum value of advance ratio, J_{max}, is also retrieved from the function of η_r as propeller efficiency reaches the value of zero

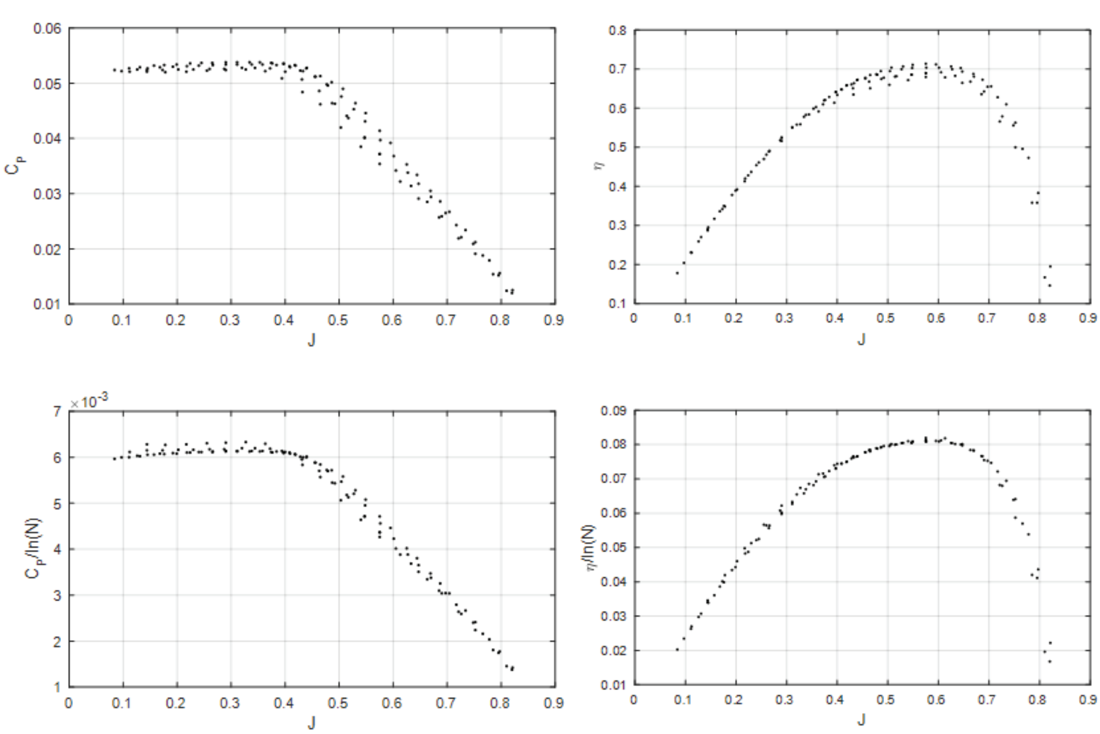

Figure 2: Example of data points for C_P, C_{Pr}, η and η_r.

before the power coefficient, and J is then divided by J_{max}. Thus, the following functions are obtained

$$\frac{C_{pr}}{C_{Pr0}}\left(\frac{J}{J_{max}}\right); \qquad \frac{\eta_r}{\eta_{rmax}}\left(\frac{J}{J_{max}}\right)$$

Examples of the scaled data points used to obtain these functions are shown in Figure 3 for the 8x4 propeller.

After this step, all propeller performance curves have practically the same limits in the *x*-axis and *y*-axis. The curve fitting procedure is the next step to construct the analytical model, using the *Curve Fitting* Toolbox in MATLAB®.

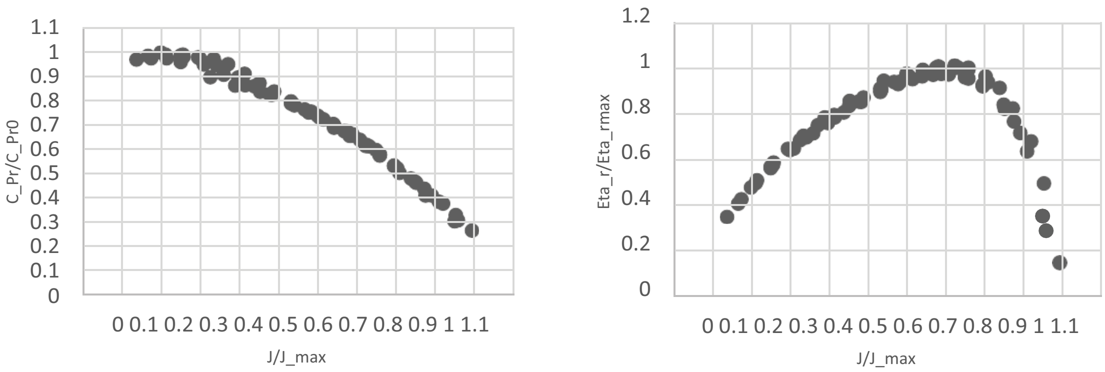

Figure 3: Example of data point scaling for the 8x4 propeller: (left) $\frac{C_{Pr}}{C_{Pr0}}(\frac{J}{J_{max}})$ and (right) $\frac{\eta_r}{\eta_{rmax}}(\frac{J}{J_{max}})$.

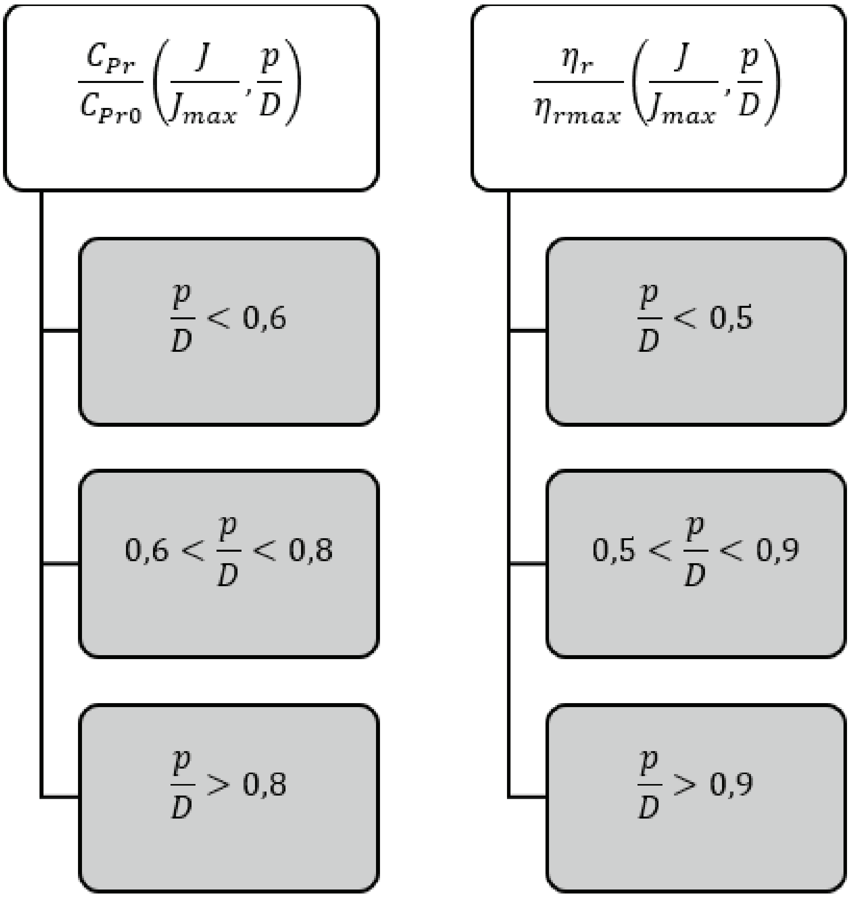

Figure 4: Separation criteria for each parameter.

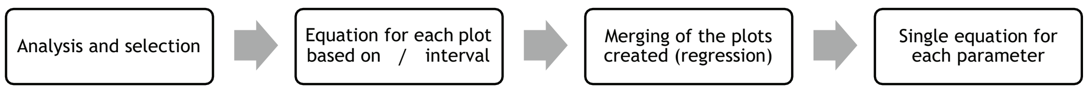

Figure 5: Process to create the analytical model equations.

The plots are analyzed in order to separate the different $C_P \times J$ and $\eta \times J$ plots into groups with a common shape distributions. Then, using regression methods, multivariable functions that describe how the performance parameters behave when plotted against J and a common variable are created. The common variable in this case is the propeller pitch-to-diameter ratio, p/D.

Based on the criteria in Figure 4, the plots are separated into the three indicated p/D intervals and different functions are calculated for each interval. Then, a single function of J/J_{max} and p/D for C_{P_r}/C_{Pr0} is computed. The same procedure is repeated for η_r/η_{rmax}. The procedure sequence is illustrated in Figure 5.

3. Results and Discussion

The obtained experimental data is analyzed in order to separate the different C_P and η plots into groups with a common variable, to create a multivariable plot, using regression methods such as LSQ [11] and the Constrained Linear Least Squares [12], that describes how the performance parameters behave when plotted against J and a common variable. The results of this analysis are verified with the measurements from UIUC (propellers with dimensions: 8x4, 8x6, 8x8, 9x4.5, 9x6, 9x7.5, 9x9, 10x5, 10x7, 11x5.5, 11x7, 11x8, 11x8.5, 11x10, 14x12, 17x12 and 19x12) [7].

A test is conducted with the analytical model, to see if the estimated performance curves match the experimental data acquired at UBI. Afterwards, this experimental data is used to further develop the analytical model, and the updated model is verified with the measurements from both UIUC and UBI (propellers with dimensions: 7x4, 13x4, 13x10, 14x10, 15x6, 15x10, 16x10, 18x8, 20x8, 20x15).

The first iteration of this analytical model is created using the UIUC propeller data, which results in two functions for the $C_{P_r}/C_{P_{r0}}$ and a single function for the η_r/η_{rmax} based on the pitch-to-diameter ratio, p/D. This model is used to obtain the estimation of the propeller efficiency for the propellers studied at UBI, and the results are compared to the real data (Table 2).

The values in Table 2 show that the model's prediction is not very accurate. This is due to the difference in the geometry between the propellers used to create the model and the samples studied at UBI. However, some of the propellers tested at UBI have very similar dimensions to the ones used to create the initial model. These predictions are very accurate. Figure 6 shows the various propeller sizes tested by both UIUC and UBI. The propeller data acquired at UBI is used to further enhance the initial analytical model, thus creating a final iteration in this work (Figure 7).

TABLE 2: Mean relative error (MRE), δ_{max} and standard deviation of the model's prediction of the propellers tested at UBI.

Propeller	C_P			η		
	MRE [%]	δ_{max} [%]	σ	MRE [%]	δ_{max} [%]	σ
7x4	20	42.75	0.0078	52.57	535.89	0.1913
13x4	756.07	1061.2	0.0994	61.31	732.76	0.1459
13x10	3.20	11.83	0.0015	28.31	404.06	0.1043
14x10	8.93	13.32	0.0042	19.25	135.04	0.1149
15x6	354.43	1080.6	0.0613	2869.3	159370	0.0958
15x10	23.67	31.34	0.0101	23.09	403.54	0.1151
16x10	30.23	42.62	0.0108	27.49	196.92	0.1445
18x8	234.84	332.88	0.0457	16.25	97.38	0.0798
20x8	734.89	971.73	0.1135	40.87	209.96	0.1754
20x15	94.40	118.54	0.0353	17.21	67.86	0.0973

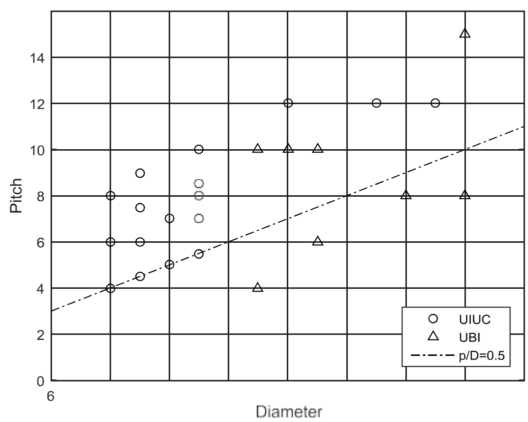

Figure 6: Representation of the propellers used to create (data acquired from UIUC, circles) and validate (data acquired at UBI, triangles) the analytical model.

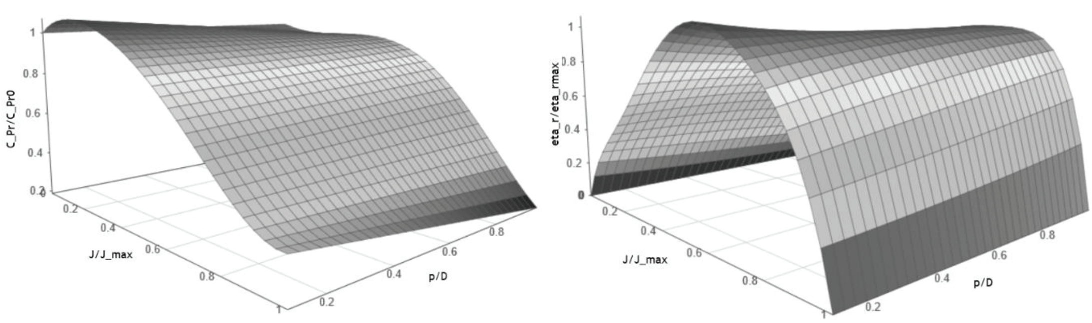

Figure 7: 3D representation of the propeller performance analytical model: (left) ($\frac{C_{Pr}}{C_{Pr0}}(\frac{J}{J_{max}}, \frac{p}{D})$) and (right) $\frac{\eta_r}{\eta_{rmax}}(\frac{J}{J_{max}}, \frac{p}{D})$.

The final analytical model consists of a set of five equations for C_{Pr}/C_{Pr0}, η_r/η_{rmax}, C_{P0}, η_{rmax} and J_{max} as shown from Eq. 7 through to Eq. 11 which are valid for $D \in [7, 20]$, $p \in [4, 15]$ and $p/D \in [0.4, 1]$.

$$\begin{aligned}
&\frac{C_{Pr}}{C_{Pr0}}\left(\frac{J}{J_{max}}, \frac{p}{D}\right) \\
&= 1 + 1.177\left(\frac{J}{J_{max}}\right) + 0.004779\left(\frac{p}{D}\right) - 5.287\left(\frac{J}{J_{max}}\right)^2 - 1.654\left(\frac{J}{J_{max}}\right)\left(\frac{p}{D}\right) \\
&+8.743\left(\frac{J}{J_{max}}\right)^3 + 6.371\left(\frac{J}{J_{max}}\right)^2\left(\frac{p}{D}\right) - 9.728\left(\frac{J}{J_{max}}\right)^4 - 6.741\left(\frac{J}{J_{max}}\right)^3\left(\frac{p}{D}\right) \\
&+4.49\left(\frac{J}{J_{max}}\right)^5 + 1.812\left(\frac{J}{J_{max}}\right)^4\left(\frac{p}{D}\right)
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\frac{\eta_r}{\eta_{rmax}}\left(\frac{J}{J_{max}}, \frac{p}{D}\right) \\
&= \left[5.702262681\left(\frac{J}{J_{max}}\right) - 38.01485015\left(\frac{J}{J_{max}}\right)^2 + 161.4243788\left(\frac{J}{J_{max}}\right)^3\right. \\
&\left.-315.6039562\left(\frac{J}{J_{max}}\right)^4 + 279.2845206\left(\frac{J}{J_{max}}\right)^5 - 92.79235578\left(\frac{J}{J_{max}}\right)^6\right] \\
&-\left(\frac{p}{D}\right)\left[8.151725438\left(\frac{J}{J_{max}}\right) - 116.4783213\left(\frac{J}{J_{max}}\right)^2 + 533.0383366\left(\frac{J}{J_{max}}\right)^3\right. \\
&\left.-1032.988581\left(\frac{J}{J_{max}}\right)^4 + 891.0573894\left(\frac{J}{J_{max}}\right)^5 - 282.7805492\left(\frac{J}{J_{max}}\right)^6\right] \\
&+\left(\frac{p}{D}\right)^2\left[5.427814965\left(\frac{J}{J_{max}}\right) - 95.74279874\left(\frac{J}{J_{max}}\right)^2 + 466.6603959\left(\frac{J}{J_{max}}\right)^3\right. \\
&\left.-938.6448505\left(\frac{J}{J_{max}}\right)^4 + 837.3278541\left(\frac{J}{J_{max}}\right)^5 - 275.0284158\left(\frac{J}{J_{max}}\right)^6\right]
\end{aligned} \tag{8}$$

The equations of J_{max}, C_{Pr0} and η_{rmax} are functions of D and p only in the form:

$$\begin{aligned}
J_{max}(D,p) &= -1.099 + 0.1789D + 0.7614p - 0.001555D^2 - 0.1169Dp - 0.01523p^2 \\
&-0.0005051D^3 + 0.005775D^2p + 0.003119Dp^2 - 0.0007585p^3 \\
&-0.000004304D^3p - 0.000273D^2p^2 + 0.0001492Dp^3 - 0.00001965p^4
\end{aligned} \tag{9}$$

$$\begin{aligned}
C_{Pr0}(D,p) &= -0.3875 + 0.1077D + 0.05883p - 0.01285D^2 + 0.002117Dp_0.01179p^2 \\
&+0.0006309D^3 - 0.0001054D^2p - 0.00006636Dp^2 + 0.0009324p^3 \\
&-0.00001161D^4 + 0.000005865D^3p - 0.00001089D^2p^2 \\
&+0.0000178Dp^3 - 0.00003423p^4
\end{aligned} \tag{10}$$

$$\begin{aligned}\eta_{rmax}(D,p) \; &= 0.0136 - 0.00469D + 0.005591p + 0.0004904D^2 - 0.0008541Dp \\ &+0.0001986p^2 - 0.00001569D^3 + 0.00003468D^2p - 0.00001612Dp^2 \\ &+0.000002577p^3\end{aligned} \quad (11)$$

To get the estimated values of C_P and η, in the form of two functions of four variables (advance ratio, propeller diameter, propeller pitch and propeller rotational speed), the following equations must be solved:

$$C_P(J,D,p,N) = \frac{C_{Pr}}{C_{Pr0}}\left(\frac{J}{J_{max}},\frac{p}{D}\right)C_{Pr0}ln(N) \quad (12)$$

$$\eta(J,D,p,N) = \frac{\eta_r}{\eta_{rmax}}\left(\frac{J}{J_{max}},\frac{p}{D}\right)\eta_{rmax}ln(N) \quad (13)$$

with $0 \leq J \leq J_{max}$.

The validation procedure of this iteration generated the results in Table 3. The experimental data of C_P and η is plotted against J, the same J values are used to calculate the estimates from the analytical model. The relative error is then calculated for every point measured for each propeller and averaged to compute a mean relative error (MRE).

As observed, the maximum value of the mean relative error for C_P is 21.27% and for η is 29.57%. The values of standard deviation show that the measured data points are close to the model's predictions. As for the lowest value of the mean relative error, for C_P it is 3.01% and for η it is 1.94%. The result for R^2 calculation is also shown, being 0.9717 for C_P and 0.8595 for η, indicating that the C_P model represents well the measured data while the η model is sufficiently adequate.

4. Conclusion

This work has presented the development and validation of an analytical model for many propellers of the APC Thin Electric family comprising the following main tasks: experimental testing of propellers belonging to this family was performed at UBI's wind tunnel; creation of an analytical model to estimate the power coefficient and the propulsive efficiency of the propellers for a range of airspeeds.

The developed analytical model is capable of estimating the performance parameters for propellers with a diameter range between [7,20], a pitch range between [4,15] and a pitch-to-diameter ratio between [0.4;1] with good accuracy. For other propeller sizes, the model should be used with caution. The procedure described, can be used in the

Table 3: Mean relative error, maximum relative error and standard deviation of the model's prediction of all propellers and total value.

Propeller	C_P			η		
	MRE [%]	δ_{max} [%]	σ	MRE [%]	δ_{max} [%]	σ
Propeller 7x4	12.09	20.41	0.0056	29.57	327.39	0.1071
Propeller 8x4	5.53	18.36	0.0016	17.90	430.70	0.0879
Propeller 8x6	4.82	21.97	0.0026	14.51	509.16	0.0641
Propeller 8x8	5.79	22.04	0.0054	2.56	32.17	0.0180
Propeller 9x4.5	4.34	27.52	0.0019	6.41	54.52	0.0283
Propeller 9x6	10.54	56.49	0.0037	7.71	366.58	0.0342
Propeller 9x7.5	3.95	14.67	0.0026	5.94	250.44	0.0290
Propeller 9x9	6.48	16.14	0.0058	4.93	86.09	0.0281
Propeller 10x5	13.42	22.71	0.0043	10.34	210.61	0.0625
Propeller 10x7	4.38	25.99	0.0018	1.94	34.90	0.0130
Propeller 11x5.5	6.72	22.18	0.0015	8.79	99.85	0.0453
Propeller 11x7	9.58	19.49	0.0041	14.48	462.19	0.0896
Propeller 11x8	4.71	29.63	0.0020	16.57	1007.6	0.0512
Propeller 11x8.5	4.32	15.88	0.0021	6.20	106.26	0.0479
Propeller 11x10	5.14	18.88	0.0032	15.81	529.31	0.1059
Propeller 13x4	12.70	28.27	0.0023	11.81	225.34	0.0350
Propeller 13x10	10.69	23.80	0.0045	5.39	23.47	0.0342
Propeller 14x10	7.79	13.99	0.0033	6.14	39.82	0.0338
Propeller 14x12	8.18	15.22	0.0044	4.66	25.13	0.0297
Propeller 15x6	10.02	48.01	0.0024	5.14	46.25	0.0311
Propeller 15x10	9.36	14.74	0.0038	8.03	144.15	0.0363
Propeller 16x10	7.54	15.80	0.0028	9	123.83	0.0566
Propeller 17x12	21.27	31.51	0.0086	28.65	1597.4	0.1014
Propeller 18x8	9.39	28.51	0.0021	9.66	28.83	0.0533
Propeller 19x12	6.3	15.54	0.0026	7.38	228.85	0.0511
Propeller 20x8	6.82	18.22	0.0012	9.04	45.18	0.0416
Propeller 20x15	3.01	7.5950	0.0017	4.86	14.95	0.0310
Average MRE		7.96			10.13	
(total)		0.9717			0.8595	

future with new propeller data to increase its applicability to a wider range of propeller sizes.

With the conducted experiments, more low Reynolds numbers propeller performance data has been collected, therefore increasing the database available to the community.

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