Conference Paper

Determination of Vertical Deflection Based on Terestrial Gravity Disturbance Data (A Case Study in Semarang City)

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Abstract

Vertical deflection can be determined by geometrical and physical measurement. In geometrical way, vertical deflection is obtained by comparing astronomical coordinate and geodetical coordinate. In physical way, vertical deflection can be computed from gravity measurement. In the past, vertical deflection was computed from gravity anomaly data. Gravity anomaly data measurement is difficult because it need reduction of gravity from surface of the earth to the geoid using orthometric height from spirit level measurement. In modern era, gravity anomaly data may be replaced by gravity disturbance data whose only required gravity and GNSS (Global Navigation Satellite System) measurement. This research aims to determine vertical deflection in Semarang City from terestrial gravity disturbance data. The gravity data were measured in March of 2016. Formula of Vening Meinesz that usually used for vertical deflection was replaced by new formula that generated from derivation of function of Hotine. Applying gravity disturbance gave vertical deflection of east-west component that were vary from -1.2" to 12.2" while north-south component were vary from -4.2" to 4.2". Comparing vertical deflection as computed from terestrial data to as computed from EGM2008 coefficients showed conformity in shape and values. It was concluded that derivation of function of Hotine could be applied for vertical deflection determination from gravity disturbance.

Keywords: vertical deflection, gravity disturbance, Hotine Function, Vening Meinesz

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1. Introduction

Vertical deflection is deviation of actual plumbline from normal plumbline [1]. Actual plumbline is perpendicular to geoid while normal plumbline is perpendicular to ellipsoid. In the flat area. the deviation value can be ignored, while in mountainous area, the deflection values are large and must be considered as systematic error. Deflection vertical value is important for many geodesy applications [2]. Transforming astronomical coordinates into geodetic coordinates, transforming astronomical azimuth into geodetic

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azimuth, reducing horizontal and vertical angles to areference ellipsoid. Deflection vertical value also can be used for predicting gravity and geoid [3]

Along with the widespread use of GNSS in determining the position of the horizontal control network, the theodolite measurement is only applied to small areas and specific areas that cannot be measured with GNSS. Vertical defection is still useful in the case of combining the theodolite and GNSS measurement results [4]. In some cases, merging the theodolite measurement data produces different coordinates with the results of the GNSS measurement. The discrepancies occur mainly in areas that have significant vertical deflection.

At present, there are almost no studies on vertical deflection in Indonesia [5]. Gravity data in large area is frequently used for geoid determination [6]. Gravity anomaly data is used to calculate geoids with the Stokes and Molodensky approach. Gravity disturbance data is used to calculate geoid by Hotine approach [7]. Outside of Indonesia, gravity disturbance data is not only used for geoid calculations but also for vertical deflection calculations.

Vertical deflection mainly occurs in areas around high mountains. The effect of mountain mass to the value of vertical deflection needs to be investigated. The characteristic of vertical deflection calculation from gravity disturbance data is also interesting to investigate. This study aims to obtain vertical deflection values calculated from gravity disturbance data in Semarang City. Semarang City was chosen as the case study area, because Semarang City is near by Ungaran Volcano as seen in Figure 1.

2. Methods

2.1. Data and Equipments

Vertical deflection calculations require terrestrial gravity data, precise geodetic position data, and Global Geopotential Model (GGM). Terrestrial gravity data was measured in 2016 using Gravimeter Scintrex CG-5. The gravity data distribution in Semarang City can be seen in Figure 1. The geodetic coordinates of the gravity measurement point are measured with TopCon HiperII and Topcon Hiper SR receivers with rapid static method to obtain the centimeter accuracy. Vertical defection is calculated by the GRAVSOFT program.

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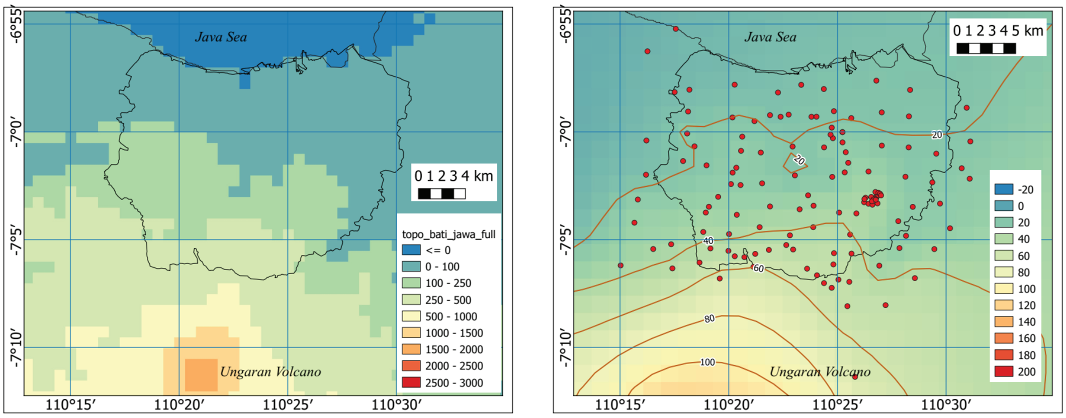

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Figure 1: Research area, topographical situation (a), terrestrial gravity disturbance data (b).

2.2. Computation of vertical deflection

Actual plumbline is a line that is perpendicular to the actual gravity equipotential plane, whereas normal plumbline is a line that is perpendicular to the normal gravity equipotential plane. Geoid is one of the actual gravity equipotential fields, while the reference ellipsoid is one of the normal equipotential fields. Every point on the surface of the earth, inside the earth, and outside the earth has a normal and actual gravitational potential value. Normal and actual gravitational potential values at a point can be different. The difference between the actual and normal potential values is called as potential disturbance (*T*) as written below:

$$T = W - U \tag{1}$$

where *W* is actual gravity potential and *U* is potential of normal gravity.

Before the GNSS era, the geoid determination must be calculated on the surface of geoid. Hence, gravity data must be downward from surface of the earth to surface of geoid using free air reduction, as shown by Figure 2. The gravity anomaly (Δg) for or geoid determination is calculated as follow:

$$\Delta g = g_Q - \gamma_0 \tag{2}$$

here g_Q is the actual gravity on geoid, γ_0 is the normal gravity on reference ellipsoid.

Normal gravity at a certain latitude (φ) on reference ellipsoid is calculated with formula of Somigliana as follow [1]:

$$\gamma_0 = \frac{a.\gamma_e.\cos^2\varphi + b.\gamma_p.\sin^2\varphi}{\sqrt{a^2.\cos^2\varphi + b^2.\sin^2\varphi}} \tag{3}$$

where, γ_e is the gravity at equator, γ_p is the gravity at poles, *a* is semi-major axis of the reference ellipsoid, *b* is semi-minor axis of the reference ellipsoid. Normal gravity values on the surface of the earth (γ_P) can be calculated by following formula:

$$\gamma_P = \gamma_0 \left\{ 1 - 2\left(1 + f + m - 2.f.\sin^2\varphi\right).\frac{h}{a} + 3.\left(\frac{h}{a}\right)^2 \right\} \tag{4}$$

where *m* is a physical constant representing comparison of centrifugal force at equator and gravity at equator, *f* is geometrical flattening of reference ellipsoid, *h* is a geodetic height of observation point above reference ellipsoid.

First derivation of potential disturbance is gravity disturbance (Δg). Theoretically, gravity disturbance as direct derivation value must be more accurate than gravity anomaly (Δg) for geoid determination. Gravity disturbance (Δg) is calculated by the following equation:

$$\delta g = g_P - \gamma_P \tag{5}$$

where is g_P is the actual gravity on arbitrary surface. For calculation of gravity disturbance on the surface of the earth, the terrestrial gravity data do not need to be reduced to geoid, as shown in Figure 2.

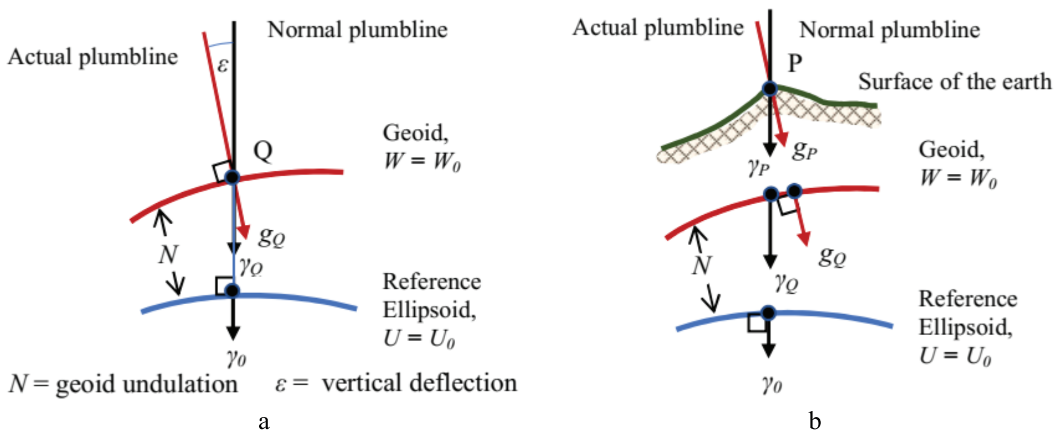

Figure 2: Actual and normal gravity data, on geoid (a), on surface of the earth (b).

Geoid undulation can be calculated from gravity anomaly data or gravity disturbance data [8]. Calculations with gravity anomaly data are known as the Stokes or Molodensky approach and use the function of Stokes [1] as follow:

$$S(\psi) = \frac{1}{\sin\frac{\psi}{2}} + 1 - 6.\sin\frac{\psi}{2} - 5.\cos\psi - 3.\cos\psi.\ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right) \tag{6}$$

where S(ψ) is function of Stokes that computed using spherical distance (ψ) from evaluation points to gridded gravity anomaly data. Geoid determination with gravity disturbance data is called as the Hotine approach using the function of Hotine [1] as follow:

$$H(\psi) = \frac{1}{\sin\frac{\psi}{2}} - \ln\left(1 + \frac{1}{\sin\frac{\psi}{2}}\right) \quad (7)$$

The calculation of vertical deflection from gravity anomaly data traditionally apply the formula of Vening Meinesz, while the calculation of vertical deflection from gravity disturbance data is not popular. The Vening Meinesz formula is the first derivative of the function of Stokes with respect to spherical distances as shown below:

$$\frac{\partial S(\psi)}{\partial\psi} = -\frac{\cos\frac{\psi}{2}}{2\sin^2\frac{\psi}{2}} + 8\sin\psi - 6.\cos\frac{\psi}{2} - 3.\frac{1-\sin\frac{\psi}{2}}{\sin\psi} + 3.\sin\psi.\ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right) \quad (8)$$

Applying similar technique, derivation of the Hotine function produces the following equation:

$$\frac{\partial H(\psi)}{\partial\psi} = \frac{\cos\frac{\psi}{2}}{2\sin^2\frac{\psi}{2}}\left(-1 + \frac{\sin\frac{\psi}{2}}{1+\sin\frac{\psi}{2}}\right) \quad (9)$$

One of the geoid calculation techniques that is often applied is R-C-R as abbreviation of Remove-Compute-Restore [9]. The principle of R-C-R is to calculate the geoid with the small gravity anomaly data by reducing the long wave and short wave components in the terrestrial gravity data. The long wave component is represented by GGM, while shortwave component is surrounding topography. Hypothetically, R-C-R can be applied for calculation of vertical deflection. The REMOVE part of R-C-R technique applied for computing residual gravity disturbance (Δg_{RES}) is only to reduce terrestrial gravity disturbance data (Δg_{P}) with gravity disturbance from EGM2008 ($\Delta g_{EGM2008}$) without reducing the topographic effect (Δg_{topo}), as shown below:

$$\delta g_{RES} = \delta g_p - \delta g_{EGM2008} \quad (10)$$

Residual gravity disturbances (Δg_{RES}) at measurement points are then interpolated to obtain continuous gridded residual gravity disturbance data (Δg_{GRID}) over calculation area.the COMPUTE part of R-C-R technique is to apply gridded residual gravity disturbance for computing residual vertical deflection in North-South (ξ_{RES}) and the East-West (η_{RES}) as follows:

$$\xi_{res}(\varphi,\lambda) = \frac{1}{4.\pi.\gamma_0}\int_{\lambda'=0}^{2\pi}\int_{\varphi'=-\pi/2}^{\pi/2} \delta g_{GRID}(\varphi',\lambda').\frac{\partial H(\psi)}{\partial(\psi)}.\cos\alpha.d\varphi'.d\lambda' \quad (11)$$

$$\eta_{res}(\varphi,\lambda) = \frac{1}{4.\pi.\gamma_0}\int_{\lambda'=0}^{2\pi}\int_{\varphi'=-\pi/2}^{\pi/2} \delta g_{GRID}(\varphi',\lambda').\frac{\partial H(\psi)}{\partial(\psi)}.\sin\alpha.d\varphi'.d\lambda' \quad (12)$$

where α is the azimuth from the evaluation point to the gridded points. North-South (ξ_{RES}) and East-West (η_{RES}) of the Vertical Deflection from EGM2008 n=2190 and m = 2160 are calculated as follows:

$$\xi_{EGM2008} = -\frac{GM}{r^2\gamma}\sum_{m=0}^{M}\left(\cos m\lambda \sum_{m=0}^{M}\left(\frac{a}{r}\right)^n \bar{C}_{nm}P_{nm}(\sin\varphi) + \sin m\lambda \sum_{m=0}^{M}\left(\frac{a}{r}\right)^n \bar{S}_{nm}P_{nm}(\sin\varphi)\right) \tag{13}$$

$$\eta_{EGM2008} = -\frac{GM}{r^2\gamma\cos\varphi}\sum_{m=0}^{M} m\left(\cos m\lambda \sum_{m=0}^{m}\left(\frac{a}{r}\right)^n \bar{S}_{nm}P_{nm}(\sin\varphi) - \sin m\lambda \sum_{m=0}^{M}\left(\frac{a}{r}\right)^n \bar{C}_{nm}P_{nm}(\sin\varphi)\right) \tag{14}$$

where *G* is Newton gravitational constant, *M* is mass of the earth, *R* is radii of the earth, $P_{n,m}$ is function of Legendre, while φ and λ is latitude and longitude of evaluation points. The value depends on degree (n) and order (m) of the function. The RESTORE part of R-C-R technique for obtaining definitive vertical deflection is calculated by equations below:

$$\xi = \xi_{RES} + \xi_{GGM} \tag{15}$$

$$\eta = \eta_{RES} + \eta_{GGM} \tag{16}$$

3. Results

Vertical defection was calculated using several modification of FORTRAN scripts in the Gravsoft program. One modified part is replacing the Vening Meinesz function with the derivative function of the Hotine function in the STOKES program. Modifications aim to calculate vertical deflection from gravity disturbance data. The vertical deflection of GGM is calculated based on the coefficient EGM2008 with n = 2190. In Semarang City, the north-south component is graded from -4 arc second to 4 arc second from northwest to southeast, while the east-west component is graded from -1 arc second to 12 arc second from Northeast to Southwest direction, as shown in Figure 3. The results show that Mount Ungaran has a significant effect on deflection of plumbline in the city of Semarang.This study also calculates vertical deflection in two ways, namely the R-C-R method and the direct method. The direct method is the process of calculating the vertical deflection by directly using the gravity disturbance data with reducing the long wave and short wave components. Calculations using the direct method produced a vertical deflection map that similar to the shape of a vertical deflection map from

EGM2008, but with a smaller vertical deflection value. The north-south component is graded from -1 arc second to 5 arc second from Northwest to Southeast, while the east-west component is graded from 4 arc seconds to 14 arc second from Northeast to Southwest, as shown in Figure 4.Calculations using the R-C-R method produce a vertical deflection map that more or less conform to the shape of a vertical deflection map from EGM2008, but with more detailed shape values. The north-south component is graded from -5 arc second to 3 arc second from Northwest to Southeast, while the east-west component is graded from -1 arc second to 14 arc second from Northeast to Southwest, as shown in Figure 5. Some hills and valleys in Semarang City significantly influenced the amount of vertical deflection.

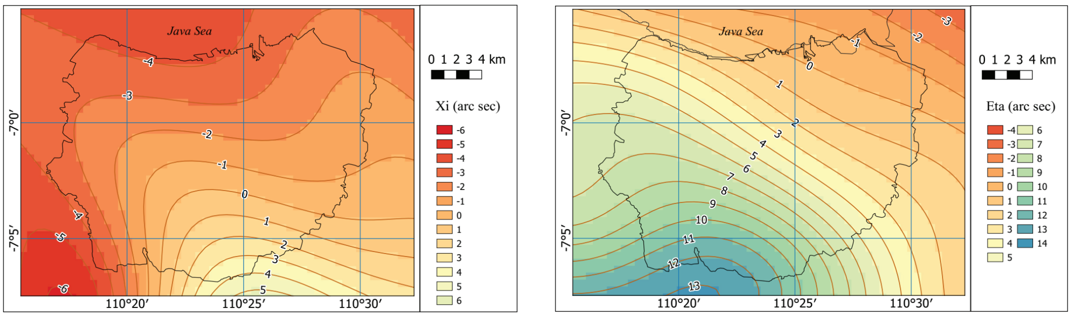

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Figure 3: Vertical Deflection from EGM2008 n=2190, north-south (a), east-west (b).

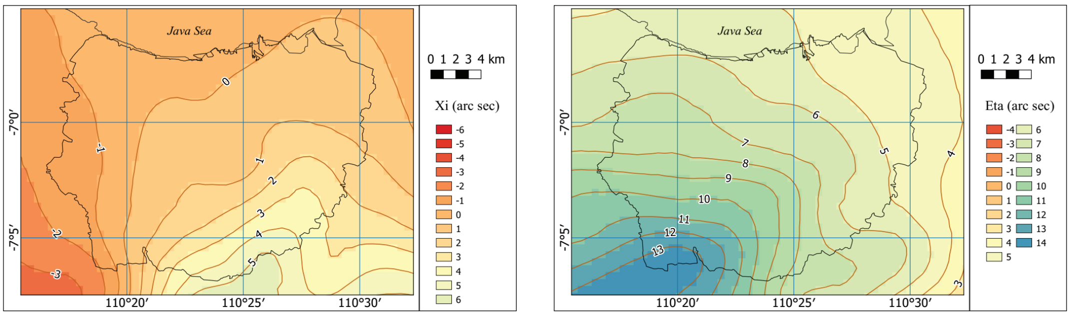

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Figure 4: Vertical Deflection from EGM2008 n=2190, north-south (a), east-west (b).

4. Conclusion

The derivation of function of Hotine introduced a new function that could be used to calculate vertical deflection from gravity disturbance data. The vertical deflection of the gravity disturbance data on the surface had the same shape as the vertical deflection of the EGM2008 data.The application of the R-C-R method for the calculation of vertical deflection with gravity disturbance data also gave satisfying results. Direct calculations

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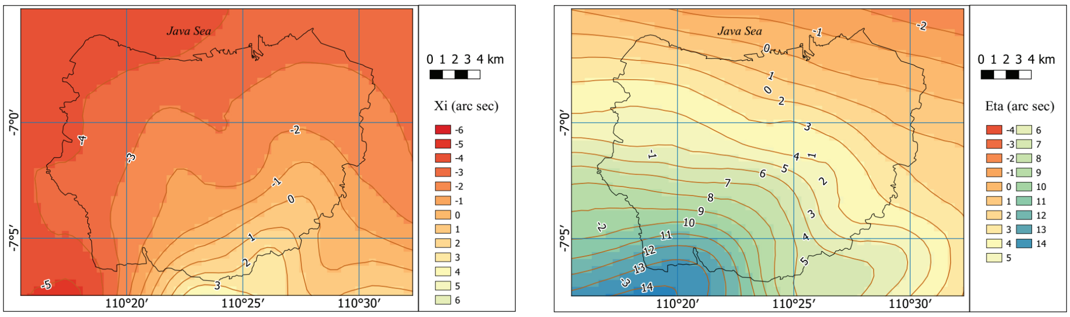

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Figure 5: Vertical Deflection from EGM2008 n=2190, north-south (a), east-west (b).

produced a precise vertical deflection map in the middle of the calculation area, whereas in the measurement boundary section, the vertical deflection value tend to differ greatly from the deflection value from R-C-R or from EGM2008. The recommendation from this study was the need to analyze the accuracy of the results of the calculation of vertical deflection.

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