Conference Paper

Mathematical Model of Limestone Calcining in the Shaft Furnace

Vladimir Shvydkii, Sergei Kudelin, and Vladislav Noskov

Ural Federal University (UrFU), Yekaterinburg, Russia

Abstract

The article reviews the operation principle of the shaft furnace for calcination of carbonate materials and the options for description of thermophysical processes occurring in these furnaces. It provides equations of heat transfer by radiation and convection, special features of thermal interaction between the gas flow and surface of solid particles, influence of thermophysical properties and content of particles on efficiency of furnace performance as well as the possible chemistry of reactions to lime dissociation parameters.

Keywords: dissociation of carbonate materials, thermophysical processes, radiation, convection, dissociation chemistry

Corresponding Author:
Vladimir Shvydkii
v.s.shvydkiy@urfu.ru

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1. Problem Statement

The problem [1] has been stated as follows. Carbonate materials in the quantity of G_{m}^{o}, kg/s with the initial temperature of t_0, C are charged into the shaft furnace. Fuel combustion products in the quantity of V_{comb}, m^3/s with the temperature of T_0, C are delivered to the furnace bottom (cooling zone is not provided for). When the surface temperature of the descending material lump reaches t, the solid-state reaction of thermal decomposition of limestone starts in this lump. It is necessary to determine the change in the process variables throughout the height of the furnace if the (volumetric) coefficient of convective heat exchange between gas and material considering the internal heat resistance of the lump is equal to αV, W/(m^3.K) and the limestone dissociation reaction heat is ΔH, J/kg. The heat losses into the environment are characterized by the heat-transfer coefficient $k^* = k.F/V = 4k/D$ as a fraction of the bed volume unit, where k is a coefficient of heat transfer through the wall reflecting intensity of heat transfer from furnace gases to the ambient air; F is an area of the internal surface of the furnace walls; V is a volume of the furnace work space and D is a furnace diameter.

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2. Mathematical Model

Close to the stockline level, one cubic meter of the bed contains $\rho^{o}_{m}(1-\epsilon)$, kg of solid materials where $\rho^{0}_{M}(1-\epsilon)C_0$, kg is a quantity of limestone; here $\rho^{o}_{m}$is an apparent density of the charged material (lump); ϵ is a fractional void volume of the bed; $C_0$is a content of $CaCO_3$ in the materials charged in the furnace.

According to the reaction of thermal decomposition $CaCO_3 \rightarrow CaO + CO_2 + \Delta H$, carbon dioxide is released in the quantity of 43.97 kg per 100 kg of calcium carbonate. In this case, the source intensity of gas is determined by the following equation:

$$q_g = 0.4397\rho^{o}_{m}(1-\epsilon)C_0\frac{d\,\xi}{d\,\tau}kg/(m^3.s),$$

where ξ is a degree of limestone decomposition; τ is a current time of calcination.

The outflow intensity of the material mass will be the same regarding the absolute value. For the steady-state rate of the calcining process, the current time is expressly connected with the time required for the material to reach this bed level z. If the coordinate z is directed away from the stockline level along the material flow, the proportion $d\tau = dz/w_m$ will be true, where $w_m$is a material flow velocity. Within the one-dimension model, the material flow velocity *wm* depends only on the furnace cross-section area S as w_mS = const; otherwise, the flow continuity will be broken. Based on the general equation of the local mass balance of the moving medium [2, 3], the following equations can be written:

$$\frac{d(\rho V_g)}{dz} = -0.4397C_0G^{o}_{m}\frac{d\xi}{dz} = \frac{dG}{dz}; \tag{1}$$

$$\frac{dG_m}{dz} = -0.4397C_0G^{o}_{m}\frac{d\xi}{dz}, \tag{2}$$

where ρ is a density of furnace gases being a function of z; V_g is a local volumetric gas flow rate; $G = \rho V_g = \rho\epsilon|w_g|S$ and $G_m = \rho_M(1-\epsilon)w_mS$ are mass flow rates of gas and material; w_g is a gas velocity in the bed space between the lumps; S is a furnace cross-section area; ρ_m is a local apparent density of material lumps.

Considering that $\xi = 0$ at $z = 0$, we integrate the equation (2) in the range from 0 to z. As only the degree of dissociation ξ depends on z, we have the following result:

$$G_m(z) - G^{o}_{m} = -0.4397C_0G^{o}_{m}\xi(z); G_m(z) = [1-0.4397C_0\xi(z)], \tag{3}$$

or

$$\rho_m(z) = \rho^{o}_{m}[1-0.4397C_0\xi(z)]. \tag{3,a}$$

For two successive bed levels z_1 and z_2 the equation (3) can be written as

$$G_m(z_2) - G_m(z_1) = -0.4397C_0G_m^0[\xi(z_2) - \xi(z_1)]. \tag{4}$$

In the same way, integrating the equation (1) in the range from z to H (with the degree of limestone decomposition being changed from ξ to ξ^* where ξ^* is a degree of limestone decomposition on exit from the calcining zone), we get

$$\begin{aligned} G(z) &= \rho(z)V_g(z) = \rho_{comb}V_{comb} + 0.4397C_0G_m^0[\xi * -\xi(z)] = \\ &= \rho_{comb}V_{comb} + G_m(z) - G_m(H), \end{aligned} \tag{5}$$

where ρ_{comb} is a density of injected combustion products; H is a bed height (point of gas injection).

When writing the heat transfer equations, it is necessary to consider that generally the process of thermal decomposition runs in the volume of a lump and, consequently, it is reasonable to assign the heat input for dissociation to the material. Then, the heat outflow intensity in the material $q_{t.m.}$ will be

$$q_{\text{t.m}} = -\rho_m^0(1-\epsilon)C_0\Delta H\frac{d\,\xi}{d\,\tau} = -\frac{G_m^0C_0\Delta H}{S}\frac{d\,\xi}{d\,z}. \tag{6}$$

Consequently, the following equations can be written as

$$c_mG_m\frac{dt}{dz} = \alpha_{\Sigma V}S(T-t) - G_m^0C_0\Delta H\frac{d\xi}{dz}; \tag{7}$$

$$cG\frac{dT}{dz} = \alpha_{\Sigma V}S(T-t) + k^*S(T - T_{\text{OK}}), \tag{8}$$

with the following conditions being met

$$\left.\begin{array}{lll} t = t_0 = \text{const} & \text{at} & z = 0, \\ T = T_0 = \text{const} & \text{at} & z = H. \end{array}\right\} \tag{9}$$

Here t is a bulk temperature of the material lump, °C; T_{amb} is an ambient (air) temperature, °C and T is a temperature of furnace gases, °C; c and c_m are specific heat of gas and material, J/(kg.K).

The system (1) – (9) is closed by the equation of thermal decomposition. As an earlier analysis has shown, the degree of limestone dissociation shall be calculated by two equations. At $\xi \le 0.1$ (when the dissociation front is close to the external surface of the lump) the following equation is used:

$$\xi = 0.8\frac{S}{G^{\mathrm{o}}_{\mathrm{m}}C_0}\int_{z_i}^{z}\frac{\alpha_V}{\Delta H}(T - t_{\mathrm{p}})dz, \tag{10}$$

where z_i is a bed level where the surface temperature of the lump reaches the value of t; α_V is a volumetric coefficient of heat transfer between gas and the lump surface.

The reaction temperature is set in accordance with recommendations given by N. P. Tabunschikov [4], that is, it is determined by the following equation:

$$t = 740 + 0.148 \cdot T + 0.13 \cdot CO_2, \tag{11}$$

where CO_2 is a content of carbon dioxide in furnace gases.

At $\xi \ge 0.1$ the limestone dissociation equation obtained by us from the condition of thermal flow density equality to zero in the lump center is used. This equation is approximate and needs to be validated experimentally. It is written as

$$\frac{d\xi}{dz} = \frac{12\,\lambda_e(1-\epsilon)S}{G^{\mathrm{o}}_{m}C_0\Delta H d_l^2}\,\frac{(1-\xi)^{2/3}}{1-(1-\xi)^{1/3}} \times \left\{(t_s - t_c)(1-\xi)^{2/3} + (t_{\mathrm{p}} - t_c)\left[\frac{2}{(1-\xi)^{1/3}} - 3\right]\right\}, \tag{12}$$

where λ_e is an effective conductivity of the lump; d_l is a lump diameter; t_s is a surface temperature and t_c is a lump center temperature.

The values of the surface temperature and lump center temperature required for the equation (12) can be determined in accordance with recommendations given in [5]. They are determined by the following equations:

$$t_s = T - 1.25\frac{\overline{c_m\rho_m}}{\rho^{\mathrm{o}}_m\alpha_V}\,\frac{G^{\mathrm{o}}_m}{S}\,\frac{dt}{dz}; \tag{13}$$

$$t_s - t_{\mathrm{c}} = \frac{\overline{c_m\rho_m}G^{\mathrm{o}}_m d_l^2}{24\,\lambda_e\rho^{\mathrm{o}}_m(1-\epsilon)S}\,\frac{dt}{dz} \tag{14}$$

or

$$t_s - t_c = 2.5(T - t) - 3.125\frac{\overline{c_m\rho_m}G^{\mathrm{o}}_m}{\rho^{\mathrm{o}}_m\alpha_V S}\,\frac{dt}{dz}. \tag{14,a}$$

Here the line above the value of $c_m\rho_m$ means averaging over the lump volume.

3. Mathematical Model Solution

The described mathematical model is highly nonlinear and, therefore, it can be solved only by numerical iterative methods [1, 2]. For numerical solutions, it is always more convenient to work with values commensurable to one [1]. In this regard, we reduced the system of the main equations to a nondimensional form.

We have introduced a nondimensional coordinate of $Z = z/H$ and temperature:

$$\theta = \frac{T - t_0}{T_0 - t_0}, \quad \vartheta = \frac{t - t_0}{T_0 - t_0}, \quad \theta_{\text{OK}} = \frac{T_{\text{OK}} - t_0}{T_0 - t_0}.$$

Then, the heat-transfer equations (7) and (8) can be written as follows:

$$\frac{d\vartheta}{dZ} = \frac{\alpha_{\Sigma V} V}{c_m G_m}(\theta - \vartheta) - \frac{C_0 G_m^0 \Delta H}{c_m G_m (T_0 - t_0)} \frac{d\xi}{dZ}; \tag{15}$$

$$\frac{d\theta}{dZ} = \frac{\alpha_{\Sigma V} V}{cG}(\theta - \vartheta) + \frac{k^* V}{cG}(\theta - \theta_{\text{OK}}). \tag{16}$$

The calculation of the heat-transfer coefficient through the wall k depends on the furnace dimensions. If the furnace diameter is large enough and the thickness of the wall layers is relatively small, we can use a formula that is true for a multilayer flat wall:

$$k = \left(\frac{1}{\alpha_F} + \sum_{i=1}^{n} \frac{S_i}{\lambda_i} + \frac{1}{\alpha_{ext}} \right)^{-1}, \; W/(\mathrm{m}^2 \cdot). \tag{17}$$

Here α_F and α_{ext} are heat-transfer coefficients on the internal and external surfaces of the wall; S_i is a thickness of the i-th brickwork layer and λ_i is its heat conductivity coefficient; n is a total number of the brickwork layers.

In those cases when the wall thickness is commensurable to the furnace diameter, it is necessary to consider the wall roundness:

$$k = \left(\frac{1}{\alpha_F d_1} + \frac{1}{2} \sum_{i=1}^{n} \frac{1}{\lambda_i} \ln \frac{d_{i+1}}{d_i} + \frac{1}{\alpha_{ext} d_{n+1}} \right)^{-1}. \tag{18}$$

At the first stage of the solution, we will assume that the function $\Xi(Z)$ is known. When going to finite differences in the equations (15) and (16), we will use the simplest form of approximation [1, 2], that is, the right difference, compensating first-order errors of approximation by decreasing the step size of ΔZ. For calculation of variables, we will take temperatures and degrees of dissociation from the previous step. Then, instead of equations (15) and (16) we can write as follows:

$$\frac{\vartheta_{i+1}-\vartheta_i}{\Delta Z}=\left(\frac{\alpha_V V}{c_m G_m}\right)_i(\theta_i-\vartheta_i)-\frac{C_0 G_m^{\mathrm{o}}}{(T_0-t_0)}\cdot\left(\frac{\Delta H}{c_m G_m}\right)_i\frac{\xi_{i+1}-\xi_i}{\Delta Z}, \tag{19}$$

$$\frac{\theta_{i+1}-\theta_i}{\Delta Z}=\left(\frac{\alpha_V V}{c\,G}\right)_i(\theta_i-\vartheta_i)+\left(\frac{k_* V}{c\,G}\right)_i(\theta_i-\theta_{\mathrm{OK}}), \tag{20}$$

or in an expanded form:

$$\begin{aligned}\vartheta_{i+1}=&\left[1-\left(\frac{\alpha_V V}{c_m G_m}\right)_i\Delta Z\right]\vartheta_i+\left(\frac{\alpha_V V}{c_m G_m}\right)_i\Delta Z\cdot\theta_i-\\&-\frac{C_0 G_m^{\mathrm{o}}}{(T_0-t_0)}\cdot\left(\frac{\Delta H_8}{c_m G_m}\right)_i(\xi_{i+1}-\xi_i),\end{aligned} \tag{19,a}$$

$$\begin{aligned}\theta_{i+1}=&\left[1+\left(\frac{\alpha_V V}{c\,G}\right)_i\Delta Z+\left(\frac{k_* V}{c\,G}\right)_i\Delta Z\right]\theta_i-\left(\frac{\alpha_V V}{c\,G}\right)_i\Delta Z\cdot\vartheta_i-\\&-\left(\frac{k_* V}{c\,G}\right)_i\Delta Z\cdot\theta_{\mathrm{OK}}.\end{aligned} \tag{20,a}$$

When the temperature of the material increases at the previous step, its temperature cannot decrease at the next step. Therefore, there are restrictions for the step size of ΔZ resulting from the equation (19,a):

$$1-\left(\frac{\alpha_V V}{c_{\mathrm{m}} G_m}\right)_i\Delta Z\geq 0\text{ or }\Delta Z\ \leq 1/\left(\frac{\alpha_V V}{c_m G_m}\right)_i. \tag{21}$$

We introduce the following designations:

$$A1G_i=\left(\frac{\alpha_V V}{c_m G_m}\right)_i\Delta Z;\quad A1M_i=1-\left(\frac{\alpha_V V}{c_m G_m}\right)_i\Delta Z=1-A1G_i;$$

$$A2M_i=\left(\frac{\alpha_V V}{c\,G}\right)_i\Delta Z;\quad D1_i=\frac{C_0 G_m^{\mathrm{o}}}{(T_0-t_0)}\cdot\left(\frac{\Delta H_8}{c_m G_{\mathrm{m}}}\right)_i(\xi_{i+1}-\xi_i);$$

$$D2_i=\left(\frac{k_* V}{c\,G}\right)_i\Delta Z\cdot\theta_{\mathrm{OK}};$$

$$A2G_i=1+\left(\frac{\alpha_V V}{c\,G}\right)_i\Delta Z+\left(\frac{k_* V}{c\,G}\right)_i\Delta Z=1+A2M_i+D2_i/\theta_{\mathrm{OK}}.$$

Then, the equations (19,a) and (20,a) can be rewritten in a compact form:

$$\vartheta_{i+1} = A1M_i.\vartheta_i + A1G_i.\theta_i - D1_i; \tag{22}$$

$$\theta_{i+1} = -A2M_i.\vartheta_i + A2G_i.\theta_i - D2_{i.} \tag{23}$$

We will solve the system (22), (23) by the sweep method [3]. Let's assume that

$$\vartheta i = \rho i.\theta i + \beta i, \tag{24}$$

with ρ_0 = 0, β_0 = 0. Putting (24) in the equation (22), we will obtain

$$\begin{aligned} \rho_{i+1}\theta_{i+1} + \beta_{i+1} &= A1M_i(\rho_i\theta_i + \beta_i) + A1G_i\theta_i - D1_i = \\ &= (A1M_i \cdot \rho_i + A1G_i)\theta_i + (A1M_i \cdot \beta_i - D1_i). \end{aligned} \tag{25}$$

But from the equation (23) we find

$$\begin{aligned} \theta_{i+1} &= -A2M_i(\rho_i\theta_i + \beta_i) + A2G_i \cdot \theta_i - D2_i = \\ &= (A2G_i - A2M_i \cdot \rho_i)\theta_i - (A2M_i \cdot \beta_i + D2_i). \end{aligned}$$

Consequently, we have

$$\begin{aligned} &\rho_{i+1}(A2G_i - A2M_i \cdot \rho_i)\theta_i - \rho_{i+1}(A2M_i \cdot \beta_i + D2_i) + \beta_{i+1} = \\ &= (A1M_i \cdot \rho_i + A1G_i)\theta_i + (A1M_i \cdot \beta_i - D1_i), \end{aligned}$$

wherefrom we obtain two equations:

$$\begin{aligned} &\rho_{i+1}(A2G_i - A2M_i \cdot \rho_i) = A1G_i + A1M_i \cdot \rho_i; \\ &\beta_{i+1} = A1M_i \cdot \beta_i - D1_i + \rho_{i+1}(A2M_i \cdot \beta_i + D2_i). \end{aligned} \tag{26}$$

Thus, the coefficients of the direct sweep are determined by the following equations:

$$\rho_{i+1} = \frac{A1G_i + A1M_i \cdot \rho_i}{A2G_i - A2M_i \cdot \rho_i}; \quad \rho_1 = \frac{A1G_1}{A2G_1} < 1; \tag{27}$$

$$\begin{aligned} &\beta_{i+1} = A1M_i \cdot \beta_i - D1_i + \frac{A1G_i + A1M_i \cdot \rho_i}{A2G_i - A2M_i \cdot \rho_i}(A2M_i \cdot \beta_i + D2_i); \\ &\beta_1 = -D1_1 + \frac{A1G_1}{A2G_1} \cdot D2_1. \end{aligned} \tag{28}$$

In the formulas (27), (28) *i* = 1, 2, 3, …, *N* − 1. At the last step of the direct sweep we find ρ_N, β_N and $\vartheta_N = \rho_N .\theta_N + \beta_N = \rho_N + \beta_N$. Consequently, *by the reverse sweep* for *i* = *N* − 1, *N* − 2, …, 1 from the equation system

$$\begin{aligned} A1M_i \cdot \vartheta_i + A1G_i \cdot \Theta_i &= \vartheta_{i+1} + D1_i\,, \\ -A2M_i \cdot \vartheta_i + A2G_i \cdot \Theta_i &= \Theta_{i+1} + D2_i \end{aligned} \tag{29}$$

we can calculate *successively* all *gas* temperatures, and using the formula (24) – the temperature of the material. In this case

$$\begin{aligned} \theta_i &= \frac{A1M_i(\theta_{i+1} + D2_i) + A2M_i(\vartheta_{i+1} + D1_i)}{A1M_i \cdot A2G_i + A2M_i \cdot A1G_i}\,; \\ \vartheta_i &= \rho_i \theta_i + \beta_i\,. \end{aligned} \tag{30}$$

The analysis of the obtained equations shows that if the condition (21) is satisfied all the sweep coefficients and all nondimensional temperatures are positive and do not exceed 1. A solution example is shown in Figure 1.

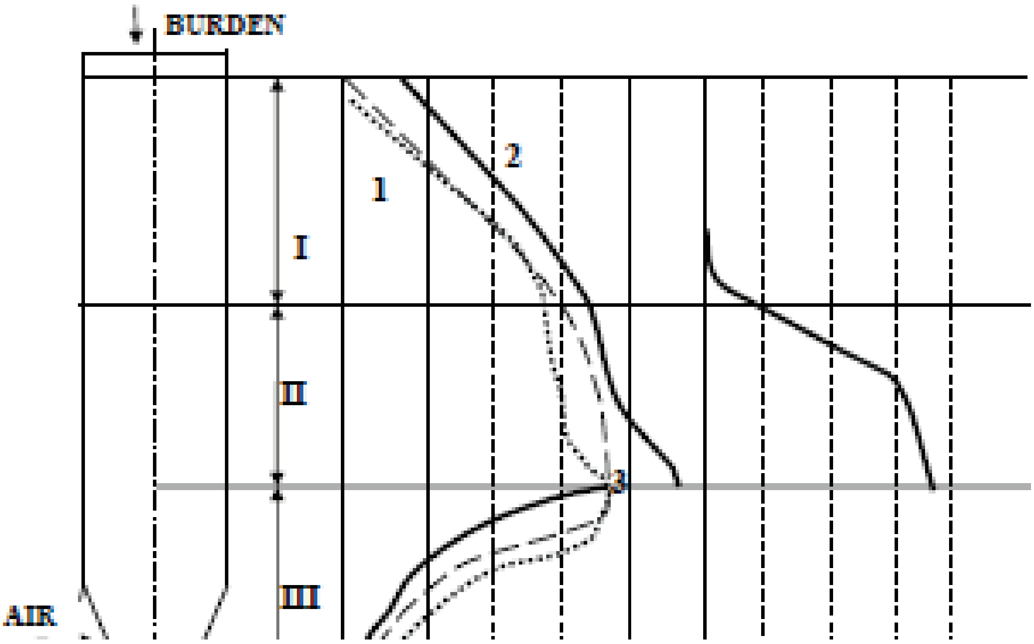

Figure 1: A solution example:I = lump heating zone, II = surface calcining zone, III = cooling zone;1 = center temperature, 2 = temperature, 3 = gas temperature.

4. Conclusion

We have developed a mathematical model of limestone calcination (dissociation) in the shaft furnace. The model makes it possible to explore different patterns of gas mechanics in the furnace, optimize its design and operating parameters, reduce power consumption for limestone production. As an initial model approximation, any experimental data or solution of a one-dimensional problem of heat transfer and dissociation can be used.

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