





Conference Paper

Spin-orbital Conversion of Bessel Light Beams By Liquid Crystal Elements

S.A. Nazarov¹, D.V. Gorbach¹, E.A. Melnikova¹, S.N. Kurilkina², and A.L. Tolstik¹

¹Belarusian State University, Minsk, Belarus ²B.I. Stepanov Institute of Physics, NAS of Belarus, Minsk, Belarus

Abstract

For the first time, the spin-orbital conversion of the linearly polarized Bessel beams in the process of their propagation in the electrically-controlled liquid crystal cell has been realized experimentally. Variations in the polarization, phase, and spatial structure of the beam have been analyzed. It has been shown that, when a Bessel beam is propagating along the liquid crystal director, the generated beam is orthogonally polarized, whereas the topological charge is varied by two unities.

Keywords: Bessel light beams, spin-orbital conversion; liquid crystals

1. Introduction

The notion of spin-orbital conversion in optics is used in studies of light beams with phase singularity. The spin angular momentum is associated with polarization of a light beam and the orbital angular momentum – with the wavefront profile and with the topological charge [1, 2]. When a light beam is propagating in a homogeneous isotropic medium, these two momenta are independent of each other. In the case of multiwave interactions in nonlinear media, there is a possibility for conversion of the spin and orbital angular momenta that is exhibited as variations in polarization of light beams and/or of their topological charges [3 – 5]. The spin-orbital interaction consisting in simultaneous variations of polarization and of the topological charge is realized in inhomogeneous or anisotropic media [6 - 8]. The features of the spinorbital conversion of the angular momentum for the light beams propagating along the optical axis of a uniaxial crystal have been studied in detail both theoretically [7, 8] and experimentally [9, 10]. However, to realize the effective conversion, one should use a crystal having a thickness of a few millimeters or centimeters. Because of this, of a particular interest is to use liquid crystals (LC) whose anisotropy is higher by twothree orders of magnitude. In so doing, due to the Fredericks effect associated with

Corresponding Author: A.L. Tolstik tolstik@bsu.by

Received: 28 January 2018 Accepted: 15 March 2018 Published: 25 April 2018

Publishing services provided by Knowledge E

© S.A. Nazarov et al. This article is distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use and redistribution provided that the

redistribution provided that the original author and source are credited.

Selection and Peer-review under the responsibility of the PhIO Conference Committee.



the LC director reorientation under the effect of an electric field, one is enabled to implement the control of the spin-orbital interaction.

This paper presents a study into the features of conversion of the spin and orbital angular-momentum components for a Bessel light beam propagating in the electrically controlled LC cell with a LC layer thickness as low as 20 μ m.

2. Experimental setup

The LC cell used (see a scheme in Fig. 1) is a sandwich-type structure comprising two glass plates (1) with the layers of indium oxide (ITO) (2) approximately 100 nm thick, step-by-step applied to the inner sides of the plates, and of a photopolymeric orientant (3) about 400 nm thick. The initial planar orientation of the LC director was attained by nonpolarized UV irradiation of the preliminary rubbed photopolymeric layer (3) [11]. In the process of work we have used the positive nematic LC 1289. Optical anisotropy of this LC at the wavelength λ =633 nm comes to $\Delta n = 0.16$ (the refractive index for an ordinary wave being n_o =1.53 and for an extraordinary wave – n_e =1.69). A thickness of the LC layer in a flat capillary, determined by the size of the diameter-calibrated polymeric fibers (4), is 20 µm. In this geometry of the molecular orientation the planar oriented LC layer is similar to the uniaxial crystal with the optical axis lying within the cell plane (Fig. 1, *a*).



Figure 1: Schematic of an LC cell and the characteristic orientation of the LC director without (*a*) and with the voltage applied (*b*).



The application of an external electric field with a strength exceeding the threshold value initiates the director reorientation process, whereas the LC molecules tend to line up along the control-field strength lines. The molecular reorientation process begins from the center of the LC layer, extending to the wall layers with the growing field strength. When the field strength is approximately ten times higher than the threshold (the voltage threshold for the LC used comes to U=1.1 V), practically all the molecules over the LC layer thickness line up along the field E to lead to the homeotropic orientation of the LC director. A homeotropically oriented layer in the LC cell is similar to the classical uniaxial crystal with the optical axis directed perpendicular to the cell plane (Fig. 1, b). In this way we can implement a device capable to control the optical axis orientation of an anisotropic medium by means of the applied voltage.

3. Theoretical description

Let a linearly-polarized Bessel light beam be incident on the homeotropically oriented LC cell. We neglect the diffraction effects in the central (axial) region of the Bessel beam as a thickness of the cell is too low, while the Bessel-beam cross-section is comparatively large. Then, in analogy with the uniaxial crystal case [12], the zero-order Bessel light beam with a circular polarization to the right (to the left), normally incident on the LC cell, is exciting in the medium a superposition of the *o*- and *e*-type Bessel beams and we have

$$E^{\pm}(r,\phi,z) = \left[E_o^{\pm}(r)\exp(\frac{i\Delta kz}{2}) + E_e^{\pm}(r)\exp(-\frac{i\Delta kz}{2})\right]\exp[i(k_z z + m\phi)],$$
(1)

$$E_{o,e}^{+}(r) = A\sigma \left[J_{m-1}(qr)e_{+} \exp(-i\phi) \pm J_{m+1}(qr)e_{-} \exp(i\phi) \right],$$
 (2)

$$E_{o,e}^{-}(r) = A\sigma \left[J_{m+1}(qr)e_{-}\exp(i\phi) \pm J_{m-1}(qr)e_{+}\exp(-i\phi) \right],$$
(3)

where $J_m(x)$ - *m*-th order Bessel function; *A*-constant; the parameter σ equals 1 or $\cos \gamma_e \approx 1$ for the *o*- or *e*-type Bessel beams, respectively; $q = k_0 n_r$ - radial wave number; $n_r = n_e(\gamma_e) \sin \gamma_e = n_o \sin \gamma_o = n_1 \sin \gamma_1$, n_1 - refractive index of an isotropic medium adjoining the LC cell; $n_e(\gamma_e)$ - refractive index of the *e*-type partial wave formed within the cell; $k_0 = (2\pi/\lambda)$, γ_1 , γ_o , γ_e - cone angles of the incident Bessel beam, *o*-type and *e*-type Bessel beams within the cell, respectively; $e_{\pm} = (e_1 \pm e_2)/\sqrt{2}$ - unit vectors of circular polarizations to the right and to the left, respectively; $k_z = (k_{ze} + k_{zo})/2$, $\Delta k =$



 $k_{zo} - k_{ze}, k_{zo} = (2\pi/\lambda)\sqrt{k_0^2 n_o^2 - q^2}, k_{ze} = (2\pi/\lambda)(n_o/n_e)\sqrt{k_0^2 n_e^2 - q^2}$. Then, for the electric strength vector at the output of the cell with the thickness D, we obtain the following:

$$E^{\pm}(r,\phi,D) = \tilde{A} \left[J_{m\mp1}(qr) \cos(\frac{\Delta kD}{2}) e_{\pm} \exp(\mp i\phi) + i J_{m\pm1}(qr) \sin\left(\frac{\Delta kD}{2}\right) e_{\mp} \exp(\pm i\phi) \right] \exp[i(k_z D + m\phi)].$$
(4)

Here $\tilde{A} = 2At$, $t = t_o \approx t_e$ - amplitude reflection factor. Expression (4) makes it possible to study the features of the conversion of a Bessel light beam in a LC cell if the incident Bessel beam is circularly polarized or represented by a superposition of circularly polarized beams. In so doing, for a special case of the incident nonvortex Bessel beam circularly polarized to the right (left), in relation (4) we should assume that m = 1 (m = -1).

As follows from (4), when a zero-order linearly polarized Bessel light beam is incident on a LC cell, the field E(r) at the cell output may be given as

$$E(r, D) = \tilde{A} \left[J_0(qr) \cos(\Delta k D/2) e_1 + i J_2(qr) \sin(\Delta k D/2) (e_1 \cos 2\phi + e_2 \sin 2\phi) \right] \times$$

$$\times \exp[ik_z D].$$
(5)

Here e_1 – unit vector collinear with the transmission direction of a polarizer; ϕ – azimuthal angle in the output plane of the cell.

The linearly polarized Bessel light beam is represented as a superposition of Bessel beams circularly polarized to the right and to the left. The angular momentum of the incident zero-order Bessel light beam circularly polarized to the right (left) has the spin component $\sigma^{\pm} = \pm \hbar$ (for each photon) only. In the general case at the output face of the cell two beams are formed: the zero-order Bessel beam circularly polarized to the right (left) carrying the spin angular momentum $\sigma^{\pm} = \pm \hbar$ (for each photon) and the second-order Bessel beam circularly polarized to the left (right) possessing both spin $\sigma^{\mp} = \mp \hbar$ and orbital $l^{\pm} = \pm 2\hbar$ angular momenta (for each photon). In such a manner in the linearly polarized Bessel light beam propagating within a LC cell one can simultaneously observe two types of spin-orbital conversion. In the first case the zero-order Bessel beam circularly polarized to the right, with the angular momentum $\sigma^+ = \hbar$ (for each photon), is converted to the second-order Bessel beam circularly polarized to the left, with the total angular momentum $\sigma^{-} + l^{+} = \hbar$ (for each photon). At the same time, the Bessel beam circularly polarized to the left, with the angular momentum $\sigma^{-} = -\hbar$ (for each photon), is converted to the second-order Bessel beam circularly polarized to the right, with the total angular momentum $\sigma^+ + l^- = -\hbar$ (for each photon).



As follows from equation (5), in the case when a Bessel light beam is passing a system including crossed polarizer and analyzer with a LC cell between them, the radiation intensity is given by the following expression:

$$I_{+2}(r) \sim A^2 J_2^2(qr) \sin^2(\Delta k d/2) \sin^2 2\phi.$$
 (6)

As seen from expression (6), at the output of the above-mentioned system we should observe a spatial four-lobe interference structure pointing to the spin-orbit conversion in the LC cell [12, 13]. Note that I_{+2} is dependent on the path difference Δk of two (*o*- and *e*-type) Bessel light beams within the cell, being maximal on fulfillment of the following condition:

$$\Delta kd = (2l+1)\pi, l = 0, \pm 1...$$
(7)

4. Results and discussion

An experimental study of the above conversion processes of a Bessel light beam has been performed using a scheme shown in Fig. 2. Radiation of a helium-neon laser 1 with the wavelength 632.8 nm was transmitted through a system of spatial filtering 2 – 4. Objective 2 focused radiation to a pinhole 3 with the diameter 30 µm. The generated Gaussian beam was collimated by a lens 4 forming the beam ~ 2 cm in diameter. Linear polarization of the beam was set by a polarizer 5. The polarized beam was incident on an axicon 6 with a base angle of ~ 30°. The formed zero-order Bessel light beam with a cone angle of ~ 18° traversed a LC cell 7 with a thickness of ~ 20 µm. The control AC voltage at the frequency 1 kHz was applied to the cell. Radiation was focused by the second axicon 8 to pass through an analyzer 9. The far-field image of the formed beam was recorded by a camera 11 with the help of an objective 10.



Figure 2: Schematic of an experimental setup.

To study the features of the Bessel light beam conversion depending on the applied control voltage, variations in the output-field intensity distribution in the far-field region have been analyzed using the scheme with crossed polarizer and analyzer. In the absence of an electric field, we have observed the low-intensity circular distribution characteristic for the zero-order Bessel beam (Fig. 3, a). With the voltage increased



up to 6 V, the circular distribution has been retained but the intensity was growing significantly due to changes in the polarization state (Fig. 3, b). At the voltage above 8 V, the circular distribution became discontinuous, having two (upper and lower) breaks (Fig. 3, c, d), whereas at the voltage above 12 V, the formation of four sectors was observed (Fig. 3, e, f), pointing to the beginning of the spin-orbital conversion and to the fact that the orbital angular momentum of the beam became equal to two. With further increase in voltage, the sectors became more symmetric and, beginning from 20 V, their position was actually invariable (Fig. 3, g, h). Such an azimuthal intensity modulation in the form of four sectors characteristic for the second-order Bessel light beam was observed with the use of uniaxial crystals [12, 13].



Figure 3: Transverse intensity distribution of a Bessel light beam in the far field at crossed polarizer and analyzer under the applied voltage U = 0 (a), 6 (b), 8 (c), 10 (d), 12 (e), 15 (f), 20 (g), and 40 V (h).

This scenario of variations in the intensity spatial distribution agrees well with the plot for the energy factor of conversion to the orthogonal polarization component measured at crossed polarizers (Fig. 4).

The measured function is typical for transmission of a LC cell at crossed polarizers and describes the transient processes on reorientation of the LC director. As seen, peaks of the conversion factor are observed even for voltages of a few volts, being indicative of variations in polarization of the light beam exiting the LC cell. Passing to the plateau at voltages above 20 V bears witness to actually complete reorientation of the LC director. Note that the measured value of the conversion factor $\eta \cong 0.18$ is in a good agreement with the theoretical computations performed with the use of equations (4) for the above-mentioned experimental conditions.

5. Conclusion

In this way, the obtained results demonstrate the potentialities of using liquid crystals for conversion of the polarization, phase, and spatial structure of Bessel light beams. The main advantages of LC cells used to form the described fields are the following:





Figure 4: The parameter η as a function of the voltage applied to the cell.

small thickness of an LC layer ($\sim 20 \ \mu m$) and possibility for electric switching. The conversion of a part of the zero-order Bessel beam to the second-order beam, with their subsequent interaction, is of particular interest for solving problems of optical microscopy during the formation of diffractionless light beams with the desired properties.

References

- Allen L., Beijersbergen M.W., Spreeuw R.J.C., Woerdman J.P. Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes // Phys. Rev. 1992. A 45. P. 8185–8189.
- [2] *Vasnetsov M., Staliunas K. (Eds.)*. Optical Vortices. V.228 of Horizons of World Physics. Huntington, N.Y.: Nova Science, 1999. 313 p.
- [3] *Romanov O.G., Gorbach D.V., Tolstik A.L.* Transformation of optical vortices by polarization dynamic holograms // Optics and spectroscopy. 2013. V. 115. № 3. P. 335-339.
- [4] Gorbach D.V., Nazarov S.A., Romanov O.G., Tolstik A.L. Polarization transformation of singular light beams upon four- and six-wave mixing // Nonlinear phenomena in complex systems. 2015. V. 18. № 2. P. 149-156.



- [5] Tolstik A.L. Singular dynamic holography // Russian Physics Journal. 2016. V. 58. №
 10. P.1431–1440.
- [6] *Liberman V., Zeldovich B.* Spin-orbit interaction of the photon in an inhomogeneous medium // Phys. Rev. 1992. A 46. P. 5199.
- [7] *Ciattoni A., Cincotti G., Palma. C*. Circularly polarized beams and vortex generation in uniaxial media // J.Opt.Soc.Am. A. 2003. V. 20. P. 163-171.
- [8] Pogrebnaya, A. O.; Rybas, A. F. Evolution of a circularly polarized beam bearing an optical vortex with fractional topological charge in a uniaxial crystal // Journal of Optical Technology. 2017. V.83. P.586-589.
- [9] Brasselet E., Elzdebskaya Y., Shvedov V., Desyatnikov A.S., Krolikowski W., Kivshar Y.S. Dynamics of optical spin-orbit coupling in uniaxial crystals // Opt. Lett. 2009.
 V. 34. P. 1021-1023.
- [10] Loussert C., Brasselet E. Efficient scalar and vectorial singular beam shaping using homogeneous anisotropic media // Opt. Lett. 2010. V. 35. P. 7–9.
- [11] Mahilny U., Trofimova A., Stankevich A., Tolstik A., Murauski A., Muravsky A. New photocrosslinking polymeric materials for liquid crystal photoalignment // Nonlinear Phenomena in complex systems. 2013. V. 16. P. 79-85.
- [12] *Khilo N.A., Petrova E.S, Ryzhevich A.A.* Transformation of the order of Bessel beams in uniaxial crystals // Quantum electronics. 2001. V. 31. P. 85-89.
- [13] Belyi V.N., Khilo N.A., Kazak N.S., Ryzhevich A.A., Forbes A. // Optical Engineering.2011. V. 50. P.059001.