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Field Distribution into Binary Linear Waveguide Array

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Abstract

The binary (two-component) linear waveguide array that is formed either from identical waveguides but with alternating distances between adjacent waveguides, or waveguides with a positive refractive index, but having distinctions in the refractive index or the waveguide thickness. The exact solution of the coupled wave equations describing the field distribution over waveguides is found. These solutions describe the discrete diffraction in the model under consideration.

Keywords: coupled waveguides, binary array, discrete diffraction, generation function method.

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1. Introduction

The one dimensional array (or chain) of the waveguides in the linear approximation represents the system for discrete diffraction realization. In this paper we will consider the binary linear waveguide array. The unit cell contains two kind of the waveguide, i.e., A-type and B-type. In slowly varying envelopes of the electric field of the continue wave radiation in waveguide is governed by the system of equations for coupled waves [1, 2]. If we would like to consider only discrete diffraction [2, 3] then the nonlinear properties of the waveguides may be neglected. It cab be considered as the first approximation.

The system of the equations describing the electromagnetic wave propagation in the coupled wave approximation takes the following form

$$i\partial_\zeta A_n = B_n + B_{n-1}, \qquad i\partial_\zeta B_n = A_n + A_{n+1}, \tag{1}$$

where A_n and B_n are normalized amplitudes of the electric field in waveguide from n-th unit cell, ζ is the normalized coordinate [1]. We assume that the phase mismatch is zero.

2. Analytical solution of the base equations

To obtain the solution of the system of equations (1) we can use the generation function method. Let us introduce the following functions

$$P_A(\zeta, y) = \sum_n A_n(\zeta)e^{iyn}, \qquad P_B(\zeta, y) = \sum_n B_n(\zeta)e^{iyn}$$

Using the equations (1) we can find the equations for the generation functions P_A and P_B:

$$i\partial_\zeta P_A = (1 + e^{iy})P_B, \qquad i\partial_\zeta P_B = (1 + e^{-iy})P_A \tag{2}$$

From (1) it follows equation $\partial^2_\zeta P_A + \Omega^2 P_A = 0$, where $\Omega^2 = 4\cos^2(y/2)$. The solution of this equation takes the form

$$P_A(\zeta, y) = C_1 e^{i\Omega(y)\zeta} + C_2 e^{-i\Omega(y)\zeta}$$

The expression for the generation function P_B follows from the first equation of (2). If the initial conditions for the amplitudes A_n and B_n are known, we can define the initial conditions for the generation function

$$P_A(0, y) = P_{A0} = \sum_n A_n(0)e^{iyn}, \qquad P_B(0, y) = P_{B0} = \sum_n B_n(0)e^{iyn}.$$

The initial conditions for P_A and P_B allows us to determine integration constants C_1 and C_2. Thus the generation function can be written as

$$P_A(\zeta, y) = P_{A0}(y)\cos\Omega(y)\zeta - ie^{iy}P_{B0}(y)\sin\Omega(y)\zeta, \tag{3}$$

$$P_B(\zeta, y) = P_{B0}(y)\cos\Omega(y)\zeta - ie^{-iy}P_{A0}(y)\sin\Omega(y)\zeta, \tag{4}$$

where $\Omega = 2\cos(y/2)$. Using the orthogonality condition

$$\int_{-\pi}^{\pi} e^{iky}dy = 2\pi\delta(k),$$

the solution of the initial system of equations (1) can be written as

$$2\pi A_n(\zeta) = \int_{-\pi}^{\pi} e^{-iny}P_A(\zeta, y)dy, \qquad 2\pi B_n(\zeta) = \int_{-\pi}^{\pi} e^{-iny}P_B(\zeta, y)dy. \tag{5}$$

3. Particular examples of the field distribution over waveguides

Let us consider the following initial condition that corresponds to strong focusing radiation at $\zeta = 0$: $A_n(0) = A_0\delta_{n0}$ and $B_n(0) = 0$. In this case $P_{A0} = A_0$ and $P_{B0} = 0$. By the use the expressions (3), (4) and (5) we can write

$$2\pi A_n(\zeta) = A_0 \int_{-\pi}^{\pi} e^{-iny} \cos \Omega(y)\zeta dy, \qquad 2\pi B_n(\zeta) = -iA_0 \int_{-\pi}^{\pi} e^{-iny-iy/2} \sin \Omega(y)\zeta dy.$$

To compute the integrals in these expressions we may use the formula by Anger, which in this case is looking like

$$\cos(\eta \cos y/2) = J_0(\eta) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(\eta) \cos ky, \tag{6}$$

$$\sin(\eta \cos y/2) = -2\sum_{k=1}^{\infty}(-1)^k J_{2k-1}(\eta) \cos[(2k-1)y/2]. \tag{7}$$

Substitution of the equation (6) into (5) results in the following expression (here we will use the term $\eta = 2\zeta$)

$$2\pi A_n(\zeta) = A_0 J_0(\eta) \int_{-\pi}^{\pi} e^{-iny} dy + A_0 \sum_{k=1}^{\infty}(-1)^k J_{2k}(\eta) \int_{-\pi}^{\pi} e^{-iny}\left(e^{iky} + e^{-iky}\right) dy.$$

Hence, at n=0 we have $A_0(\zeta) = A_0 J_0(2\zeta)$. At $n \geq 1$ it follows that

$$A_n(\zeta) = (-1)^n A_0 J_{2n}(2\zeta). \tag{8}$$

The second equation of (1) may be used to obtain the amplitudes $B_n(\zeta)$. By using the expression (8) for $A_n(\zeta)$, one can write

$$A_{n+1} + A_n = A_0(-1)^n \left[J_{2n}(\eta) - J_{2n+2}(\eta)\right].$$

As

$$\frac{dJ_n(z)}{dz} = \left[J_{n-1}(z) - J_{n+1}(z)\right],$$

the equation for $B_n(\zeta)$ can be rewritten as

$$i\partial_\zeta B_n = 2i\frac{d}{d\eta}B_n = A_0(-1)^n \frac{d}{d\eta}J_{2n+1}.$$

It follows that

$$B_n(\zeta) = -iA_0(-1)^n J_{2n+1}(2\zeta). \tag{9}$$

Thus the distribution of the field amplitudes over waveguides in array under considered initial conditions is presented by the expressions (8) and (9). These equations describe the discrete diffraction in binary waveguide array under the condition of the strong focusing at $\zeta = 0$.

Let us consider the case where the both waveguides in unit cell $n = 0$ are illuminated. Initial conditions are following $A_n(0) = A_0\delta_{n0}$ and $B_n(0) = B_0\delta_{n0}$. In this case we have $P_{A0} = A_0$ and $P_{B0} = B_0$. The generation function can be written as

$$P_A(\zeta, y) = A_0 \cos\Omega(y)\zeta - ie^{iy}B_0 \sin\Omega(y)\zeta,$$

$$P_B(\zeta, y) = B_0 \cos\Omega(y)\zeta - ie^{-iy}A_0 \sin\Omega(y)\zeta.$$

Using the (5) we can write

$$2\pi A_n(\zeta) = A_0 \int_{-\pi}^{\pi} e^{-iny} \cos\Omega(y)\zeta\, dy - iB_0 \int_{-\pi}^{\pi} e^{-iny+iy/2} \sin\Omega(y)\zeta\, dy,$$

$$2\pi B_n(\zeta) = B_0 \int_{-\pi}^{\pi} e^{-iny} \cos\Omega(y)\zeta\, dy - iA_0 \int_{-\pi}^{\pi} e^{-iny-iy/2} \sin\Omega(y)\zeta\, dy$$

Two incoming here the integrals have been found previously

$$\int_{-\pi}^{\pi} e^{-iny} \cos\Omega(y)\zeta\, dy = 2\pi \left[J_0(\eta)\delta_{n0} + (-1)^n J_{2n}(\eta)\right],$$

$$\int_{-\pi}^{\pi} e^{-iny-iy/2} \sin\Omega(y)\zeta\, dy = 2\pi(-1)^n J_{2n+1}(\eta).$$

The third integral can be defined by the similar way. It results in following expression

$$\int_{-\pi}^{\pi} e^{-iny+iy/2} \sin\Omega(y)\zeta\, dy = 2\pi(-1)^{n+1} J_{2n-1}(\eta), n \geq 1.$$

So can immediately write down expressions for the distributions of the electric field amplitudes over waveguides for the selected initial conditions

$$A_n(\zeta) = A_0 J_0(2\zeta)\delta_{n0} + (-1)^n A_0 J_{2n}(2\zeta) + i(-1)^n B_0 J_{2n-1}(2\zeta),$$

$$B_n(\zeta) = B_0 J_0(2\zeta)\delta_{n0} + (-1)^n B_0 J_{2n}(2\zeta) + i(-1)^n A_0 J_{2n+1}(2\zeta),$$

where $n = \pm 1, \pm 2, \ldots$. The third terms in these expressions represent the interference phenomena in waveguide array.

4. Conclusion

This distribution of field strengths describes discrete diffraction in a binary array of waveguides. For more complex cases of the initial conditions the expressions for the electric fields in waveguides contain terms that account for interference fields in the neighboring waveguides. It should be noted that if the initial conditions are selected as $A_n(0) = (-1)^n A_0$ and $B_n(0) = (-1)^n B_0$, then the diffraction is absent. The equations (1) show that under these conditions the fields in waveguides are invariants. However, the flat band [5–7] in the spectrum of the linear waves is absent, as the number of nodes in the unit cell is less than three. In the spectrum there are only two branches that meet dispersive waves along the chain.

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