





Conference Paper

Polariton Propagation in Imperfect Resonantly Absorbing Bragg grating

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Abstract

The nonlinear polariton transmission through resonantly absorbing Bragg grating (RABG) with randomly varying lattice spacing is studied. In this work are presented the results of numerical simulation of propagation stability of polaritonic solitary wave consisting of matter wave coupled with counter propagating light waves which propagate through dielectric medium, containing periodically placed in the dielectric waveguide thin dielectric films with metallic nanoparticles (or quantum dots, nanoagregates with nonlinear dielectric properties). The influence of lattice spacing deviations from mean value (average) of the lattice spacing is described in the model equations by random multiplicative noise set in the phases of forward and backward electric components of electromagnetic wave. As the initial condition it was used the solitary wave solution of the model describing nonlinear wave propagation in perfect nonlinear RABG. The results of numerical simulations show that phase fluctuations lead to periodic oscillations (with the period 2π) in the ratio of the amplitude and the width of polariton. This ratio is a constant in the perfect periodic nonlinear Bragg grating. Oscillations in the ratio of the amplitude and the width of polaritonic solitary wave decrease with its propagation in imperfect nonlinear Bragg grating.

Keywords: solitons, polaritons, Bragg grating.

1. Introduction

The beginning of the last century was the fruitful period that has arisen the new branches of physics concerned in the structure of the matter, for example atomic and quantum physics, condensed matter physics. The interest to very promising properties of semiconductor materials resulted in development of physics of semiconductors that in turn caused the origin of the era of electronics. It is hardly imagine our daily life without devices containing integrated circuits. Revolutionary invention of the laser in the 1957 and the following progress in the integrated optics and optical fiber fabrication gave rise to the photonics. One of its branches, namely optical fiber communication,

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developed considerably during more than 30 years. In the beginning of conventional fiber optics the principal requirement to the fiber media was its ability to guide light pulses without distortion for the most long distance. The other inherent characteristics such as inhomogeneity, dispersion and nonlinearity mostly were considered as side and often undesirable kinds. The modern technology of fiber fabrication successfully overcomes all the difficulties caused by the material properties of the fiber: the profile of the fiber can be made uniform, the losses are very low (~0.2 dB/km) and can be compensated by amplifiers, and the dispersion can be managed.

In the conventional fibers light propagates due to the total internal reflection which is implies that the refractive index of the core is high comparing with the cladding. The common hollow core fibers are leaky for the light waves, however the improvement of their waveguide properties based on the Bragg reflection of the transverse modes was proposed [1]. The idea was to create a fiber with refractive index periodically changing in the cross section along the radius of the fiber. The solid-core concentric structure made using method of modified chemical vapor deposition (MCVD) was reported [2]. A detailed review related to the state of art in the fabrication and applications of the photonic crystals is given in [3]. The interesting idea of hollow core photonic bandgap fiber fabrication was reported in [4]. The photonic crystal fiber (PCF) preform was constructed from a bunch of stacked cylinder silica capillaries. The silica canes were removed from the center of the preform to create a hollow core structure. After being fused and drawn down the fiber with perforated cross section and the hole in the center was obtained. Another technique to produce PCF from the material with low melting temperature is the extrusion. Comparing to the previously described method the extrusion can provide a huge variety of the fiber cross-section profiles. The typical materials here are the polymers [5] as well as tellurite glass or chalcogenides. The advantages of hollow core PCF are in the enhancing of the nonlinear processes at high intensities (harmonic generation, stimulated Raman and Brillouin scattering). Having intrinsic filtering properties they also might serve as spectral filtering devices. Filling the hollow core of fibers with gases, molecular aggregates, bacteria or viruses opens implementations of hollow core PCF in the biology and in chemistry. The guidance of the dielectric polystyrene particles of micron sizes under radiation pressure of argon ion laser in the hollow core photonic bandgap fiber was reported in [6]. Advantages of this non-intrusive technique are self-evident.

The presence of the gap in the permitted range of the linear wave spectra that is arises due to the structure periodicity offered an incentive in fabrication of the fiber waveguides with its refraction index periodically varying along the fiber length. Fiber





Bragg gratings are produced using phase mask created by electron beam lithography. In [7] the stitching error-free 100 mm phase mask was constructed for production of the fiber Bragg gratings. The new technique for simultaneous laser writing [8] of Bragg gratings by UV laser on photosensitive germanium doped silica was reported in [9]. The review on technology of fiber Bragg grating fabrication is given in [10].

The methods for fabrication the periodic structure with nanometer-scale thin films containing metallic nanoparticles or molecules are the electron beam deposition and layer-by-layer adsorption technique [11].

Though the linear properties of light coupled with excitations of the medium inspire many researchers in the optics of nanomaterials, the consideration of nonlinear response of the medium will lead to understanding of much more complex and intriguing optical phenomena. In [12] it was considered the phenomenon of second harmonic generation in the nonlinear medium containing resonant metallic nanoparticles, and it was found the solitary wave solution for the wave of medium polarization coupled with electric field of the light pulse. In the papers [13-15] the model is developed describing the ultrashort (comparing to the relaxation time in the medium) light pulse propagating through the ideal periodic Bragg grating with resonant metallic nanoparticles. The nonlinear dielectric properties of metallic nanoparticles implanted into the thin dielectric films, which periodically placed in the optical wavequide, are described by cubically nonlinear Duffing equation. This model of nonlinear oscillator also properly describes many other nanoaggregates having dielectric properties such as ensembles of quantum dots, atoms and dipole molecules. The authors of the works [13-14] found the solitary wave solution for the electric field of the light pulse coupled with nonlinear polarization of the medium. The solution exists even for the case of inexact resonance between the frequency of dimensional quantization of nanoparticles and carrier frequency of the electromagnetic wave, as well as for inexact resonance between the wave vector of reciprocal lattice - Bragg vector and the wave vector of the light wave. The excited state of the medium polarization coupled with the light wave is named polariton [16-17]. The dispersion relation for the model considered in [13-14] corresponds to polaritonic spectrum.

Recently there was a splash of interest in the localization of light in the disordered media [18-20]. It is noteworthy that dispersion relation will give the answer concerning the possibility of light localization in the medium. For example in the one dimensional model the presence of the defect in the lattice will gives the peak in the density of states. As the group velocity is defined by the formula $v_g = \frac{\partial \omega}{\partial k}$, where ω is the frequency and k is the wave vector of the electromagnetic wave, the points where

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 $v_g \rightarrow 0$ are the frequencies of the localized modes. Light pinning by the defect in a nonuniform resonant structure is reported in [21]. Depending on the strength of the defect the pulse can pass through with low radiation or to localize in the defect. The action of the second pulse could lead to depinning of the initial pulse or to trapping both of them in the defect. Trapping Bragg solitons by a pair of localized defects is demonstrated in [22]. This problem is very interesting for the experiments on "standing light" and from the viewpoint of working up the devices with light by light controlling.

In the paper [23] a model of distributed Bragg grating was considered with taking into account inhomogeneity of density of resonant atoms in nonlinear medium to investigate numerically the propagation of polaritons, coupled solitary waves of polarization and electric field, in such inhomogeneous medium. In the numerical simulation a solitary wave solution derived by the authors of papers [13-14] is used as the initial condition to study the propagation and scattering of the solitary waves at the density defects. The three types of defects: microcavity - linear medium without resonant nanoparticles, groove - the defect span with reduced density of nanoparticles, and stripe - the defect with high density of resonant atoms were discussed. The nonlinear polariton transmission and scattering at defects, trapping of solitary wave in microcavity placed in the resonantly absorbing Bragg grating are demonstrated by numerical simulations.

The Bragg gratings are usually fabricated by laser writing on the photoresist with subsequent etching of the unexposed regions, or by the chemical layer deposition methods. In the fabrication process the inaccuracy of measurement of the distance between layers and inexactitude of the layer deposition is always exists. There are two types of the errors can occur: the former is when the error is determined only by measurement of the whole sample (laser writing). In this case during the fabrication process the error in the distance between resonant films is corrected in such a way that the error at the length of the sample is comparable to the measurement error. In the second case the error accumulates all deviations of lattice period if consequent deposition of the layers continues without control of the length of the waveguide, for example in layer-by-layer adsorption using method of modified chemical vapor deposition. If the layers are deposited one by one and the width of each dielectric layer is checked, the error in the total length will grow as a square root of the length of the sample. In this technological process the width of one layer is not correlated with the neighbors.

The purpose of the numerical simulation discussed in this work is to investigate the evolution of the ultrashort electromagnetic pulse propagating through the Bragg





grating formed by the layers of resonant nanoparticles almost periodically placed in the host dielectric waveguide. As the method of the grating fabrication by the laser is more precise, here it is considered the case when the layers of resonant nanoparticles are arranged so that the distance between layers can be slightly small or large comparing to the average value of the lattice spacing, but the total error is not exceeds the error of measurement. The influence of such slight deviations is described by random multiplicative noise in the phases of the forward and backward components of the electric field of the electromagnetic wave.

2. Methods

It is performed numerical simulation of ultrashort electromagnetic pulse propagation in the waveguide medium consisting of almost periodically placed in the host dielectric waveguide thin dielectric films with metallic nanoparticles. Periodicity of the waveguide results in strong scattering of the incident electromagnetic wave propagating in the Bragg grating. In the model equations both forward and backward components of electric field of electromagnetic wave are taken into account. The length of the ultrashort electromagnetic pulse with picosecond duration is equal to hundreds of double periods of the grating. The wavelength of the carrier wave of the light pulse is comparable with a period of the grating, which is defined by the distance between the adjacent thin films with resonant nanoparticles. Thus the condition of Bragg resonance is almost fulfils. The model equations describing polaritonic wave propagating in the periodic Bragg grating in the approximation of the slowly varying amplitudes were obtained in [13]:

$$i\left(\frac{\partial e_1}{\partial \zeta} + \frac{\partial e_1}{\partial \tau}\right) + \delta e_1 = -p, \tag{1.1}$$

$$i\left(\frac{\partial e_2}{\partial \zeta} - \frac{\partial e_2}{\partial \tau}\right) - \delta e_2 = p, \tag{1.2}$$

$$i\frac{\partial p}{\partial \tau} + \Delta p + \mu |p|^2 p = -(e_1 + e_2).$$
 (1.3)

Dimensionless variables $e_{1,2}$ are the slowly varying amplitudes of the electromagnetic field and p is the slowly varying polarization of the medium, which is determined by the periodic resonant inclusions of layered nanoparticles. Here $\Delta = 2\sqrt{\epsilon}(\omega_d - \omega_0)/\omega_p$ is the detuning of a nanoparticle's frequency of dimensional quantization from the carrier frequency of ultrashort electromagnetic pulse. $\delta = 2\Delta q_0 c/\omega_p$ is a wave vector detuning, with $\Delta q_0 = 2k_0 - q_0$, where $q_0 = 2\pi/a_0$ is the wave vector of the lattice



and a_0 is a lattice constant. $k_0 = \omega_0 \sqrt{\epsilon}/c$ is a wave vector of the light wave in the media with permittivity ϵ , ω_0 is the carrier wave frequency, ω_p is the plasma frequency defined by concentration of metallic nanoparticles, ω_d is the frequency of dimensional quantization of nanoparticles.

The solitary wave solution found in [13] reads as

$$e_{1}(\eta) = -0.5(1+\alpha)f_{s}(\eta)\exp(i\delta\tau),$$

$$e_{2}(\eta) = -0.5(1-\alpha)f_{s}(\eta)\exp(i\delta\tau), \ p(\eta) = q(\eta)\exp(i\delta\tau),$$
(2)

 $\eta = \tau - \alpha \zeta$, α is a solitary wave parameter, $\beta = 2/(\alpha^2 - 1)$, $f_s = u \exp(i\phi)$, $q = r \exp(i\psi)$,

$$u^{2}(\eta) = \frac{4\beta\sqrt{\beta}}{\cosh[2\sqrt{\beta}(\eta - \eta_{0})]}, r^{2}(\eta) = \frac{4\sqrt{\beta}}{\cosh[2\sqrt{\beta}(\eta - \eta_{0})]}.$$
(3)

$$\phi(\eta) = \phi_0 \pm \arctan \tanh[\sqrt{\beta}(\eta - \eta_0)], \psi(\eta) = \psi_0 \pm 3 \arctan \tanh[\sqrt{\beta}(\eta - \eta_0)].$$
(4)

Initial phases are set in a such way that $\Phi_0 = \phi_0 - \psi_0 = \pi/2$ at $\eta \to -\infty$.

The model which describes ultrashort electromagnetic pulse propagation in perfectly periodic structure with spatially localized in the grating layer with inhomogeneity of density of resonant atoms in nonlinear medium was considered in the paper [23]. In the numerical simulation an exact solitary wave solution defined by equations (2) and (3) was used as an initial condition to study the propagation and scattering of the solitary waves at the density defects. The model equations were the following:

$$i\left(\frac{\partial e_1}{\partial \zeta} + \frac{\partial e_1}{\partial \tau}\right) + \delta e_1 = -\gamma(\zeta) p, i\left(\frac{\partial e_2}{\partial \zeta} - \frac{\partial e_2}{\partial \tau}\right) - \delta e_2$$

$$= \gamma(\zeta) p, i\frac{\partial}{\partial \tau}p + \Delta p + \mu |p|^2 p = -(e_1 + e_2).$$
(5)

Distribution of nanoparticles density in different types of defects was defined by the parameter $\gamma(\zeta)$. A constant value $\gamma(\zeta) = 1$ at the length of the whole grating corresponds to Bragg grating without defects. Microcavity defect is defined by $\gamma(\zeta) = 0$, $\gamma(\zeta) < 1$ corresponds to groove defect, $\gamma(\zeta) > 1$ corresponds to stripe defect.

It was shown in [23] that the slow solitary wave propagating through nonlinear RABG can be captured in a wide defect - microcavity placed in the Bragg grating. Inside the microcavity the light pulse propagates by reflecting from the boundaries of the defect, where its radiation partly scatters.

A thorough derivation of the continuous equations for the disordered media is a distinct and difficult problem to solve. Here is used the phenomenological model which accounts for the phase variations of the slowly varying amplitudes of forward



and backward electric components of ultrashort electromagnetic pulse. These random phases assumed to appear due to randomness of deviations of the lattice constant. The equations of the model describing the ultrashort electromagnetic pulse propagation in the imperfect Bragg grating with resonant absorption are the following:

$$i\left(\frac{\partial}{\partial\zeta} + \frac{\partial}{\partial\tau}\right)e_1 + \delta e_1 = -p\exp\left(i\phi\left(\zeta\right)\right) \tag{6.1}$$

$$i\left(\frac{\partial}{\partial\zeta} - \frac{\partial}{\partial\tau}\right)e_2 - \delta e_2 = p\exp\left(-i\phi\left(\zeta\right)\right)$$
(6.2)

$$i\frac{\partial p}{\partial \tau} + \Delta p + \mu |p|^2 p = -\left(e_1 \exp\left(-i\phi\left(\zeta\right)\right) + e_2 \exp\left(i\phi\left(\zeta\right)\right)\right).$$
(6.3)

Wave vector detuning $\delta = 2(c/\omega_p)\Delta q_0$ with $\Delta q_0 = 2k_0 - q_0$, $q_0 = 2\pi/a_0$ is mean wave vector of the lattice with mean lattice spacing a_0 . $\phi(\zeta)$ is the random phase, defined as

$$\phi(\zeta) = \frac{2\pi}{a_0^2} \int_0^L \delta a(\xi) d\xi << 1.$$
(7)

The noise in the phase is considered as Gaussian and delta-correlated, correlation between the phases of wave at the points ζ_1 and ζ_2 is $\langle \phi(\zeta_1) \phi(\zeta_2) \rangle = D\delta(\zeta_1 - \zeta_2)$. *D* is the dispersion of the random phase, $D = \sigma^2$, σ is standard deviation.

In numerical simulations the condition of the exact resonance is assumed: $\delta = 2k_0 - q_0 = 0$ for the propagation constant (wave vector) k_0 of the electric field of the light pulse and mean wave vector of the lattice $q_0 = 2\pi/a_0$. The condition $\Delta = 0$ of the exact resonance between the frequency of dimensional quantization of nanoparticles and electromagnetic wave carrier frequency is also fulfils. Nonlinearity parameter in numerical simulations is $\mu = 1$. Solitary wave parameter $\alpha = 1.5$ characterizes solitary wave used in numerical simulations as the initial pulse set at $\tau = 0$. It was implemented implicit finite difference scheme to solve the model equations. The steps in time and the coordinate were the same and equal to $2.5 * 10^{-4}$. To avoid the reflection on the boundaries the large computational window was used. The computation over normalized coordinate variable ζ was performed at the interval $[\zeta_0, \zeta_L] = [0, 40]$, for the normalized time variable τ the computational interval is [0, 40]. The center of the initial pulse characterized by solitary wave parameter $\alpha = 1.5$ is at $\zeta = 5$.



3. Results

In the numerical statistical experiment the parameters (the amplitude, width – pulse duration at the half of the maximum, and position of the pulse maximum) of solitary polaritonic wave propagating in imperfect Bragg grating were measured. It was performed a set of 370 of realizations of the random grating characterized with the standard deviation $\sigma = 0.1$ and 200 realizations with the standard deviation $\sigma = 0.3$ in the phase (7). At the Fig. 1 is illustrates one realization of solitary polaritonic wave propagation in the random grating which provides standard deviation $\sigma = 0.3$ for the phase of the electric field components e_1 and e_2 .



Figure 1: Three dimensional plots illustrating the solitary wave (characterized by parameter $\alpha = 1.5$) propagation through the disordered Bragg grating leading to the standard deviation $\sigma = 0.3$ in the phases of the counter propagating waves e_1 and e_2 in the system of the equations (6). Electric field of the forward wave e_1 is illustrated by the fig. (a), backward wave e_2 corresponds to fig. (b), polarization p corresponds to fig. (c).

One can see that the scattering of solitary wave occurs on numerous fluctuations of the grating, while the polariton propagates in the random Bragg grating. To investigate the evolution of the nonlinear polariton in the random Bragg grating, the two moments (the average value and the standard deviation) of following parameters of solitary polaritonic wave were measured: amplitude and the width (defined as the pulse duration at the half of the amplitude maximum), position of the pulse maximum for the forward and backward electric field components e_1 and e_2 , as well as for the polarization component p.

The amplitude and the width of the pulse propagating through the random Bragg grating were calculated and processed for all the realizations. It was found that the averaged values of the pulse amplitude and the width are oscillating with decay to certain value. In the set of pictures of Figs. 2-4 are represented the plots showing the evolution over time τ of the averaged over random grating realizations amplitude, width and location of the pulse maximum of the electric components e_1 and e_2 , as well as polarization p of polariton propagating through random ($\sigma = 0.1$ (dot line) and $\sigma = 0.3$ (solid line)) Bragg grating comparing to the time dependence of these parameters for the polaritonic solitary wave propagating in the perfect Bragg grating



(dashed line). Fig. 2 corresponds to forward wave e_1 , Fig. 3 – to the backward wave e_2 , and the Fig. 4 – to the wave of polarization p.



Figure 2: The plot illustrating evolution over time τ for averaged over random grating realizations of amplitude (a), width (b) and position of the pulse maximum (c) of the forward wave e_1 .



Figure 3: The plot illustrating evolution over time τ for averaged over random grating realizations of amplitude (a), width (b) and position of the pulse maximum (c) of the backward wave e_2 .



Figure 4: The plot illustrating evolution over time τ for averaged over random grating realizations amplitude (a), width (b) and location of the pulse maximum (c) of polarization p component of polariton, propagating through the random Bragg grating.

The amplitude and width of the pulse are oscillating with the decay to certain value. The next plots at the Fig. 5 are representing the time evolution of the ratio $\delta = 1 - \frac{a_{stoch}w_0}{a_{stoch}}$ $a_0 w_{stoch}$ between amplitude a_{stoch} and width w_{stoch} of the solitary wave propagating in random lattice and the amplitude a_0 and width w_0 of the initial pulse corresponding to exact solitary wave solution with $\alpha = 1.5$ for the perfectly regular lattice.

The ratio of the averaged over realizations amplitude and the pulse width tends to the initial ratio (corresponding to solitary wave in the model of perfect periodic Bragg grating) for the forward wave e_1 both for $\sigma = 0.1$ and $\sigma = 0.3$ for the random phase in the equations of the system (6). The settled values of ratios of amplitude and width of the solitary wave for the backward e_2 and polarization p waves seem to be close by their absolute values. The difference of ratio of the amplitude and width for the polariton components in the random media comparing to the unperturbed initial solitary wave is in 10 % for the backward wave e_2 and wave of polarization p, and in 5 % for the forward wave e_1 at $\sigma = 0.3$. At $\sigma = 0.1$ the distinction is almost negligible.





Figure 5: The ratio $\delta = 1 - \frac{a_{stoch}w_{sol}}{a_{sol}w_{soch}}$ between amplitude and width of the solitary wave depending on time τ . The left panel (a) corresponds to $\sigma = 0.1$ for the random phase, right panel (b) corresponds to $\sigma = 0.3$. Thick solid line corresponds to the component of electric field of the wave e_1 , thin solid line corresponds to p_2 .

At the Fig. 6 is illustrated the behavior of the second moments - standard deviations of amplitude and width of the pulse from their average values (first moments) calculated previously.

From the plots shown at the Fig. 6 one can see that for standard deviation in the phase $\sigma = 0.3$ for the electric field components e_1 and e_2 the standard deviation of the amplitude and width of the forward wave (component e_1 of nonlinear polariton) seems to grows with the time, the same deviations for backward wave (component e_2) and polarization p comes out to a constant (about 0.01) with the time. Of cause the better statistics of realizations should be accumulated for more exact plots but even these results show that the polariton can evolve in the grating with weak random fluctuations of lattice spacing. The nonlinear polariton propagates in the Bragg grating as a solitary wave with fluctuations along the distance in its amplitude and width.

The probability density function (PDF) does not significantly depend on the number of the realizations for neither $\sigma = 0.1$ (Fig. 7) nor $\sigma = 0.3$ (Fig. 8) for all components of polariton.

4. Conclusion

In this paper are presented the results of numerical modeling of polaritonic solitary wave propagation in the Bragg grating with random lattice spacing. It is shown that random fluctuations in the phases of electromagnetic waves resulting from variation of lattice spacing lead to periodic oscillations (with the period 2π) in the amplitudes and the widths of the solitary pulses. The amplitude of these oscillations grows with the





Figure 6: The plot for standard deviations over random grating realizations of amplitude (left panels) and the width (right panels) of electric fields e_1 (figs. (a) and (b)) and e_2 (figs. (c) and (d)) of solitary polaritonic wave and polarization p (figs. (e) and (f)) in the random media leading to the standard deviation in the phase $\sigma = 0.3$ (solid line) and $\sigma = 0.1$ (dashed line).



Figure 7: The plot for PDF of amplitude for the forward wave $e_1(a)$, backward $e_2(b)$ and the polarization p (c) of polariton depending on the number N of the random grating realizations for $\sigma = 0.1$.

increase of the amplitude of the random phase. Oscillations decay with the distance, and probably the stationary regime (without periodic energy swapping between the component of polariton) will sustain. The random weak fluctuations of the lattice spacing that are responsible for the random fluctuations in the phase of electric components of polariton, do not disrupt the polaritonic wave binding light photons with the lattice phonons, thought the radiation occurs during its evolution in the grating, and





Figure 8: The plot for PDF of amplitude for the forward wave $e_1(a)$, backward $e_2(b)$ and the polarization p (c) of polariton depending on the number N of the random grating realizations for $\sigma = 0.3$.

the amplitude decreases. Velocity of the pulse changes slightly even for a standard deviation $\sigma = 0.3$ in the phase of the electric components of polariton. The ratio between amplitude and width of the residuary pulse is close to constant at large times (and propagation distances). This ratio measured at the end of computational time interval differs from the initial (corresponding to solitary solution in the case of exact resonance) in 10 % for the backward wave and wave of polarization and in 5 % for the forward wave at $\sigma = 0.3$. At $\sigma = 0.1$ the distinction between polaritons propagating in the perfect and in the weakly disordered Bragg grating is almost negligible.

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