

## Conference Paper

# Formation of Surface Plasmon-Polariton Vortices at Reflection from Curvilinear Boundary

V. S. Pereskokov and I. V. Dzedolik

Physics and Technology Institute, V. I. Vernadsky Crimean Federal University (4, Vernadsky av., Simferopol, 295007, Russia)

## Abstract

We present the results of simulation of interference of surface plasmon-polaritons (SPPs) which are falling and reflecting from the curvilinear boundary of inhomogeneity area in the metal layer. The plasmon vortices with a screw phase dislocation appear in the singular points of the field as a result of the SPP interference after reflection from the boundary of inhomogeneity in the dovetail form. The position of the plasmon vortices on the surface of metal layer can be controlled by means of the external electrostatic field. Negative charges localized at the control probes cause the change of the boundary curvature of the permittivity of inhomogeneity area on the metal layer, which leads to displacement of the vortex localization points. When the vortex is localized under the readout nanowire probe with angular thread, the maximum or minimum of the signal takes place in the probe depending on the helicity of the thread and the topological charge of the vortex.

**Keywords:** surface plasmon-polariton, plasmon vortex, nanowire.

## 1. Introduction

In the last decade surface plasmon polaritons (SPPs) attract attention of researchers in connection with controlling of the electromagnetic fields of optical frequencies by the SPP-devices and creating logic gates for optical processors, spasers, other devices and elements of plasmon technology [1-10]. The SPPs with the dependence of field components in the form  $\sim \exp[-\alpha x + i(\beta z - \omega t)]$  can be excited on the interface of metal with a negative real part of the permittivity  $Re\epsilon_M < 0$ , and a dielectric medium with the permittivity  $Re\epsilon_0 > 0$ . When  $Re\epsilon_M < 0$  the SPPs propagation takes place on the surface of the metal; but with positive values  $Re\epsilon_M > 0$  the SPPs can not propagate because the boundary conditions  $\epsilon_M \alpha_0 = -\epsilon_0 \alpha_M$  are not satisfied, where  $\alpha_0 > 0$  and  $\alpha_M > 0$  are the decrements along the normal axis  $x$  to the metal surface (Figure 1).

Corresponding Author:  
 V. S. Pereskokov  
 pereskokow@gmail.com

Received: 28 January 2018  
 Accepted: 15 March 2018  
 Published: 25 April 2018

Publishing services provided by  
 Knowledge E

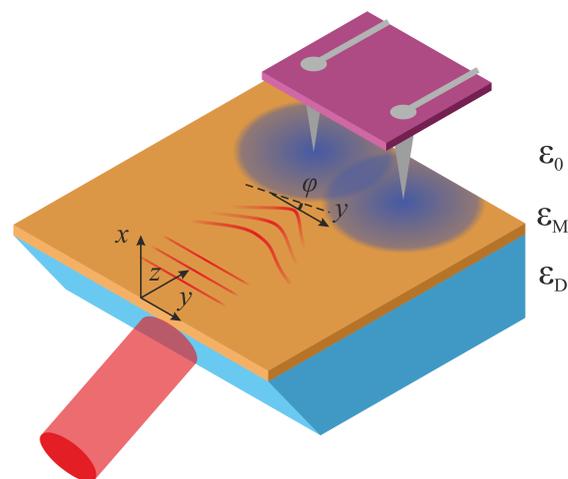
© V. S. Pereskokov and I. V. Dzedolik. This article is distributed under the terms of the [Creative Commons](#)

[Attribution License](#), which permits unrestricted use and redistribution provided that the original author and source are credited.

Selection and Peer-review under the responsibility of the PhI0 Conference Committee.

 OPEN ACCESS

The curvature of the wavefront and the direction of propagation of the SPPs change while reflecting from the curvilinear boundary of the inhomogeneity in the metal layer. Scattering of SPPs on the inhomogeneities of various configurations at the boundary between the dielectric and metal leads to enrichment of the mode composition of the SPPs, as well as to the interconnection of modes in microwave guides and microcavities, and to radiation from the metal and dielectric interface of bulk electromagnetic waves. In this field of research a large volume of theoretical and experimental works are devoted to scattering of the SPPs on various inhomogeneities in the metal layers [11-14], to focusing and controlling the SPPs by electromagnetic fields, and to various dielectric and metallic nanostructures on a chips and plasmon lenses [15-26].



**Figure 1:** Generation of the SPPs in the metal layer according to the Kretschmann scheme, and reflection of the SPPs from the boundary of permittivity inhomogeneity in the layer:  $\phi$  is the angle between the tangent to the boundary of the inhomogeneity and the transverse axis  $y$ .

It is known, that the optical vortices may appear when the waves with wavefronts of different configurations formed under reflection, refraction or diffraction of waves have interference [27, 28]. Optical vortices are still actively investigated in connection with a wide field of their application [29]. Interference of the SPPs can also lead to the formation of plasmon-polariton vortices under certain conditions. The plasmon-polariton vortices are excited when the SPPs pass through plasmon lenses which are curved slits, or cavities and protrusions in a metal layer [30-33], as well as under normal incidence of an optical vortex beam on the metal surface [34]. The singular points with a screw phase dislocation arise on the surface of the metal layer, in which plasmon-polariton vortices are formed at the interference of incident and reflected SPPs from the inhomogeneity of permittivity with curvilinear boundary [35-37]. The plasmon vortices do not arise in case of superposition of the modes reflected from a straight line boundary of the inhomogeneity.

The boundary of inhomogeneity in the form of dovetail (Figure 1) can be formed in the metallic layer as a result of the action of an electrostatic field of the negative charges that localized on the control probes which are located above the metal layer. In this case, the negatively charged probes can change the permittivity of metal in such a way that it acquires positive values  $\epsilon'_M > 0$  in the optical frequency range. Then, in the area of the electrostatic field the boundary propagation conditions are violated, and the SPPs are scattered at the boundary of the artificially created inhomogeneity of the metal permittivity. It is possible to control the position of the SPP vortices by changing the radius of the circles of the dovetail varying the intensity of the electrostatic field.

The purpose of our work is to find the conditions for the appearance of SPP vortices formed on the interface of the metal and dielectric when the SPPs are reflected from the inhomogeneity of permittivity in the metal layer in the form of dovetail. By varying the values of the negative charges on the control probes it is possible to change the form of the inhomogeneity area of the metal permittivity. This change of the boundary curvature leads to the change of the topology of the interference field of the incident and reflected SPPs. Thus, it is possible to control the configuration of the vortex lattice of the SPPs by means of the external electrostatic field arising between the probes with negative charges and the metal layer.

## 2. Surface plasmon-polariton modes

The permittivity of a metal at optical frequencies is a complex quantity with a negative real part  $\epsilon_M = -\epsilon'_M + i\epsilon''_M$ . Therefore, the propagation constants of the SPPs  $\beta = \beta' + i\beta''$  are also complex quantities. Their imaginary parts characterize the attenuation of the SPPs along the axis  $z$ , i.e. they determine the propagation length of the SPPs along the lower and upper surfaces of the metal layer  $L = 1/2\beta''$  [3]. Generally, the decrements  $\alpha_0 = (\beta^2 - c^{-2}\omega^2\epsilon_0)^{1/2}$  and  $\alpha_M = (\beta^2 - c^{-2}\omega^2\epsilon_M)^{1/2}$  are complex quantities as well as the SPP propagation constant. Thus, when the SPP propagates along the metal surface, the components of its field oscillate at the frequency  $\omega$  attenuating both along the longitudinal axis  $z$  and along the normal axis  $x$  to the media interface.

The TM-mode of the SPPs with the field components  $E_x$ ,  $E_z$ ,  $B_y$  is formed at the homogeneous interface of the non-magnetic metal and dielectric [3]. The TM-mode propagates along the homogeneous metal surface with the mode propagation constant  $\beta$  that is parallel to the axis  $z$ ; the electric vector of the TM-mode rotates in the plane  $(x, z)$ , and the wavefront of the surface wave is flat. But the surface plasmon-polariton wave is reflected from the inhomogeneity boundary if there is an

inhomogeneity in the metal layer, for example, if the metal layer is broken off or the boundary conditions  $-\alpha_0\epsilon_M = \alpha_M\epsilon_0$  are violated. The real part of the permittivity  $Re\epsilon_M = -\epsilon'_M < 0$  of the metal at optical frequencies is negative. However, under the influence of the external electrostatic field of the negative charge located at the control probe above the metal layer, the area with positive permittivity  $Re\epsilon_M = \epsilon'_M > 0$  can be formed in the metal. The SPPs will be scattered on such area of inhomogeneity of the metal permittivity.

There are the evanescent waves directed from the boundary to the inhomogeneity area in the direction of the axis  $z$ , and the SPPs directed back from the boundary of inhomogeneity against the axis  $z$ . In this case, the mode composition of the reflected SPP wave is enriched; the modes with field components  $E_x, E_y, E_z, B_y, B_z$  are formed [35-37]. However, because of the boundary conditions, the normal component of the magnetic vector  $B_x$  does not arise from the inhomogeneity reflection of the TM mode, and the rotation plane of the electric vector turns about the axis  $x$ . The singular points localized in the field minima may arise in result of the interference of the incident TM-mode and reflected modes of the SPPs. At these points, the corresponding components of the SPP field are zero, the phase of the interference field is not determined, and the interference fringes of the SPP field are split. The Poynting vector  $S = (c/4\pi) E \times B$  of the SPPs at such singular points has three components and it precesses around an axis normal to the plane of the metal layer, i.e. plasmon-polariton vortices arise.

We consider the excitation of vortices upon reflection of the SPPs from the inhomogeneity boundary in the metal layer in the form of a dovetail. We assume that the laser beam with frequency  $\omega$  falls on the side face of the prism with dielectric permittivity  $\epsilon_D$  (Figure 1). The metal layer with the thickness  $h$  and permittivity  $\epsilon_M$  is located on the dielectric prism; the metal and dielectric media are nonmagnetic. A bulk electromagnetic wave having passed through the prism excites the SPPs on the upper surface of the metal layer bordering upon the air  $\epsilon_0$  under the condition  $\beta = k_z$ . Solutions of the system of Maxwell's equations for nonmagnetic media  $\nabla \times B = -ic^{-1}\omega\epsilon E, \nabla \times E = ic^{-1}\omega B$  describe the SPPs generated by the laser beam before scattering at the boundary of the inhomogeneity in the metal, and their plane wavefront that does not depends on the transverse coordinate  $y$ . The TM-mode of the SPPs is formed with the field components at the metal-air interface

$$E_{1x} = \frac{c\beta A}{\omega\epsilon_0} \exp \varphi_T, E_{1z} = -i\frac{c\alpha_0 A}{\omega\epsilon_0} \exp \varphi_T, B_{1y} = A \exp \varphi_T, \quad (1)$$

where  $A = const, \varphi_T = -\alpha_0 x + i\beta z$ . The dispersion equations of the SPPs have the form  $c^{-2}\omega^2\epsilon_0 + \alpha_0^2 - \beta^2 = 0$  in the air, and  $c^{-2}\omega^2\epsilon_M + \alpha_M^2 - \beta^2 = 0$  in the metal. From the condition

of continuity of the tangential components of the electric field  $E_z$  on the surface of the metal layer, the boundary condition  $\alpha_M \epsilon_0 = -\alpha_0 \epsilon_M$  holds for the TM-mode of the SPPs at  $Re \epsilon_M < 0$ .

The SPPs of TM-mode are falling to the inhomogeneity in the metal and are reflected from the boundary of the inhomogeneity. Reflecting from the boundary of the inhomogeneity in the metal layer, the wave vector of the SPP directed along the axis  $z$  turns by some angle  $2\phi$ ; the projection of mode propagation constant to the axis  $z$  is equal  $\tilde{\beta} = -\beta \cos 2\phi$ , and the projection to the axis  $y$  is  $\kappa = \beta \sin 2\phi$  (Figure 1). The field components appear in the scattered SPP on the boundary of the inhomogeneity in the metal

$$E_{2x} = \frac{c\beta A}{\omega\epsilon_0} \exp \varphi_R, E_{2y} = -i \frac{c\alpha_0 A \cos \phi}{\omega\epsilon_0} \exp \varphi_R, E_{2z} = -i \frac{c\alpha_0 A \sin \phi}{\omega\epsilon_0} \exp \varphi_R, \quad (2)$$

$$B_{2y} = A \sin \phi \exp \varphi_R, B_{2z} = -A \cos \phi \exp \varphi_R,$$

where  $\varphi_R = -\alpha_0 x + i\beta (y \sin 2\phi - z \cos 2\phi)$ , and the normal to the surface component of the magnetic field  $B_x = 0$  remains equal to zero [35-37]. The number of modes of reflected SPPs depends on the shape of the inhomogeneity boundary, i.e. the angles of reflection of the SPPs are  $2\phi$ . The same modes are formed in the metal layer near the upper surface, but in this case the decrement in the expressions for the field components must be replaced  $-\alpha_0 \rightarrow \alpha_M$ , and the permittivity is also replaced  $\epsilon_0 \rightarrow \epsilon_M$ . The boundary conditions for the reflected SPPs are not violated, and the transverse component of the SPP wavevector  $\kappa$  is added to the dispersion equations at the boundary with the air  $c^{-2}\omega^2\epsilon_0 + \alpha_0^2 - \kappa^2 - \tilde{\beta}^2 = 0$ , and in the metal layer  $c^{-2}\omega^2\epsilon_M + \alpha_M^2 - \kappa^2 - \tilde{\beta}^2 = 0$ , where  $\kappa = \beta \sin 2\phi$ ,  $\tilde{\beta} = \beta \cos 2\phi$ , i.e.  $\kappa^2 + \tilde{\beta}^2 = \beta^2$ . The propagation constant at the boundary of the metal layer and the air has the real value  $\beta = k_0 [\epsilon_M / (1 + \epsilon_M)]^{1/2} = k_0 \left\{ (\epsilon_M' + \epsilon_M'') / [(1 - \epsilon_M')^2 + \epsilon_M''^2] \right\}^{1/2}$ , where  $k_0 = \omega/c$ .

The distribution of the field on the metal surface ( $x = 0$ ) during the SPP mode interference depends on the angle of reflection  $2\phi$  of the TM-mode from the boundary of the inhomogeneity in the metal, i.e. from the curvature of the boundary. The maxima and minima of the components of the electric and magnetic vectors of the SPPs arise in the area of the existence of the SPPs as a result of the mode interference; accordingly, the Poynting vector  $S_j = S_{ja} \exp(i\varphi_j)$  of the SPP has the maxima and minima, where  $S_{ja} = \left[ (Re S_j)^2 + (Im S_j)^2 \right]^{1/2}$  is the amplitude,  $\varphi_j = \arctan (Im S_j / Re S_j)$  is the phase of the interference field,  $j = x, y, z$ . At least three plasmon-polariton waves arrive to the zero points of the SPP interference field: the incident wave and two reflected waves at different angles  $2\phi$  from the curvilinear boundary of the inhomogeneity,

then the screw dislocation takes place in the phase of interference field. At these points the SPP interference fringes are split, and plasmon-polariton vortices arise. Modern methods of apertureless near-field microscopy with the resolution of units of nanometers [38-40] are based on the detection of the normal component  $E_x$  of the electric vector; therefore we will analyze hereinafter the distribution of  $E_x$ .

### 3. Discussion of the proposed experiment

Negatively charged control probes (Figure 1) create the areas of inhomogeneity of the metal permittivity in the form of circles with radii  $r_0$  with the positive "mirror" charges in the metal that arise by the displacement of conduction electrons. We estimate the parameters of the experimental device assuming that a point charge  $-q$  is placed at the tip of the probe, creating a field strength  $E_0 = -q(4\pi\epsilon_0\epsilon_p h_p^2)^{-1}$ , where  $\epsilon_0 = 8.84 \times 10^{-12} F/m$ . On the boundary of the circle with the radius  $r_0$ , i.e. at a distance  $\xi = (h_p^2 + r_0^2)^{1/2}$  from the charge, the intensity of the electrostatic field will decrease as a ratio  $E_\xi/E_0 = h_p^2/\xi^2$ . The intensity of the electrostatic field decreases as the ratio  $E_\xi/E_0 \approx 10^{-5}$  at the boundary of the inhomogeneity area of the permittivity at the radius  $r_0 = 10 \mu m$  with the height of the probe  $h_p = 30 nm$  above the surface of the layer. We obtain the ratio  $E_\xi/E_0 \approx 10^{-3}$  at the radius  $r_0 = 1 \mu m$ , and the ratio  $E_\xi/E_0 \approx 10^{-1}$  at the radius  $r_0 = 100 nm$ . The voltage at the control probe placed at the height  $h_p$  is  $U = E_0 h_p$ . With the voltage at the probe  $U = 1 mV$  the field intensity under the probe will be  $E_0 = 33.3 \times 10^3 V/m$ , and the magnitude of the positive "mirror" charge is equal to  $q = 4\pi\epsilon_0\epsilon_p h_p U = 0.111 \times 10^{-12} C$ . It is possible to increase the "mirror" charge at constant voltage, if dielectric medium with permittivity  $\epsilon_0 \rightarrow \epsilon$  is placed between the control probe and the metal layer. It leads to the electrocapacity increasing in the space between the control probe and the metal layer, then the charge value is  $q = C(\epsilon) U$ .

The scattering of the SPPs at the boundary of the inhomogeneity area is inelastic. However, in this case the reflected SPPs directed against the axis  $z$  are generated, and they interfere with the SPPs falling along the axis  $z$ . One can change the radius  $r_0$  of the area of permittivity inhomogeneity of the metal by varying the voltage  $U$  at the control probe  $U_2/U_1 = q_2/q_1 = (r_{02}^2 + h_p^2)^{1/2} (r_{01}^2 + h_p^2)^{-1/2}$  at the fixed probe height  $h_p$  above the surface of the metal. Then the radius of the inhomogeneity area varies as  $r_{02} = [g^2 r_{01}^2 + (g^2 - 1) h_p^2]^{1/2}$ , where  $g = U_2/U_1$ , i.e. the radius  $r_0$  varies in proportion to the voltage between the control probe and the layer.

Figure 2 shows the normalized distribution of the normal component of the electric vector  $E_x = E_{xa} \exp(i\varphi_x)$ , where  $E_{xa} = [(Re E_x)^2 + (Im E_x)^2]^{1/2}$  is the amplitude,  $\varphi_x =$

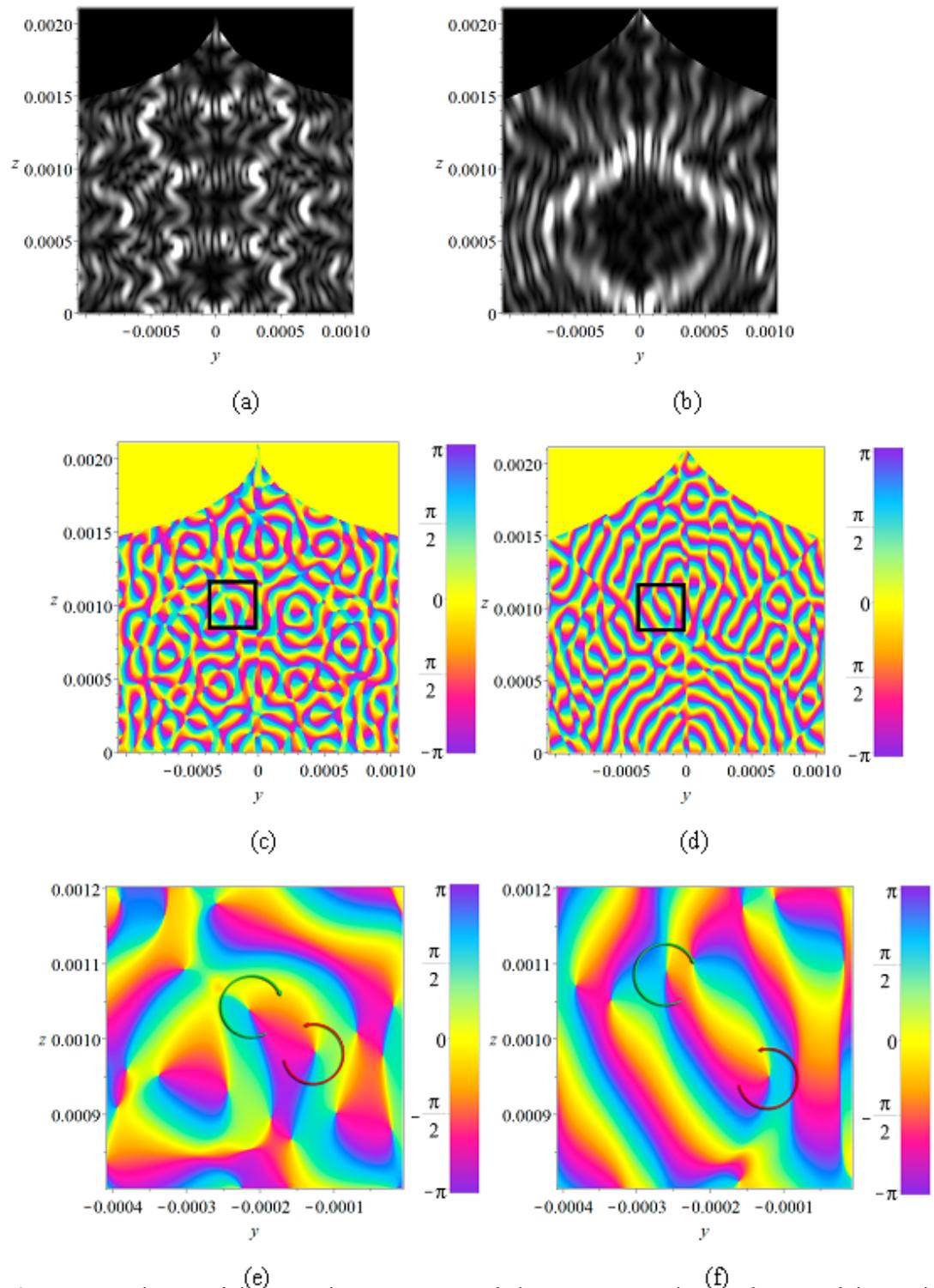
$\arctan (ImE_x/ReE_x)$  is the phase of the SPP interference field at a certain time for the superposition of TM-modes at the air-metal interface:  $\epsilon_0 = 1$  and  $\epsilon_M = -\epsilon'_M + i\epsilon''_M = -12.64 + i 1.40$  (the layer of polycrystalline gold with the thickness  $53 \text{ nm}$ ) [41], the laser beam has the wavelength in the air  $\lambda_0 = 630 \text{ nm}$ . In the case under consideration the propagation constant of the SPP is equal  $\beta = 1.08 \times 10^5 \text{ cm}^{-1}$ , then the wavelength of the SPP is  $\lambda = 2\pi/\beta = 581 \text{ nm}$ . The decrements of the SPP have values:  $\alpha_0 = 4.19 \times 10^4 \text{ cm}^{-1}$  in the air, and  $\alpha_M = 3.71 \times 10^5 \text{ cm}^{-1}$  in the metal layer, that corresponds to the distances along the axis  $x$  normal to the surface:  $h_0 = 238 \text{ nm}$ , and  $h_M = 26.9 \text{ nm}$ , where the SPP is attenuating.

As a result of SPP interference the plasmonic vortices arise at the points of splitting of the interference fringes of the field minima, when the SPPs are reflected from the curvilinear boundary of the inhomogeneity in the metal layer, (Figure 2 (a), dark lines). The change of the boundary curvature leads to a shift in the minima of the SPP field. The varying of the potential of the control probe over the metal surface leads to decreasing or increasing of the radius  $r_0$  of the permittivity inhomogeneity area in the metal, which causes to the vortices shifting from their original positions (Figure 2 (c) and (d)).

#### 4. Excitation of the SPP modes in nanowire

If the readout probe is placed above the point of localization of the SPP vortex on the metal surface, then surface plasmon modes can be excited in the nanowire of the probe [42-43]. To excite the SPP modes in the nanowire, it is necessary to match the normal component of the electric field  $E_{xM}$  of the SPP vortex on the metal surface and the longitudinal mode component  $E_{zw}$  on the nanowire surface.

Consider the process of formation of surface plasmon-polariton modes upon excitation of a nonmagnetic metal nanowire with a circular radius  $a$  of cross section by monochromatic electromagnetic radiation with frequency  $\omega$ . Suppose the boundary of the nanowire is corrugated in the form of a spiral with the deep  $d$  and step  $\Lambda$  along the axis  $z$  of the nanowire, then the radius of spiral is  $a - d = \text{const}$ . Corrugation of the nanowire boundary leads to disturbance of its permittivity  $\epsilon_d = \epsilon + \Delta\epsilon(z)$ , where  $\epsilon = \text{const}$ . The perturbation of the permittivity of the nanowire is represented in the cylindrical coordinate system as  $\Delta\epsilon = -\epsilon \frac{d}{a} \exp\left(is\frac{2\pi}{\Lambda}z\right)$ , then  $\epsilon_d = \epsilon [1 - \bar{d} \exp(isKz)]$ , where  $\bar{d} = d/a$ ,  $K = 2\pi/\Lambda$ , and  $s = \pm 1, \pm 2, \dots$  is the index which characterizes the direction of rotation (helicity) and the number of corrugation branches.



**Figure 2:** Distribution of the normal component  $E_x$  of electric vector at the interference of the incident and reflected SPPs from the inhomogeneity boundary in the metal layer in the form of dovetail: (a) the interference fringes of the amplitude at the radius of curvature boundary  $r_0 = 5 \mu\text{m}$ ; (b) the interference fringes of the amplitude at the radius of curvature boundary  $r_0 = 10 \mu\text{m}$ ; (c) the phase distribution,  $r_0 = 5 \mu\text{m}$ ; (d) the phase distribution,  $r_0 = 10 \mu\text{m}$ ; (e) the SPP vortices with topological charge  $\ell_M = +1$  (red arrow, anti-clockwise) and with topological charge  $\ell_M = -1$  (green arrow, clockwise) in the fence,  $r_0 = 5 \mu\text{m}$ ; (f) the SPP vortices in the fence,  $r_0 = 10 \mu\text{m}$ ; the values along the axes ( $y, z$ ) are marked in micrometers.

In this case the equation for the electric field components follows from the system of equations  $\nabla \times \nabla \times E = -\nabla^2 E + \nabla(\nabla E) = k_0^2 \epsilon_d E$ , where  $\nabla E = -(\epsilon_d^{-1} \nabla \epsilon_d) E = -(\partial \ln \epsilon_d / \partial z) E_z$ ,  $k_0 = \omega/c$ . The equation for the longitudinal component of the SPP mode in the nanowire in the cylindrical coordinate system has the form  $\nabla_{\perp}^2 E_z + \frac{\partial^2 E_z}{\partial z^2} + \frac{\partial \ln \epsilon_h}{\partial z} \frac{\partial E_z}{\partial z} + \left( \frac{\partial^2 \ln \epsilon_d}{\partial z^2} + k_0^2 \epsilon_d \right) E_z = 0$ , where  $\nabla_{\perp}^2 \rightarrow \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2}$ . Substituting the derivatives  $\frac{\partial \ln \epsilon_d}{\partial z} = -\frac{is\bar{d}K}{1-\bar{d}\exp(isKz)}$  and  $\frac{\partial^2 \ln \epsilon_d}{\partial z^2} = \frac{\bar{d}^2 K^2 \exp(isKz)}{(1-\bar{d}\exp(isKz))^2}$  in this equation, we obtain the equation for the longitudinal component of the electric field at the nanowire

$$\nabla^2 E_z - \frac{is\bar{d}K}{1-\bar{d}\exp(isKz)} \frac{\partial E_z}{\partial z} + \left[ k_0^2 \epsilon (1-\bar{d}\exp(isKz)) + \frac{\bar{d}^2 K^2 \exp(isKz)}{(1-\bar{d}\exp(isKz))^2} \right] E_z = 0. \tag{3}$$

To obtain analytical solutions in the first approximation, we neglect the term  $\bar{d}\exp(isKz)$  in the denominators of the terms in equation (3), we believe  $\bar{d}^2 K^2 \ll \bar{d}k_0^2 \epsilon$ , and obtain the equation

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} - is\bar{d}K \frac{\partial E_z}{\partial z} + k_0^2 \epsilon [1-\bar{d}\exp(isKz)] E_z = 0. \tag{4}$$

The variables are separated after factoring of the solution  $E_z = F(r, \phi) X(z)$  in the equation (4), we get two equations:  $\frac{\partial^2 X}{\partial z^2} - is\bar{d}K \frac{\partial X}{\partial z} - \bar{d}k_0^2 \epsilon \exp(isKz) X = -\beta^2 X$  and  $\frac{\partial^2 F}{\partial r^2} + \frac{1}{r} \frac{\partial F}{\partial r} + \frac{1}{r^2} \frac{\partial^2 F}{\partial \phi^2} + k_0^2 \epsilon F = \beta^2 F$ , where  $\beta^2 > 0$  is the separation constant of the variables. The solution of the equation for  $X(z)$  has the form

$$X(z) = \exp\left(is\frac{\bar{d}Kz}{2}\right) J_{\nu}\left(s\frac{2k_0\sqrt{\bar{d}\epsilon}}{K} \exp\left(is\frac{Kz}{2}\right)\right),$$

where  $\nu = \sqrt{4\beta^2 K^{-2} + \bar{d}^2} \approx 2\beta/K$  is the index of the Bessel function. The solution of the equation for  $F(r, \phi)$  we choose in the form of modes with angular dependence of the angle  $\phi$  in the form  $\exp(i\ell\phi)$ . The equation for  $F(r, \phi)$  in the metal can be represented as

$$\frac{\partial^2 F_M}{\partial r^2} + \frac{1}{r} \frac{\partial F_M}{\partial r} + \left(k_M^2 - \frac{\ell^2}{r^2}\right) F_M = 0, \tag{5}$$

where  $k_M^2 = (\epsilon'_M - i\epsilon''_M) k_0^2 - \beta^2$ ,  $\ell = 0, \pm 1, \pm 2, \dots$  is the azimuthal number of the mode. On the surface of the metal nanowire surrounded by dielectric medium with  $|\epsilon'| > \epsilon_0$ , the equation for  $F(r, \phi)$  is

$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r} \frac{\partial F}{\partial r} - \left(k^2 + \frac{\ell^2}{r^2}\right) F = 0, \tag{6}$$

where  $k^2 = \beta^2 - k_0^2 \epsilon_0 > 0$ . We choose the solutions of (5) in the form of Bessel functions  $F_{z\omega} = A \frac{J_{\ell}(wr/a)}{J_{\ell}(w)}$  having finite values on the nanowire axis  $r = 0$ , and the solution of equation (6) in the form of Macdonald functions  $F_{z0} = A \frac{K_{\ell}(wr/a)}{K_{\ell}(w)}$  damped at  $r \rightarrow \infty$

( $r > a$ ), where  $u = ak_M$ ,  $w = ak$ ,  $A = const$ . Then we obtain expressions for the longitudinal components of the SPP modes inside and on the nanowire surface

$$E_{zw} = A \frac{J_\ell(ur/a)}{J_\ell(u)} \exp(i\ell\phi + iK_s z) J_\nu(\zeta_s), \tag{7}$$

$$E_{z0} = A \frac{K_\ell(wr/a)}{K_\ell(w)} \exp(i\ell\phi + iK_s z) J_\nu(\zeta_s), \tag{8}$$

where  $\zeta_s = s \frac{2k_0 \sqrt{\bar{d}\epsilon}}{K} \exp\left(i s \frac{Kz}{2}\right)$ ,  $K_s = s \frac{\bar{d}K}{2}$ ,  $\nu = 2\beta/K$ . The longitudinal components of the electric field (7) and (8) attenuate when the SPP modes propagate along the nanowire.

The phase of the longitudinal component of the  $\ell$ th mode of the SPPs on the nanowire surface  $r = a$  has the form  $\varphi_{z0} = \arctan [Im(f_{\ell s})/Re(f_{\ell s})]$ , where  $f_{\ell s} = \exp[i(\ell\phi + K_s z)] J_\nu(\zeta_s)$ . From the expression for the phase of the longitudinal component, we can determine the ‘‘helicity’’ of the nanowire SPP mode  $\sigma_z = \frac{1}{2\pi} \oint dr \nabla \varphi_{z0}$ , that is  $\sigma_z = \frac{1}{2\pi} \left( \int_0^{2\pi} d\phi \frac{\partial \varphi_{z0}}{\partial \phi} + \int_0^\Lambda dz \frac{\partial \varphi_{z0}}{\partial z} \right)$ . Suppose the perturbation of the permittivity of nanowire is small  $\bar{d} \ll 1$ , and the propagation mode constant of the SPP mode for the nanowire is  $\beta = K$ , i.e.  $\epsilon = \epsilon'_M$ . Then taking into account only the first term of the series for the Bessel function  $J_2(\zeta) \approx \frac{k_0^2 \bar{d}\epsilon}{2K^2} \exp(isKz)$ , we obtain the expression  $f_{\ell s} \approx \frac{k_0^2 \bar{d}\epsilon}{2K^2} \exp\left\{ i \left[ \ell\phi + s \left( 1 + \frac{\bar{d}}{2} \right) Kz \right] \right\}$ . The phase of the  $\ell$ th SPP-mode of the nanowire has the form  $\varphi_{z0} = \ell\phi + s(1 + \bar{d}/2)Kz$  in this approximation. The helicity of the longitudinal component of the  $\ell$ th SPP-mode of the nanowire in this case is equal to  $\sigma_z = \ell + s(1 + \bar{d}/2)$ . Thus, we get the maximum or minimum signal in the readout probe depending on the helicity of the nanowire and the topological charge of the vortex under the readout probe.

## 5. Conclusion

The SPPs generated at the boundary of the homogeneous dielectric medium and the metal layer form the TM-mode propagating along the surface of the metal and having the plane wavefront. The inhomogeneities of the metal layer permittivity cause the reflection of the SPPs, while the modal composition of the surface waves changes. There is the interference of the TM-modes when the SPPs are reflected from the straight line boundary, but the SPP vortices do not arise. If the boundary of the inhomogeneity area is curvilinear, then the vortex lattice arises as a result of interference of the SPP-modes.

The distribution of the singular points at the minima of the SPP interference field, in which vortices are formed on the metal surface, depends on the curvature of the

boundary of the inhomogeneity area. The curvature of the boundary of the inhomogeneity area in the metal layer can be changed by means of the external electrostatic field of negative charges at the control probes. It is possible to control the distribution of the minima of the SPP interference field by changing the voltage on the control probes located above the metal surface, i.e. varying the value of the negative charges of the control probes, we can change the configuration of the vortex grating. In the readout probes, which are nanowires with the spiral thread, the signals are effectively excited when the helicity of the thread coincides with the topological charge of the plasmon-polariton vortex.

## References

- [1] W. L. Barnes, A. Dereux, and T. W. Ebbesen, "Surface plasmon subwavelength optics," *Nature*, vol. 424, pp. 824-830, 2003.
- [2] A. V. Zayats, and I. I. Smolyaninov, "Near-field photonics: surface plasmon polaritons and localized surface plasmons," *Journal of Optics A: Pure and Applied Optics*, vol. 5, pp. S16-S50, 2003.
- [3] S. A. Maier, *Plasmonics: Fundamental and Applications*, New York: Springer Science+Business Media, 2007.
- [4] P. Berini, "Long-range surface plasmon polaritons," *Advances in Optics and Photonics*, vol. 1, 484-588, 2009.
- [5] D. K. Gramotnev, and S. I. Bozhevolnyi, "Plasmonics beyond the diffraction limit," *Nature Photonics*, vol. 4, pp. 83-91, 2010.
- [6] M. I. Stockman, "Nanoplasmonics: past, present, and glimpse into future," *Optics Express*, vol. 19, pp. 22029-22106, 2011.
- [7] L. Novotny, and B. Hecht, *Principles of Nano-Optics*, Cambridge: Cambridge University Press, 2012.
- [8] O. V. Shulika, and I. A. Sukhoivanov, *Contemporary optoelectronics: Materials, metamaterials and device applications*, Dordrecht: Springer Science+Business Media, 2016.
- [9] I. V. Dzedolik, *Solitons and nonlinear waves of phonon-polaritons and plasmon-polaritons*, New York: Nova Science Publishers, 2016.
- [10] A. B. Shesterikov, M. Yu. Gubin, M. G. Gladush, and A. V. Prokhorov, "Formation of plasmon pulses in the cooperative decay of excitons of quantum dots near a metal surface," *Journal of Experimental and Theoretical Physics*, vol. 124, no. 1, pp. 18-31, 2017.

- [11] F. Pincemin, A. A. Maradudin, A. D. Boardman, and J.-J. Greffet, "Scattering of a surface plasmon polariton by a surface defect," *Physical Review B*, vol. 50, pp. 15261-15275, 1994.
- [12] B. Hecht, H. Bielefeld, L. Novotny, Y. Inouye, and D. W. Pohl, "Local excitation, scattering, and interference of surface plasmons," *Physical Review Letters*, vol. 77, pp. 1889-1892, 1996.
- [13] P. Cheyssac, V. A. Sterligov, S. I. Lysenko, and R. Kofman, "Surface plasmon-polaritons 1. Interaction with 1D objects," *Physical Status Solidi (a)*, vol. 175, pp. 253-258, 1999.
- [14] H. Ditlbacher, J. R. Krenn, G. Schider, A. Leitner, and F. R. Aussenegg, "Two-dimensional optics with surface plasmon polaritons," *Applied Physics Letters*, vol. 81, no. 10, pp. 1762-1764, 2002.
- [15] A. V. Krasavin, A. V. Zayats, and N. I. Zheludev, "Active control of surface plasmon-polariton waves," *Journal of Optics A: Pure and Applied Optics*, vol. 7, pp. S85-S89, 2005.
- [16] V. N. Konopsky, and E. V. Alieva, "Long-range propagation of plasmon polaritons in a thin metal film on a one-dimensional photonic crystal surface," *Physical Review Letters*, vol. 97, 253904, 2006.
- [17] P. A. Huidobro, M. L. Nesterov, L. Martin-Moreno, and F. J. Garcia-Vidal, "Transformation optics for plasmonics," *Nano Letters*, vol. 10, pp. 1985-1990, 2010.
- [18] C. Zhao, J. Zhang, and Y. Liu, "Light manipulation with encoded plasmonic nanostructures," *European Physical Journal of Applied Metamaterials*, vol. 1, 6, 2014.
- [19] Y.-G. Chen, Y.-H. Chen, and Z.-Y. Li, "Direct method to control surface plasmon polaritons on metal surfaces," *Optics Letters*, vol. 39, pp. 339-342, 2014.
- [20] S.-Y. Lee, K. Kim, S.-J. Kim, H. Park, K.-Y. Kim, and B. Lee, "Plasmonic meta-slit: shaping and controlling near-field focus," *Optica*, vol. 2, no. 1, pp. 6-13, 2015.
- [21] V. Coello, C. E. Garcia-Ortiz, and M. Garcia-Mendez, "Classic plasmonics: wave propagation control at subwavelength scale," *NANO*, vol. 10, 1530005, 2015.
- [22] Q. Guo, C. Zhang, and X. Hu, "A spiral plasmonic lens with directional excitation of surface plasmons," *Scientific Reports*, vol. 6, 32345, 2016.
- [23] H. Li, Y. Qu, H. Ullah, B. Zhang, and Z. Zhang, "Controllable multiple plasmonic bending beams via polarization of incident waves," *Optics Express*, vol. 25, no. 24, pp. 29659-29666, 2017.
- [24] W.-B. Shi, T.-Y. Chen, H. Jing, R.-W. Peng, and M. Wang, "Dielectric lens guides in-plane propagation of surface plasmon polaritons," *Optics Express*, vol. 25, no. 5, pp. 5772-5780, 2017.

- [25] J. Wang, C. Chen, and Z. Sun, "Creation of multiple on-axis foci and ultra-long focal depth for SPPs," *Optics Express*, vol. 25, no. 2, pp. 1555-1563, 2017.
- [26] Z. Wang, G. Ren, Y. Gao, B. Zhu, and S. Jian, "Plasmonic in-plane total internal reflection: azimuthal polarized beam focusing and application," *Optics Express*, vol. 25, no. 20, pp. 23989-23999, 2017.
- [27] J. F. Nye, and M. V. Berry. "Dislocations in wave trains," *Proceedings of the Royal Society of London A*, vol. 336, pp. 165-190, 1974.
- [28] M. R. Dennis, K. O'Holleran, and M. J. Padgett, "Singular optics: Optical vortices and polarization singularities," *Progress in Optics*, vol. 53, pp. 293-363, 2009.
- [29] M. Soskin, S. V. Boriskina, Y. Chong, M. R. Dennis, and A. Desyatnikov, "Singular optics and topological photonics," *Journal of Optics*, vol. 19, no. 1, 010401, 2017.
- [30] H. Kim, J. Park, S.-W. Cho, S.-Y. Lee, M. Kang, and B. Lee, "Synthesis and dynamic switching of surface plasmon vortices with plasmonic vortex lens," *Nano Letters*, vol. 10, pp. 529-536, 2010.
- [31] S. V. Boriskina and B. M. Reinhard, "Adaptive on-chip control of nano-optical fields with optoplasmonic vortex nanogates," *Optics Express*, vol. 19, no. 22, pp. 22305-22315, 2011.
- [32] H. Zhou, J. Dong, Y. Zhou, J. Zhang, M. Liu, and X. Zhang, "Designing appointed and multiple focuses with plasmonic vortex lenses," *IEEE Photonics Journal*, vol. 7, 4801007, 2015.
- [33] A. M. Kamchatnov, and N. Pavloff, "Interference effects in the two-dimensional scattering of microcavity polaritons by an obstacle: phase dislocations and resonances," *European Physical Journal D*, vol. 69: 32, 2015.
- [34] G. Yuan, Q. Wang, and X. Yuan, "Dynamic generation of plasmonic Moiré fringes using phase-engineered optical vortex beam," *Optics Letters*, vol. 37, no. 13, pp. 2715-2717, 2012.
- [35] I. V. Dzedolik, and V. Pereskokov, "Formation of vortices by interference of surface plasmon polaritons," *Journal of the Optical Society of America A*, vol. 33, no. 5, pp. 1004-1009, 2016.
- [36] I. V. Dzedolik, S. Lapayeva, and V. Pereskokov, "Vortex lattice of surface plasmon polaritons," *Journal of Optics*, vol. 18, no. 7, 074007, 2016.
- [37] I. V. Dzedolik, and V. S. Pereskokov, "Topology of plasmon-polariton vortices on an adaptive mirror," *Atmospheric and Oceanic Optics*, vol. 30, no. 2, pp. 203-208, 2017.
- [38] V. N. Konopsky, "Operation of scanning plasmon near-field microscope with gold and silver tips in tapping mode: demonstration of subtip resolution," *Optics Communications*, vol. 185, pp. 83-93, 2000.

- [39] D. V. Kazantsev, and H. Ryssel, "Scanning head for the apertureless near field optical microscope," *Modern Instrumentation*, vol. 2, pp. 33-40, 2013.
- [40] D. V. Kazantsev, E. V. Kuznetsov, S. V. Timofeev, A. V. Shelaev, and E. A. Kazantseva, "Apertureless near-field optical microscopy," *Uspekhi Fizicheskikh Nauk*, vol. 187, no. 3, pp. 277-295, 2017.
- [41] D. I. Yakubovsky, A. V. Arsenin, Yu. V. Stebunov, D. Yu. Fedyanin, and V. S. Volkov, "Optical constants and structural properties of thin gold films," *Optics Express*, vol. 25, no. 21, pp. 25574-25587, 2017.
- [42] F. Ruting, F. I. Fernandez-Dominguez, L. Martin-Moreno, and F. J. Garcia-Vidal, "Sub-wavelength chiral surface plasmons that carry tunable orbital angular momentum", *Physical Review B*, vol. 86, 075437, 2012.
- [43] K. Toyoda, F. Takahashi, S. Takizawa, Y. Tokizane, K. Miyamoto, R. Morita, and T. Omatsu, "Transfer of light helicity to nanostructures," *Physical Review Letters*, vol. 110, 143603, 2013.