



#### Conference Paper

# Charge-Exchange Resonances of Tin Isotopes in Sn(<sup>3</sup>He,t)Sb reaction

#### Yuri Lutostansky

National Research Center "Kurchatov Institute", Moscow, 123182 Russia

#### Abstract

Charge-exchange resonances: the giant Gamow–Teller, analog and the so-called "pygmy" resonances have been studied in the self-consistent theory of finite Fermi systems. Microscopic and semi-classical calculations are presented for nine tin isotopes with known experimental data <sup>112,114,116,117,118,119,120,122,124</sup>Sn. These data is from Sn(<sup>3</sup>He,*t*)Sb charge-exchange reaction at the energy (<sup>3</sup>He) = 200 MeV. The average standard deviation for GTR and AR energies is  $\delta E \leq 0.30$  MeV that is close to the experimental  $E_{GTR}$  errors. The comparison of calculations with experimental data on the energies of charge-exchange pygmy resonances gives the standard deviation  $\delta E < 0.40$  MeV for microscopic numerical calculations and  $\delta E < 0.55$  MeV for calculations by semi-classical formulas, which are comparable with experimental errors. Strength function for the <sup>118</sup>Sn isotope has been calculated and the calculated resonance energies and heights of resonance peaks are close to the experimental values.

# 1. Introduction

Charge-exchange states are associated with the charge excitation branch and correspond to excited states of isobar nuclei. They are manifested in the corresponding charge-exchange reactions such as (v,e), (p,n), (n,p), ( $^{3}$ He,t), (t, $^{3}$ He), ( $^{6}$ Li,  $^{6}$ He) and others, or in  $\beta$ -transitions in nuclei. Among these states, collective resonance excitations are of the most interest. The theoretical investigation of these collective states began with the first calculations of the giant Gamow-Teller resonance (GTR) [1] and other collective states [2] long before their experimental studies in charge-exchange reactions [3, 4]. These collective states lying below the giant GTR [5] were called "pygmy" resonances (PR).

The most complete experimental studies of the entire spectrum of the chargeexchange excitations for nine tin isotopes  $^{112,114,116,117,118,119,120,122,124}$ Sn were reported in [6], where the Sn(<sup>3</sup>He,*t*)Sb charge-exchange reaction at the energy  $E(^{3}He) = 200$  MeV

Corresponding Author: Yuri Lutostansky Lutostansky@yandex.ru

Received: 25 December 2017 Accepted: 2 February 2018 Published: 9 April 2018

Publishing services provided by Knowledge E

© Yuri Lutostansky. This article is distributed under the terms of the Creative Commons Attribution License, which

permits unrestricted use and redistribution provided that the original author and source are credited.

Selection and Peer-review under the responsibility of the ICPPA Conference Committee.





was used. The excitation energies ( $E_x$ ), widths ( $\Gamma$ ), and cross sections d $\sigma$ /d $\Omega$  (mb/sr) were measured for the analog, Gamow–Teller, and three pygmy resonances.

# 2. Method of calculation

Charge-exchange excitations of nuclei are described in the microscopic theory of finite Fermi systems TFFS by the system of equations for the effective field [7]:

$$V_{pn} = e_q V_{pn}^{\omega} + \sum_{p'n'} F_{np,n'p'}^{\omega} \rho_{p'n'} V_{pn}^h = \sum_{p'n'} F_{np,n'p'}^{\omega} \rho_{p'n'}^h$$
(1)

where  $V_{pn}$  and  $V_{pn}^{h}$  are the effective fields of quasi-particles and holes in a nucleus;  $V_{pn}^{\omega}$  – is the external charge-exchange field. The system of secular equations (1) is solved for allowed transitions with the local nucleon-nucleon interaction in the Landau–Migdal form [7]:

$$F^{\omega} = C_0 \left( f'_0 + g'_0 \left( \vec{\sigma}_1 \vec{\sigma}_2 \right) \right) \left( \vec{\tau}_1 \vec{\tau}_2 \right) \delta \left( \vec{r}_1 - \vec{r}_2 \right),$$
(2)

with the parameters  $f'_0$  = 1.35 and  $g'_0$  = 1.22 as in [8].

The energies of charge-exchange excitations were calculated both in the selfconsistent TFFS and in its approximate model variant [5], which allowed obtaining analytical solutions for the most collective states. For energies  $E_{GTR}$  and  $E_{PR}$ , the solution  $\omega_k$  (k = 0 for GTR and k = 1, 2, 3 for PR1, PR2, and PR3) divided by the average energy  $E_{ls}$  of spin–orbit splitting [8] at  $\Delta E > E_{ls}$ , has the form:

$$y_{k} = \omega_{k}/E_{ls} = (a_{k} + b_{k}) \cdot g_{k}' \cdot x + \frac{b_{k}(1 + b_{k}g_{k}')g_{k}'x}{(a_{k} + b_{k})(g_{k}'x)^{2} + [1 + 2(a_{k} + b_{k})g_{k}']/3A^{1/3}}$$
(3)

where  $x = \Delta E/E_{ls}$ ,  $\Delta E = (4/3)\varepsilon_F$  (N-Z)/A MeV,  $\varepsilon_F \approx 40$  MeV,  $a_k \approx a \cdot p_k$ ;  $b_k \approx b \cdot p_k$ ;

$$p_k \approx (k+1)^{-1}, g_k = \frac{g_0'}{1 + \alpha_k \beta_k/2}, \alpha_k = \frac{p_k \beta_k}{1 + 2g_0' \beta_k}, \beta_k = \sum_{m=1}^k p_m.$$

Thus, all resonance states from GTR (k = 0) to PR<sub>3</sub> (k = 3) are described by one formula (3).

### 3. Results

The energies of five charge-exchange resonances – AR, GTR, PR1, PR2, and PR3 – calculated in the microscopic approach for tin isotopes  $^{112,114,116,117,118,119,120,122,124}$ Sn are summarized in the Table together with experimental data [6]. As shown in the Table, the standard deviations of the calculated and experimental results for the energies are



Nucleus in./final	E <sub>AR</sub>		E <sub>GTR</sub>		E <sub>PR1</sub>		E <sub>PR2</sub>		E <sub>PR3</sub>	
	exp. ± 0.03	calc.	exp. ± 0.25	calc.	exp. ± 0.25	calc.	exp. ± 0.20	calc.	exp. ± 0.20	calc.
<sup>112</sup> Sn / <sup>112</sup> Sb <sup>114</sup> Sn / <sup>114</sup> Sb <sup>116</sup> Sn / <sup>116</sup> Sb <sup>117</sup> Sn / <sup>117</sup> Sb <sup>118</sup> Sn / <sup>118</sup> Sb <sup>119</sup> Sn / <sup>119</sup> Sb <sup>120</sup> Sn / <sup>120</sup> Sb <sup>122</sup> Sn / <sup>122</sup> Sb <sup>124</sup> Sn / <sup>124</sup> Sb	6.16 7.28 8.36 11.27 9.33 12.36 10.24 11.24 12.19	6.69 6.92 8.47 11.38 9.23 12.48 10.20 11.17 12.05	8.94 9.39 10.04 12.87 10.61 13.71 11.45 12.25 13.25	9.38 9.60 10.36 12.91 10.93 13.77 11.78 12.54 13.59	4.08 4.55 5.04 7.64 5.38 8.09 5.82 6.65 7.13	4.70 4.97 5.23 7.54 5.54 8.27 6.24 6.76 7.16	2.49 2.95 3.18 5.45 3.17 5.49 3.18 3.37 3.44	3.00 2.65 2.68 5.21 3.08 5.57 3.47 3.91 3.06	1.33 1.88 1.84 3.87 1.47 3.63 1.38 1.45 1.50	1.52 1.60 1.75 3.71 1.55 4.07 0.98 1.55 2.17
δΕ	0.23		0.29		0.31		0.36		0.33	

TABLE 1: Energies (in MeV) of the analog  $E_{AR}$ , Gamow–Teller  $E_{GTR}$ , and three pygmy resonances  $E_{PR}$  according to the TFFS microscopic calculations and the experimental data from [6], as well as the standard deviations  $\delta E = \langle E_{exp} - E_{calc} \rangle$  of the calculations from the experimental data.

small:  $\delta E < 0.40$  MeV. These values are comparable with the experimental errors  $\Delta E_{exp} = \pm 0.25$  MeV and are better than in other known calculations of high-lying excitations, e.g., in the self-consistent quasiparticle random phase approximation with Skyrme forces [9].

As shown in Figure 1 (a) the experimental data for excitation spectra in  ${}^{118}$ Sn( ${}^{3}$ He,t) ${}^{118}$ Sb reaction [6] and (b) calculated charge-exchange strength function – S( $E_x$ ) for the  ${}^{118}$ Sn isotope, where  $E_x$  – is the excitation energy. Unfortunately, direct measurements of the strength function S( $E_x$ ) have not been performed, but the data on counts shown in Fig. 2a are proportional to the partial data on the function S( $E_x$ ).

As shown in Figure 2 the calculated energy differences  $E_{GTR} - E_{PR}$  between the Gamow–Teller resonance (k = 0 in Eq. (3)) and pygmy resonances (k = 1, 2, 3) as functions of the mass number *A*. As is seen in Fig. 2, the microscopic calculations for the PR1 pygmy resonance as compared to the experiment give the best accuracy: the corresponding standard deviation is  $\delta E = 0.31$  MeV (see Table 1) as compared to  $\delta E = 0.53$  MeV for calculations by Eq. (3). For the PR2 pigmy resonance, the calculation by Eq. (3) with  $\delta E = 0.26$  MeV is more accurate than the microscopic calculations and experiment is observed for <sup>116</sup>Sn, whereas the calculations by Eq. (3) are within the measurement error equal to  $\pm 0.20$  MeV. For the PR3 pygmy resonance, the microscopic calculation with  $\delta E = 0.33$  MeV is more accurate than the calculation by Eq. (3) with  $\delta E = 0.36$  MeV. Nevertheless, the calculations by two methods on average satisfactorily describe the experimental data.





**Figure** 1: (a) Experimental [6] and (b) calculated data for excitation spectra in  ${}^{118}$ Sn( ${}^{3}$ He,t) ${}^{118}$ Sb reaction.Gamow-Teller and three pygmy resonances marked as GT1, GT2, GT3 and GT4 in experiment (a) and GTR, PR1, PR2, and PR3 in calculations (b) are identified.

# 4. Conclusion

The first microscopic numerical and semi-classical calculations have been performed for ten tin isotopes with the mass numbers A = 112, 114, 116, 117, 118, 119, 120, 122, 124, and 126, for which experimental data exist [6]. Charge-exchange resonances: giant Gamow–Teller (GTR), analog (AR) resonances, and the so-called "pygmy" resonances (PR), which are lying below GTR, have been studied in the self-consistent theory of finite Fermi systems (TFFS). Microscopic numerical calculations and semiclassical calculations are presented for nine tin isotopes <sup>112,114,116,117,118,119,120,122,124</sup>Sn. The experimental data is from Sn(<sup>3</sup>He,*t*)Sb charge-exchange reaction at the energy  $E(^{3}He) = 200$  MeV [6]. The Gamow-Teller and analog resonances with the energies –  $E_G$  and  $E_A$ , dominate in the strength function of the charge-exchange excitations of atomic nuclei. The calculated energy difference  $\Delta E_{G-A} = E_G - E_A$ tends to zero with A in heavy nuclei indicating the restoration of Wigner SU(4)-symmetry [8]. The calculated





**Figure** 2: Difference between the energies of GTR and pygmy resonances PR, lying below it for Sn isotopes from the mass number A according to ( $\bullet$ ) – experimental data [6], (( $\times$ ) connected by dashed line) the TFFS numerical calculations and lines the calculations by Eg. (3). Digits 1, 2, and 3 mark groups of PR1, PR2, and PR3.

 $\Delta E_{G-A}$  values are in good agreement with the experimental data. The average standard r.m.s deviation for GTR and AR energies is  $\delta E \leq 0.30$  MeV for the nine considered Sn nuclei that is close to the experimental  $E_{GTR}$  errors (see Table 1). The comparison of calculations with experimental data on the energies of charge-exchange pygmy resonances gives the standard deviation  $\delta E < 0.40$  MeV for microscopic numerical calcula-tions and  $\delta E < 0.55$  MeV for the calculations by semi-classical formulas (3), which are comparable with experimental errors. These calculations are original.

The charge-exchange strength function of the <sup>118</sup>Sn isotope has been calculated also. It has been shown that the calculated resonance energies are close to the experimental values. The calculated and experimental relations between heights of pygmy resonance peaks are also close to each other.

#### Acknowledgements

I am grateful to I.N. Borzov, S.S. Gershtein, E.E. Saperstein, V.N. Tikhonov, and S.V. Tolokonnikov for stimulating discussions and assistance in the work. The work is supported by the Russian RFBR grants 16-02-00228, 18-02-00670 and RSF project 16-12-10161.



# References

- [1] Gaponov Yu V, and Lyutostanskii Yu S 1972 JETP Lett. 15 120.
- [2] Gaponov Yu V, and Lyutostanskii Yu S 1974 Sov. J. Nucl. Phys. 19 33.
- [3] Doering R, Galonsky A, Patterson D M, Bertsch G F 1975 Phys. Rev. Lett. 35 1691.
- [4] Galonsky A, Doering R, Patterson D M, and Bertsch G F 1976 Phys. Rev. 14 748.
- [5] Lutostansky Yu S 2017 JETP Lett. 106 7.
- [6] Pham K, Janecke J, Roberts D A et al. 1995 Phys. Rev. C 51 526.
- [7] Migdal A B Theory of Finite Fermi Systems and Applications to Atomic Nuclei (1983 Nauka, Moscow; 1967 Inter-Sci., New York).
- [8] Lutostansky Yu S, and Tikhonov V N 2015 JETP Lett. 102 7, 2016 Phys. At. Nucl. 79
   929; arXiv: 1509.02014v1 [nucl-th].
- [9] Borzov I N, and Goriely S 2000 Phys. Rev. C 62 035501.